Vortex Dynamics in Quantum Turbulence of Superfluid ^{4}He at the Turbulent Transition

Osaka City University Hideo Yano

Quantum Turbulence generated by a thin vibrating wire

1. Vortex dynamics at the laminar-to-turbulent transition
   - Seed vortices of turbulence
   - Kelvin wave instability ⇦ bridge vortices
2. Critical behaviors at the turbulent-to-laminar transition

Collaborators

Experiment: Y. Nago, K. Andachi, Y. Miura, T. Ogawa, S. Mio, M. Chiba, K. Obara, O. Ishikawa, T. Hata

Theory: S. Fujiyama, M. Tsubota

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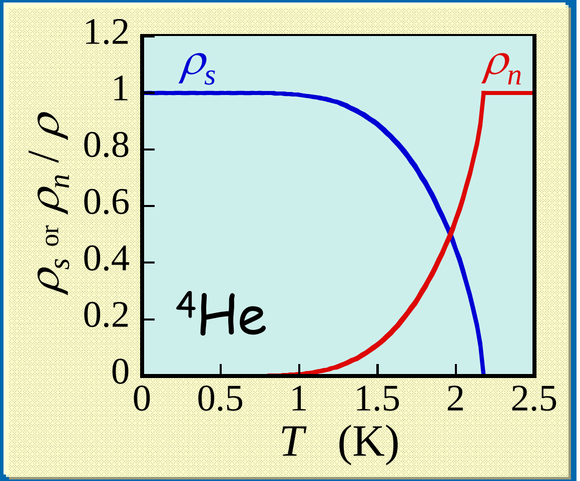
Superfluid and Quantized Vortex

- Simple superfluids (^{4}He; ^{3}He-B; cold atoms) exhibit
  - Two fluid behaviour: a viscous normal component + an "inviscid" superfluid component.
  - Normal component disappears at the 0 K limit.

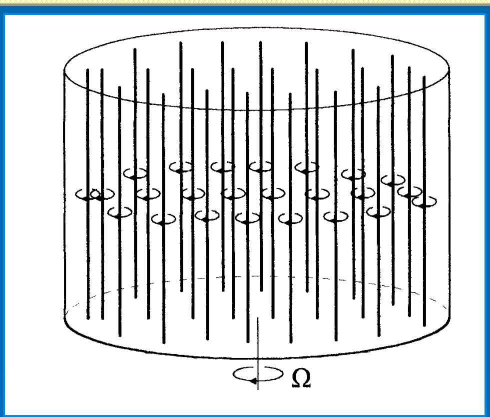


- Quantization of rotational motion: $\nabla \times v_s = 0$,
  - except on quantized vortex lines, each with one quantum of circulation

$$\kappa = \oint \mathbf{v}_s \cdot d\mathbf{r} = h/m_4 \quad : \text{circulation quantum}$$

round a core of radius (ξ ~0.05 nm for ^{4}He).

