Odd frequency pairing in spin-triplet superconductor junctions

Yukio Tanaka Nagoya University

International Symposium on Physics of New Quantum phases in Superclean Materials

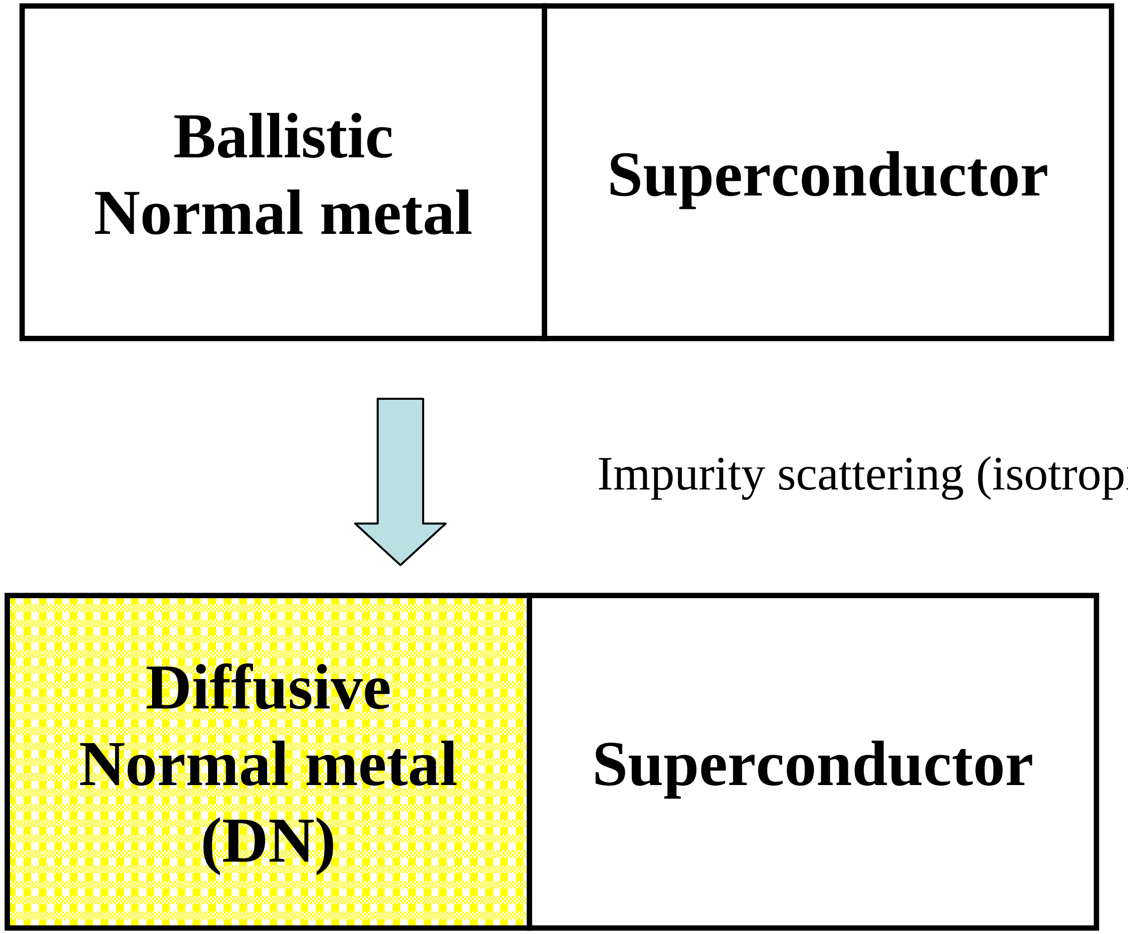
March 12 2010

Main Collaborators

A.A. Golubov	Twente University
Y. Asano	Hokkaido University
S. Kashiwaya	AIST (Tsukuba)
M. Ueda	University of Tokyo
T. Yokoyama	Tokyo Institute of Technology
Y.V. Nazarov	Delft University

Impurity scattering effect

Tanaka and Golubov, PRL. 98, 037003 (2007)



Impurity scattering (isotropic)

Only s-wave pair amplitude exists in DN

(1)ESE
(2)OTE

ESE (Even-frequency spin-singlet even-parity)
OTE (Odd-frequency spin-triplet even-parity)