  - Helium under rotation ⇨ Array of vortex lines

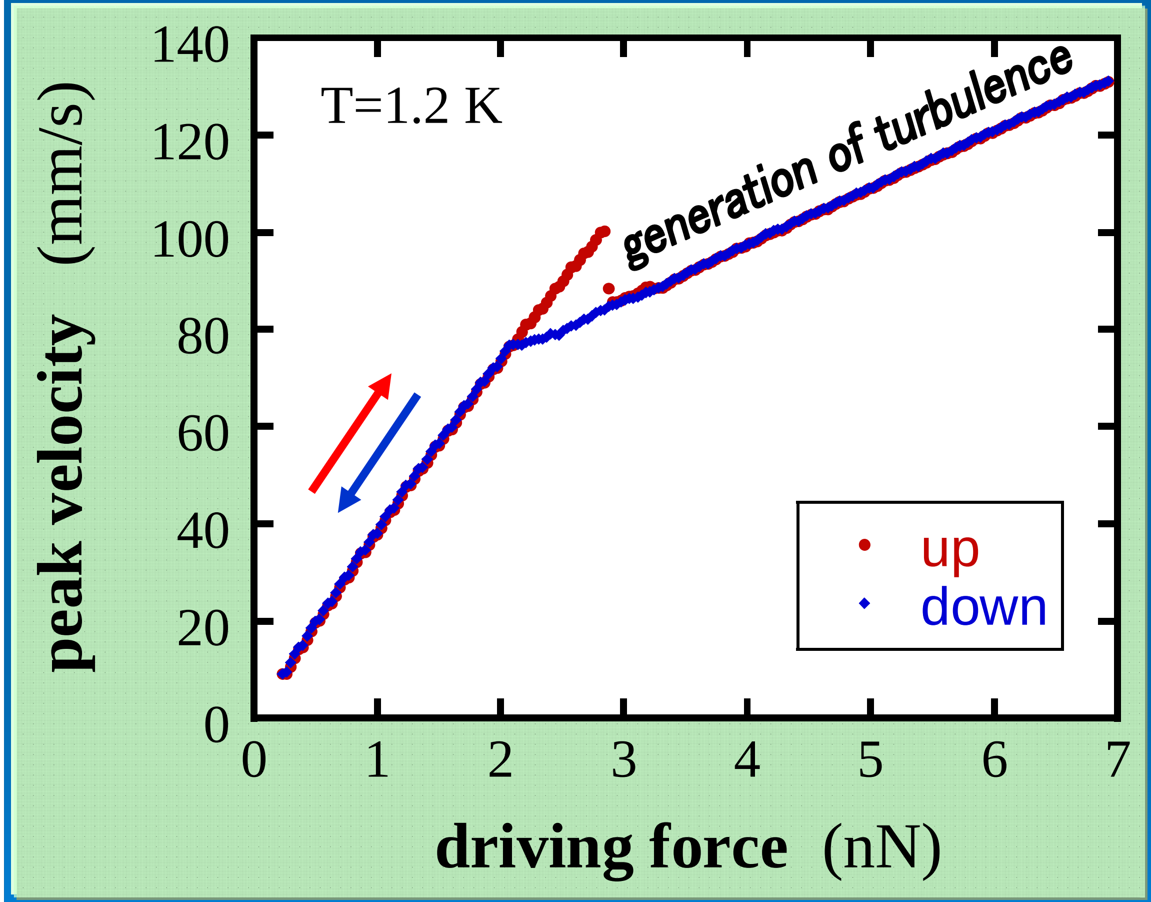


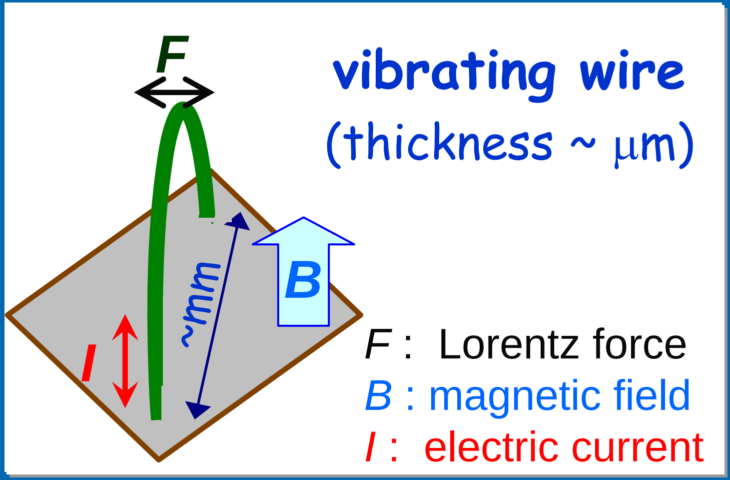
Helium under rotation

- Nucleation of vortices, during cooling through the superfluid transition
  - Remnant vortex lines are still present, attached between boundaries.

Generation of turbulence by a vibrating wire

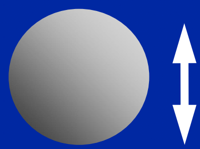
Response of a vibrating wire in superfluid ^4He



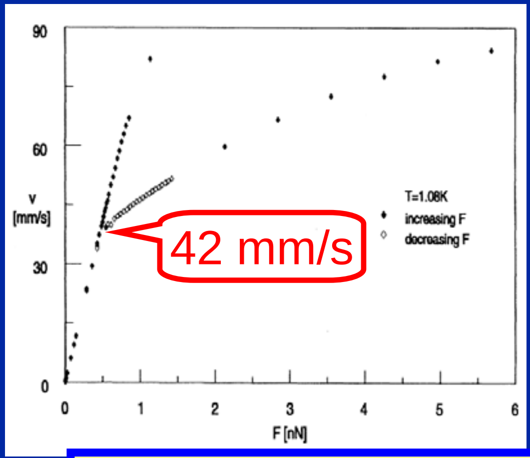


Oscillating obstacles in superfluid ^{4}He.

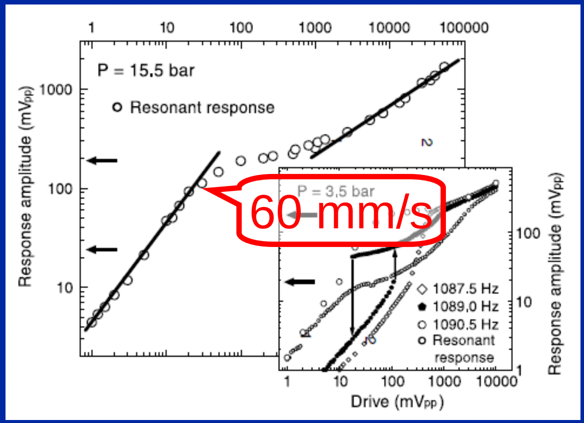
Microsphere



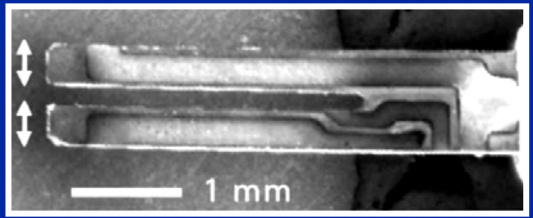
200 μm

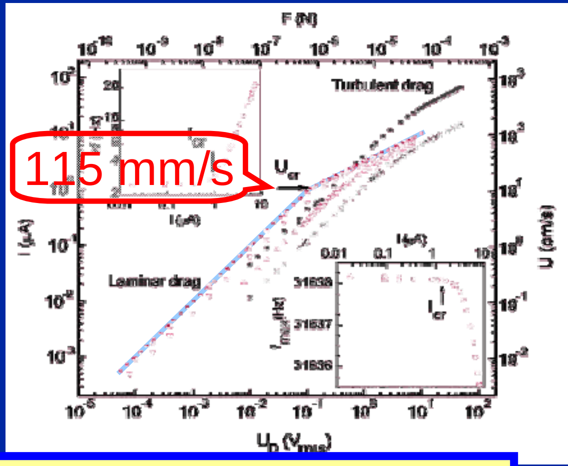


Grid



Fork





J. ... Sc... Ph... (1 ... zer, ...)

The velocity of generating turbulence (~ 50 mm/s) is much lower than an intrinsic velocity of vortex nucleation (~30 m/s).

Remanent vortices should cause the generation of turbulence !!

Study on the vortex dynamics at the laminar-to-turbulent transition

Vortex free wire in superfluid ^4He

to reduce remnant vortex lines

1. thin vibrating wire with smooth surface
2. liquefying superfluid below 100 mK

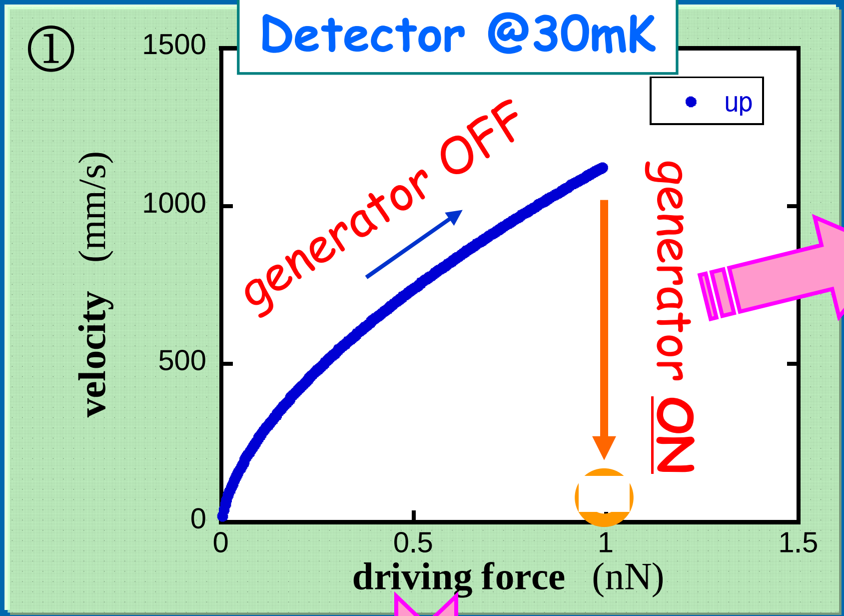
A vortex-free wire does not generate turbulence, even at a velocity above 1 m/s.

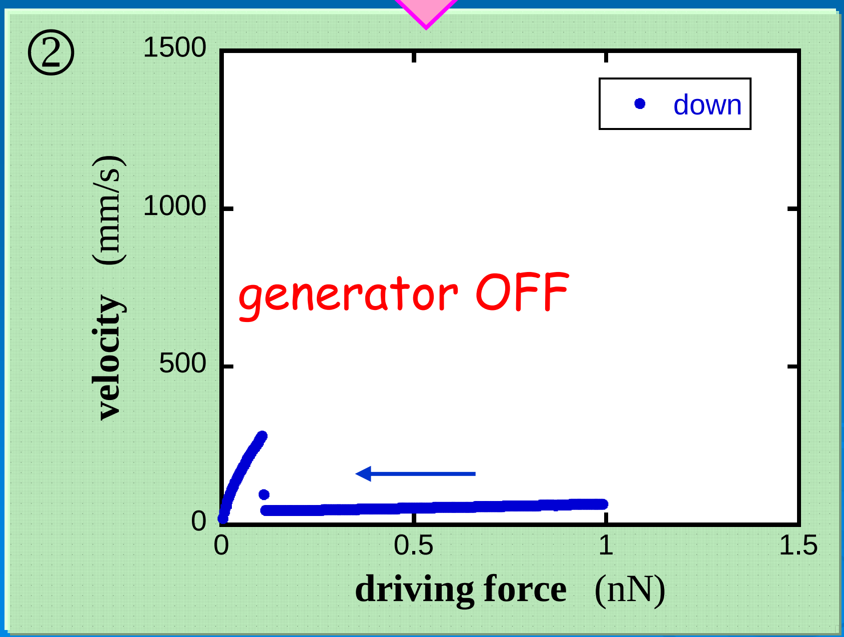
vibrating a vortex-free wire

↓ + seed vortices

Turbulence will be generated ?