Proximity effect in aerogel, Higashitani, Nagato, and Nagai, (2009)

Summary of proximity effect (No spin flip)

	Bulk state	Sign change	Interface-induced state (subdominant)	Proximity into DN
(1)	ESE(s,dx2-y2 -wave)	No	**ESE** + (OSO)	**ESE**
(2)	ESE (d_{xy}-wave)	Yes	**OSO** +(ESE)	No
(3)	ETO (p_x-wave)	Yes	**OTE** + (ETO)	**OTE**
(4)	ETO (p_y-wave)	No	**ETO** + (OTE)	No

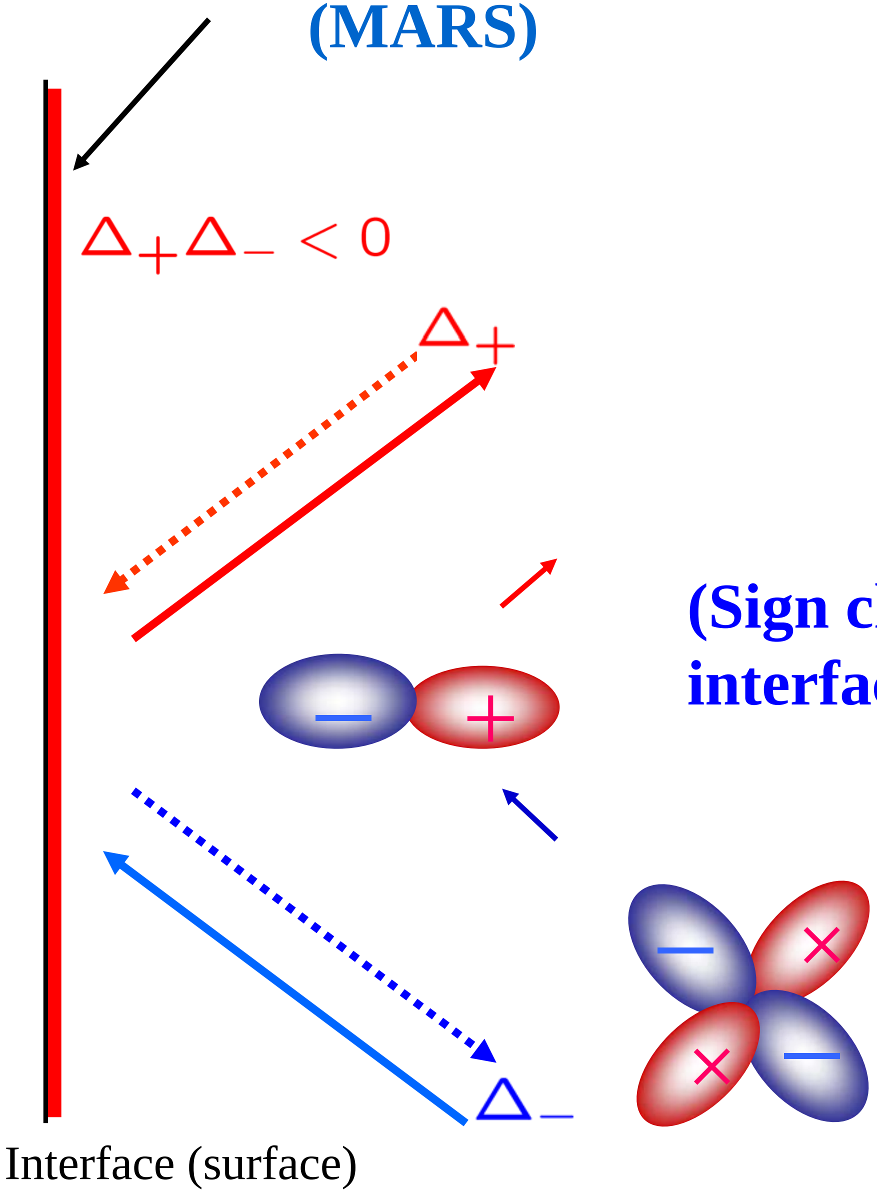
- **ESE (Even-frequency spin-singlet even-parity)**
- **ETO (Even-frequency spin-triplet odd-parity)**
- **OTE (Odd-frequency spin-triplet even-parity)**
- **OSO (Odd-frequency spin-singlet odd-parity)**