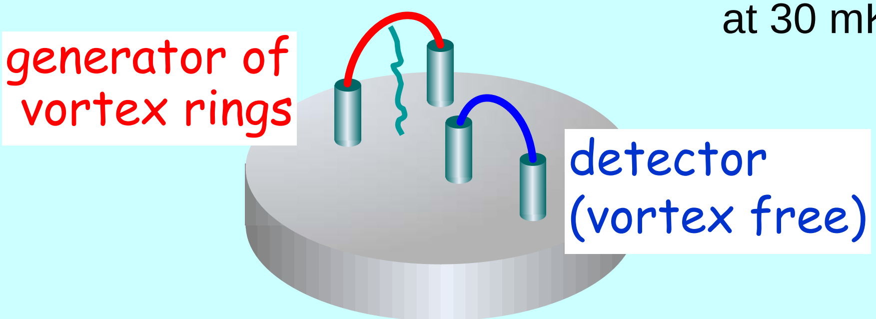
Transition to turbulence triggered by vortex rings





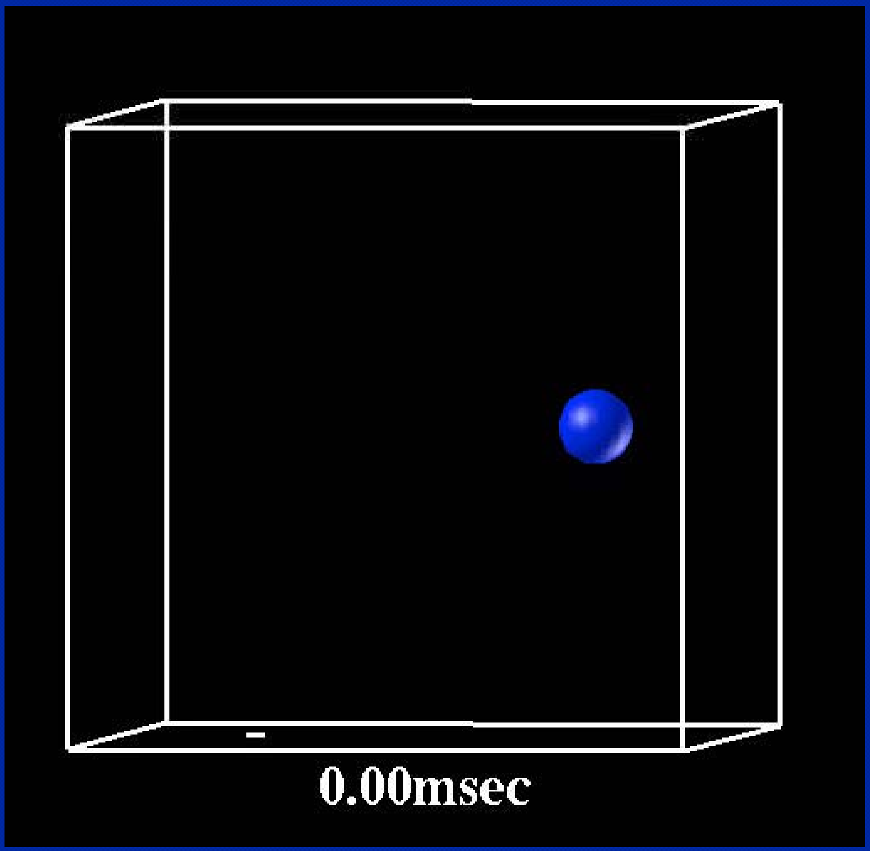
Vortex rings trigger the turbulent transition.

Vibrating wires (NbTi ϕ 3 μm) in superfluid ^{4}He at 30 mK



Simulation of turbulence triggered by vortex rings

Numerical simulation by
Fujiyama and Tsubota



oscillating obstacle: <u>**sphere 6 μm**</u>
velocity: <u>**137 mm/s**</u>
frequency: <u>**1.59 kHz**</u>

A turbulence forms in the path of the sphere.

See a joint paper: **R. Goto, S. Fujiyama, M. Tsubota, HY, *et al*, *Phys. Rev. Lett. 100, 045301 (2008)***

Study on the transition due to vortices attached to a vibrating wire

To attach vortex lines to a wire

1. Warming above T_λ
2. Cooling to 30 mK

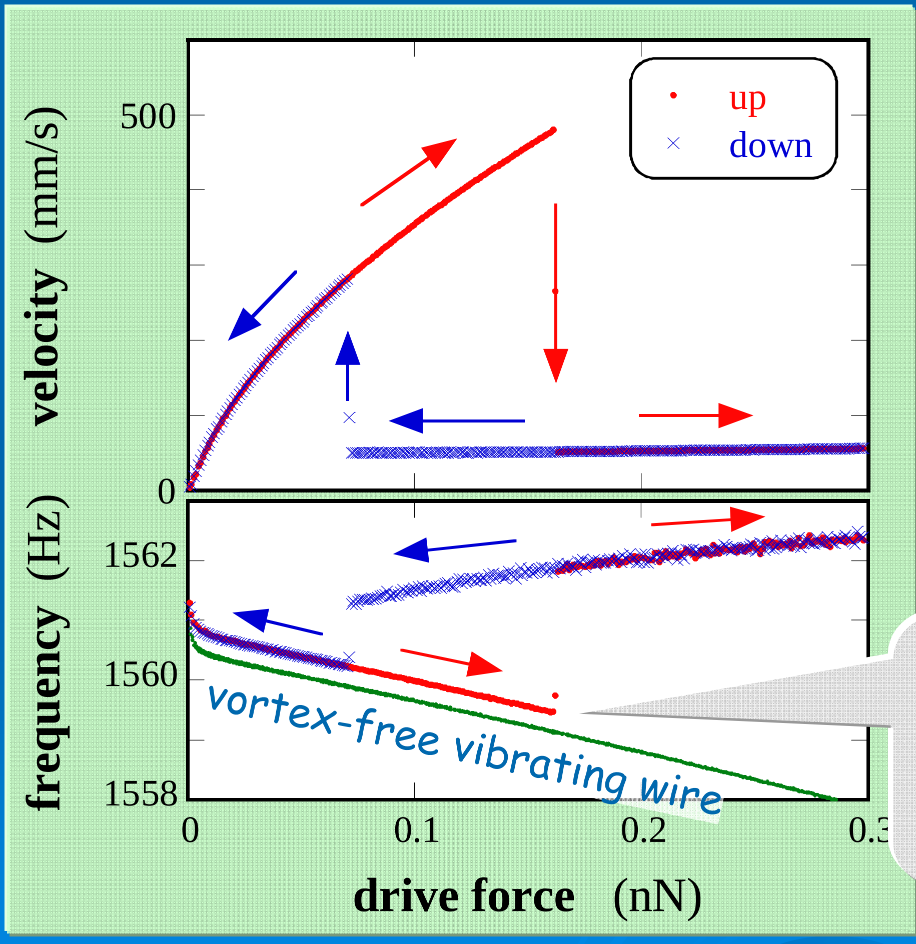
↓

How vortex lines attached to a wire cause turbulence?

- Responses of the vibrating wire

Transition to turbulence due to attached vortices

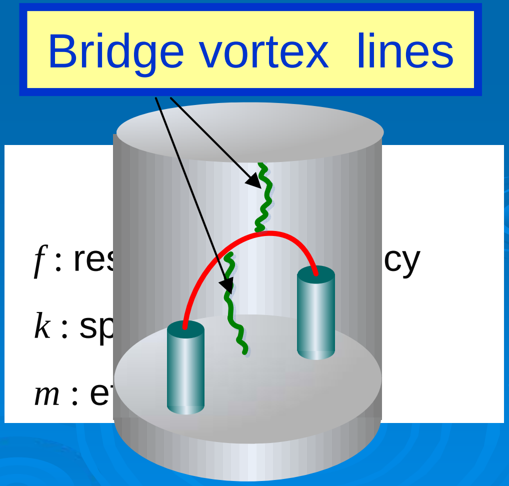
Response of a vibrating wire with attached vortices



Resonance frequency increases by 0.3 Hz

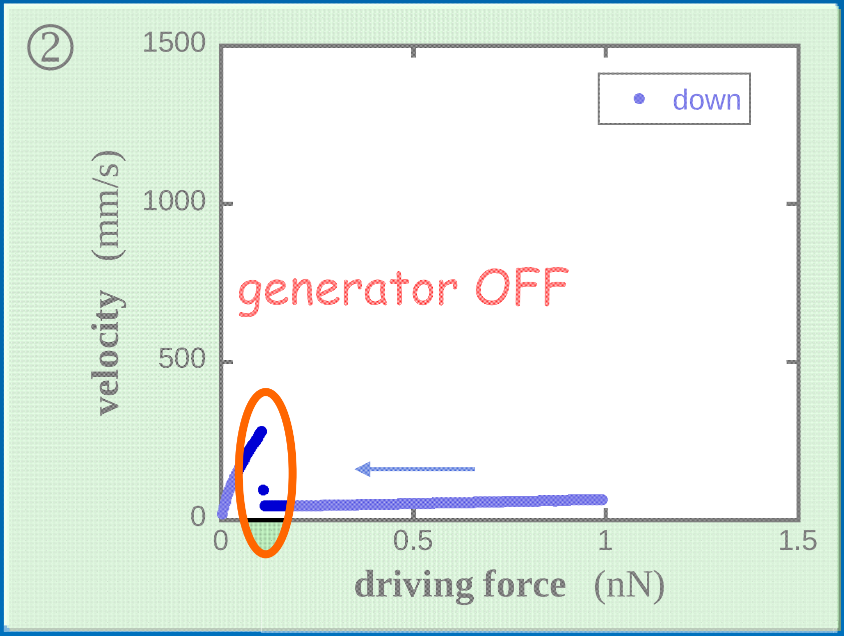
Oscillation of bridge vortex lines generates turbulence.

Kelvin wave instability causes turbulence.