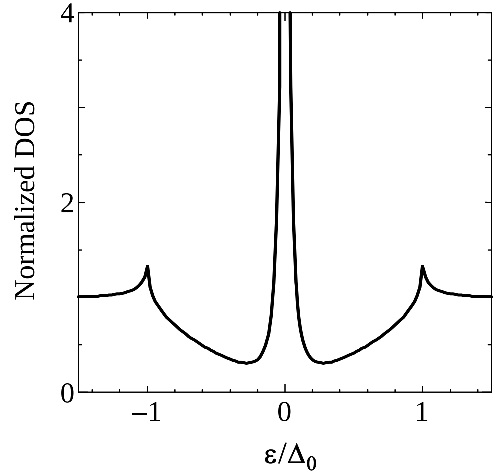
Proximity into DN (Diffusive normal metal)

even-parity (s-wave)○ Odd-parity ×

Y. Tanaka and Golubov, PRL. 98, 037003 (2007)
Y. Tanaka, et al Phys. Rev. Lett. 037005 (2007)

Mid gap Andreev resonant (bound) state (MARS) ⟺ **Odd-frequency Cooper pair**





(Sign change of the pair potential at the interface)

Summary of proximity effect (No spin flip)

	Bulk state	Sign change	Interface-induced state (subdominant)	Proximity into DN
(1)	ESE(s,dx2-y2 -wave)	No	**ESE** + (OSO)	**ESE**
(2)	ESE (d_{xy}-wave)	Yes	**OSO** +(ESE)	No
(3)	ETO (p_x-wave)	Yes	**OTE** + (ETO)	**OTE**
(4)	ETO (p_y-wave)	No	**ETO** + (OTE)	No

- **ESE (Even-frequency spin-singlet even-parity)**
- **ETO (Even-frequency spin-triplet odd-parity)**
- **OTE (Odd-frequency spin-triplet even-parity)**
- **OSO (Odd-frequency spin-singlet odd-parity)**

Proximity into DN (Diffusive normal metal)

even-parity (s-wave)○ Odd-parity ×

Y. Tanaka and Golubov, PRL. 98, 037003 (2007)
Y. Tanaka, et al Phys. Rev. Lett. 037005 (2007)

Usadel equation

Available for diffusive limit $\tau_{imp} T << 1$

$$D\nabla(\hat{g}_0^R \nabla \hat{g}_0^R) + i[\hat{H}_0, \hat{g}_0^R] = 0$$

$\hat{g}_0^R \leftarrow \hat{g}^R$ **Diffusive limit**

Angular average

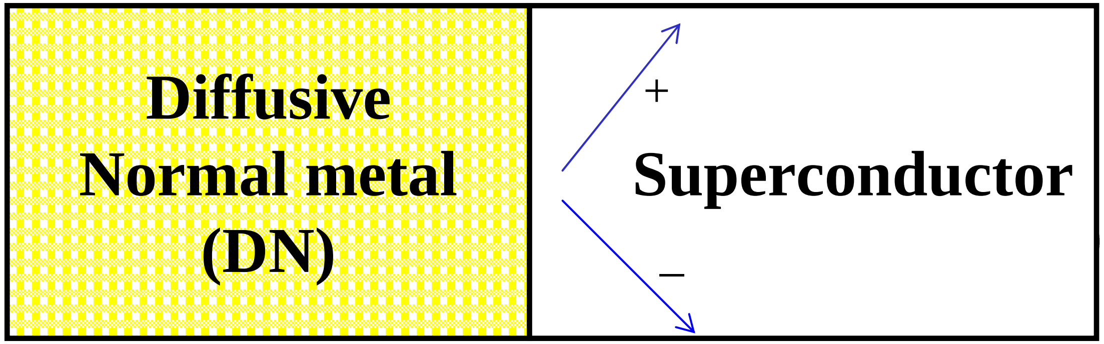
D **Diffusion constant**

Diffusive normal metal region attached to superconductor

$$\hat{H}_0 = \varepsilon \hat{\tau}_3$$

Boundary condition available for unconventional superconductors

Tanaka et al, PRL 90 167003 (2003), PRB 70 012507 (2004)



Tanaka Golubov PRL 98, 037003 (2007)