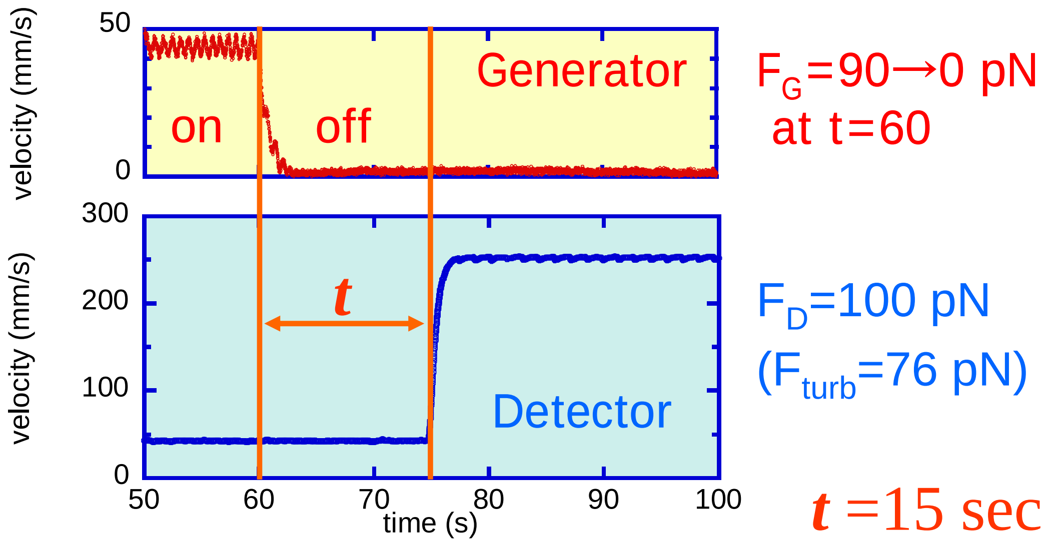


N. Hashimoto, R. Goto, HY, *et al*, Phys. Rev. B 76, 020504 (2007).

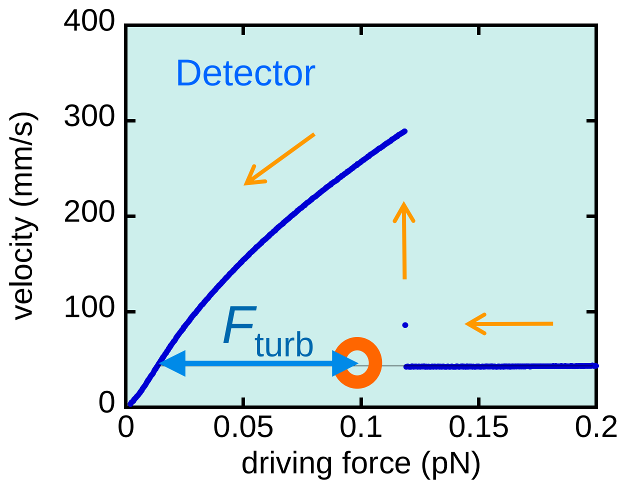
Study on the vortex dynamics at the turbulent-to-laminar transition



Turbulent-to-Laminar transition



t =15 sec



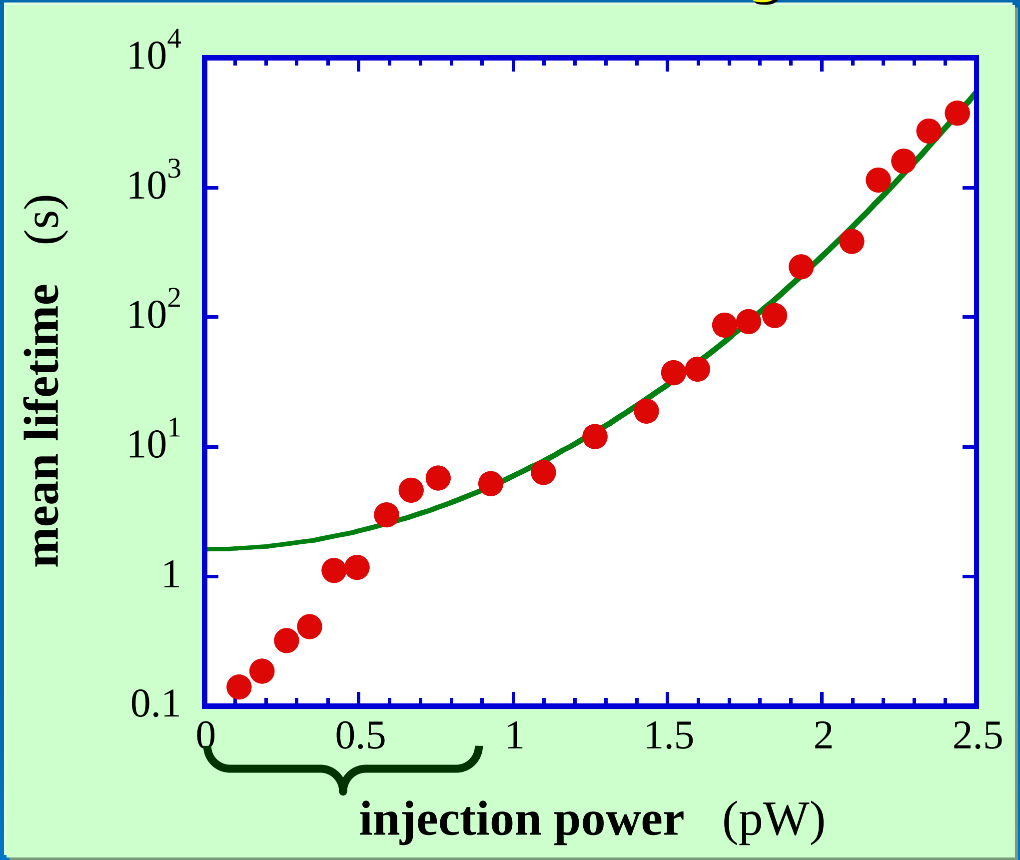
Lifetime of turbulence generation

- exponential distribution $\left(\propto \exp(-t/\tau)\right)$

⇨ mean lifetime τ of turbulence

Mean lifetime of turbulence

Mean lifetime vs. driving force



Critical behaviors of lifetime

- Above 0.9 pW, the mean lifetime τ

$$\tau = \tau_0 \exp\left(\frac{P^2}{P_0^2}\right) \qquad \begin{cases} \tau_0 = 1.5\,\mathrm{s} \\ P_0 = 0.88\,\mathrm{pW} \end{cases}$$

- Below 0.9 pW, the lifetime τ decreases greatly from the equation.

The fitting parameter P_0 reflects the critical injection energy below which the critical behaviors arise.

The lifetime is attributable to the statistical fluctuations of the vorticity [Schoepe, PRL2004].

$$\tau = \tau_0 \exp\left(\frac{\langle L^2 \rangle}{L_0^2}\right)^2 \quad (L: \text{vortex line density})$$

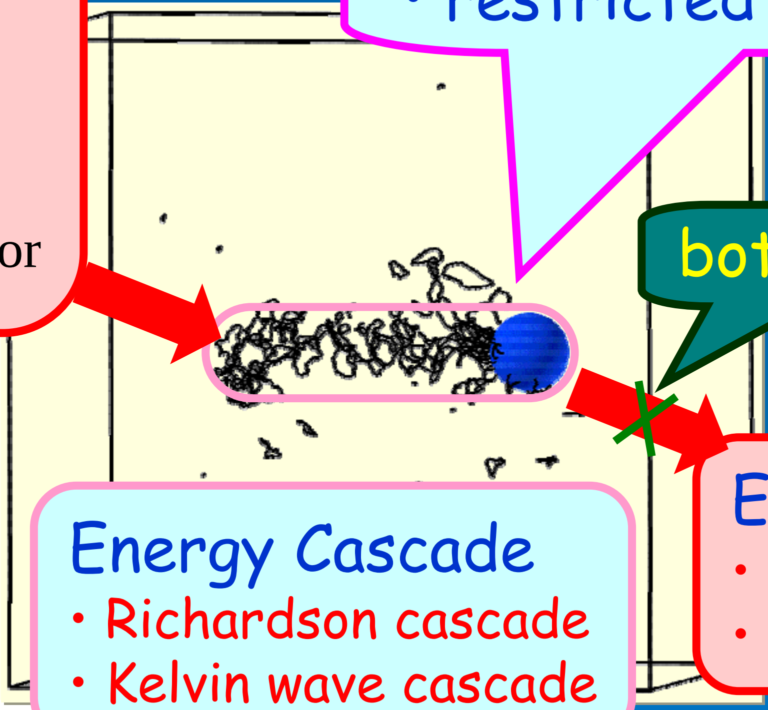
Energy flux in quantum turbulence

Injected Power

$$P = g\, F_{turb}\, v$$

$$\begin{cases} F_{turb} : \text{drag force} \\ v : \text{wire velocity} \\ g : \text{geometrical factor} \end{cases}$$