Green's function in superconductor (ballistic)

$$\hat{g}^R = g_\pm^R(\varepsilon)\hat{\tau}_3 + f_\pm^R(\varepsilon)\hat{\tau}_2 \qquad f_\pm^R(\varepsilon) = [f_\pm^R(-\varepsilon)]^*$$

Green's function in DN

Conventional proximity (even-frequency)

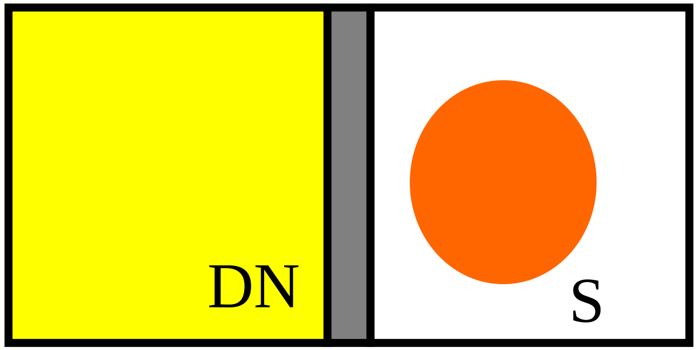
$$\hat{g}_0^R = g(\varepsilon)\hat{\tau}_3 + f(\varepsilon)\hat{\tau}_2$$

Unconventional proximity (odd-frequency)

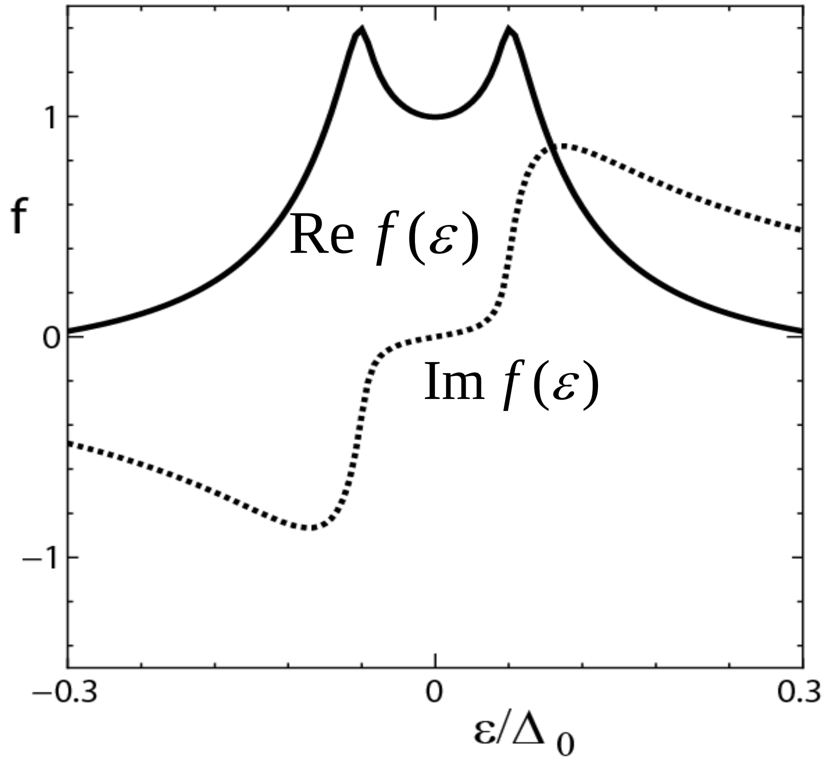
$$\hat{g}_0^R = g(\varepsilon)\hat{\tau}_3 + f(\varepsilon)\hat{\tau}_1$$

$g(\varepsilon)$ **Quasiparticle Green's function** $f(\varepsilon)$ **Pair amplitude**