- steady quantum turbulence
- restricted volume



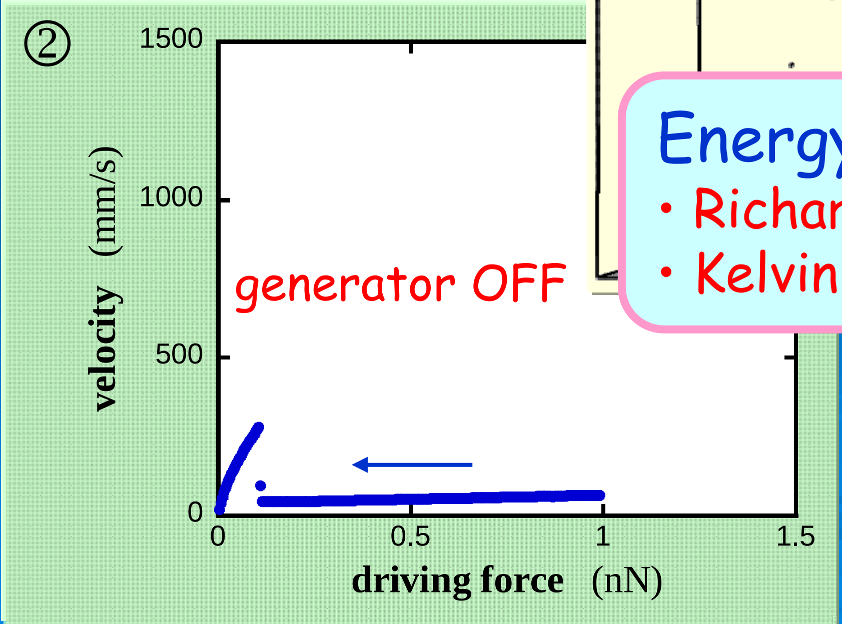
bottleneck

Energy Cascade

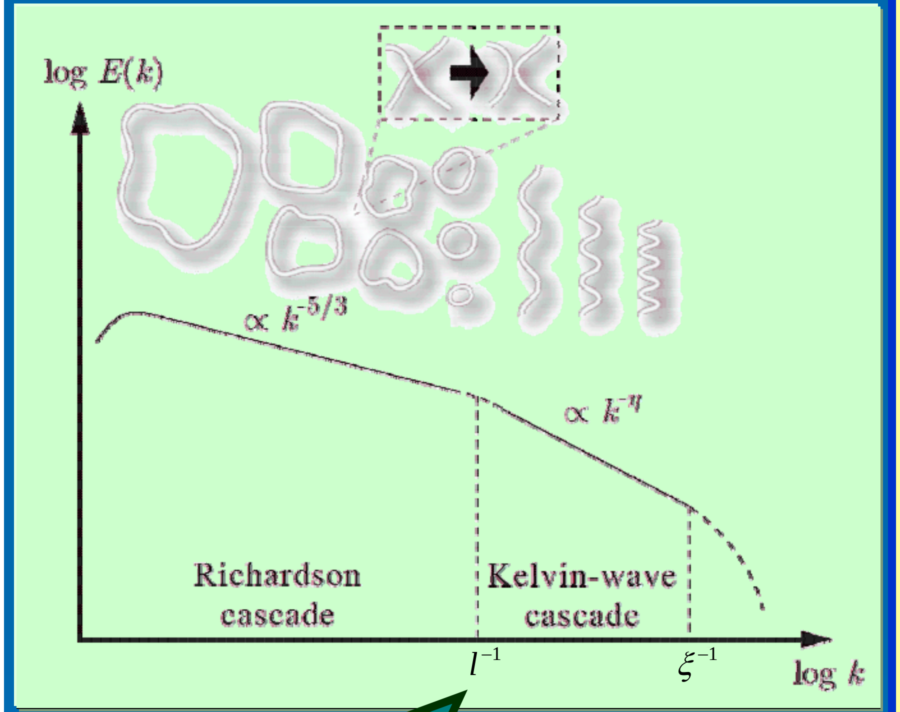
- Richardson cascade
- Kelvin wave cascade

Energy Dissipation

- vortex rings
- high energy phonons



Bottleneck of energy flux



Prediction

bottleneck of energy cascade

Vortex line density L

due to the bottleneck

$$L = \sqrt{aP / M\kappa^3}$$

$$\left(\begin{array}{ll} P = g\, F_{turb}\, \upsilon : \text{dissipated power} & l = (L)^{-1/2} : \text{vortex line spacing} \\ M : \text{mass of turbulent fluid} & k : \text{wave number} \\ \kappa : \text{circulation quantum} & \Lambda = \ln(l / a_0) \end{array} \right)$$

- $a \approx 1$: unpolarized vortex tangle (bottleneck at $kl \sim 1$)
- $a \approx \Lambda^5$: polarized vortex tangle (bottleneck at $kl \sim \Lambda^{-5/4}$, $\Lambda \approx 12$)

[V.S. L'vov, *et al*., Phys. Rev. B **76**, 024520 (2007)]

Vortex line spacing at the critical energy

Vortex line density L due to the bottleneck

$$L = \sqrt{aP / M\kappa^3} \quad \begin{cases} P = g\,F_{turb}\,\upsilon : \text{injection power} & l = (L)^{-1/2} : \text{vortex line spacing} \\ M : \text{mass of turbulent fluid} & k : \text{wave number} \\ \kappa : \text{circulation quantum} & \Lambda = \ln(l / a_0) \end{cases}$$

Vortex line spacing at the critical energy (P_0 = 0.88 pW)
(assuming unpolarized vortex tangle ($a \approx 1$) at low driving forces)

$l_0 = (L_0)^{-1/2} \approx 7\ \mu m$ ≈ oscillating amplitude 9 μm (=amp$_{p\text{-}p}$)

Turbulence ceases when vortex lines are absent in the wire path.

Summary & Future works

Quantum turbulence generated by thin vibrating wires

1. **Vortex dynamics at the turbulent transition**
    - seed vortices triggering the turbulent transition
    - turbulent transition due to Kelvin wave instability
2. **Critical behaviors of turbulence**
    - critical behaviors of mean lifetime
    - fluctuation of vortex lines
    - energy flux and its bottleneck
3. **Future works**
    - Detection of Kelvin waves (P77)
    - Vortex generation at high temperatures (P72)
    - Critical behaviors using high frequency oscillators (P73)