Conventional proximity effect



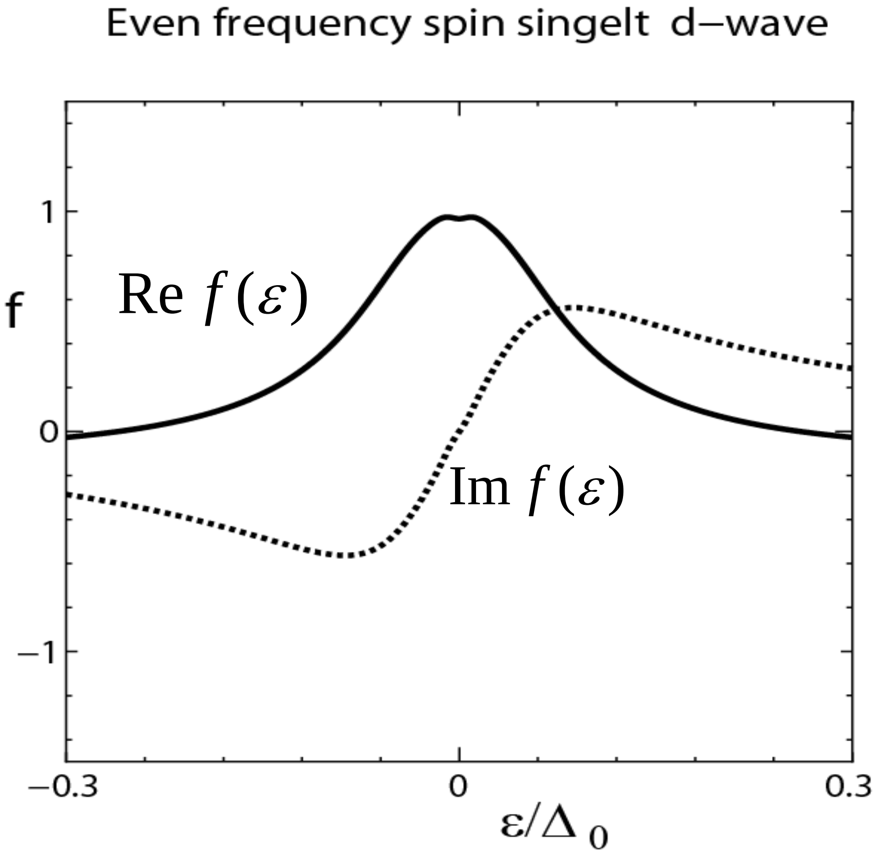
Even frequency spin singlet s–wave

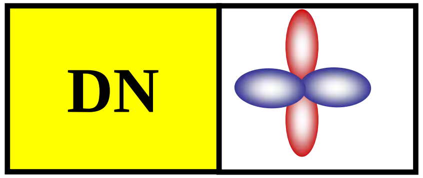


$$f(\varepsilon) = f^{*}(-\varepsilon)$$

Even frequency spin singlet
s-wave (ESE) pair is induced in DN.

Conventional proximity effect in spin-singlet d-wave junction (similar to s-wave)





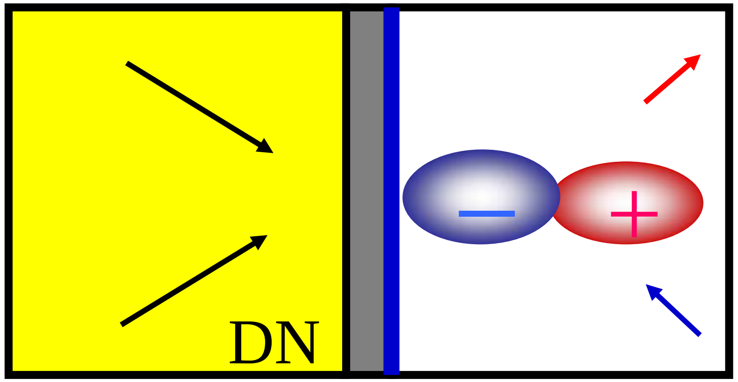
$$f(\varepsilon) = f^*(-\varepsilon)$$

Purely Even frequency s-wave component in DN

PRL 98, 037003 (2007)

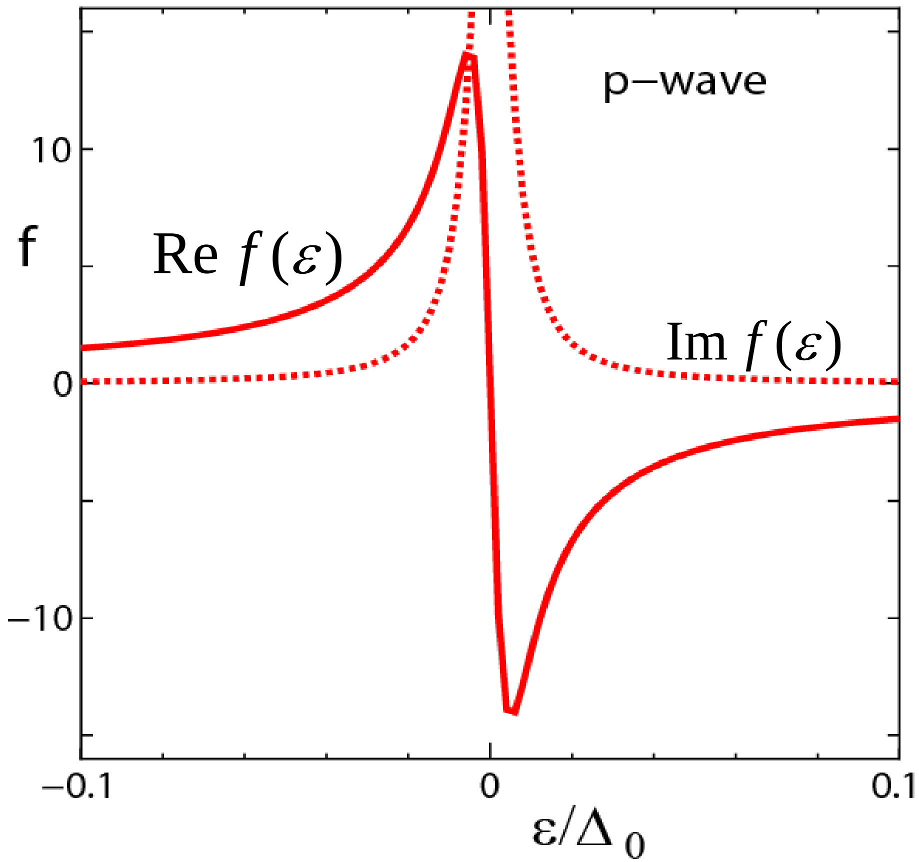
New type of proximity effect

P_x-wave case



$$f(\varepsilon) = -f^*(-\varepsilon)$$

Even frequency spin triple p–wave

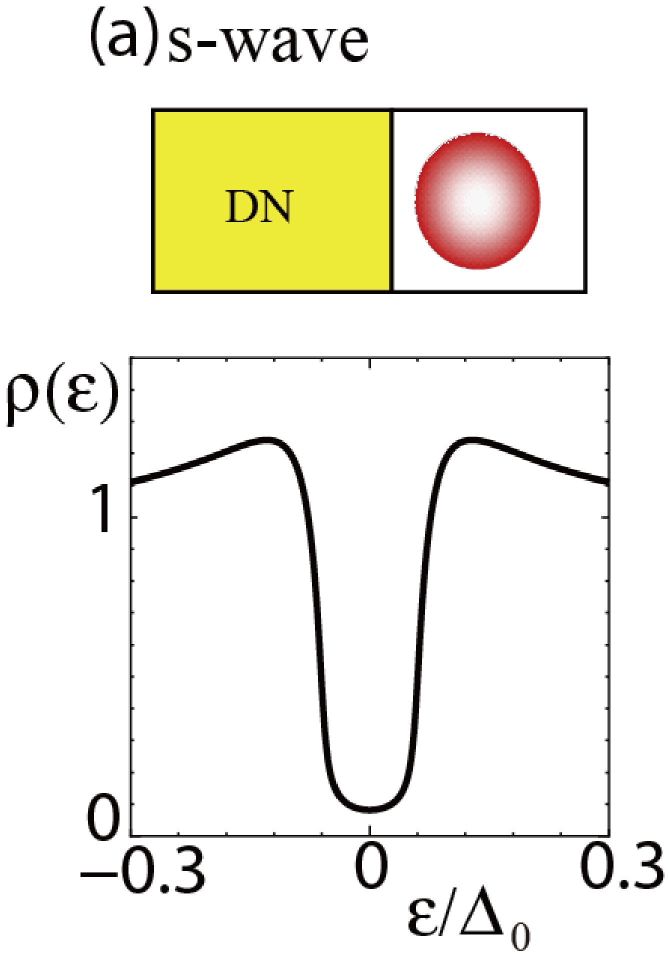


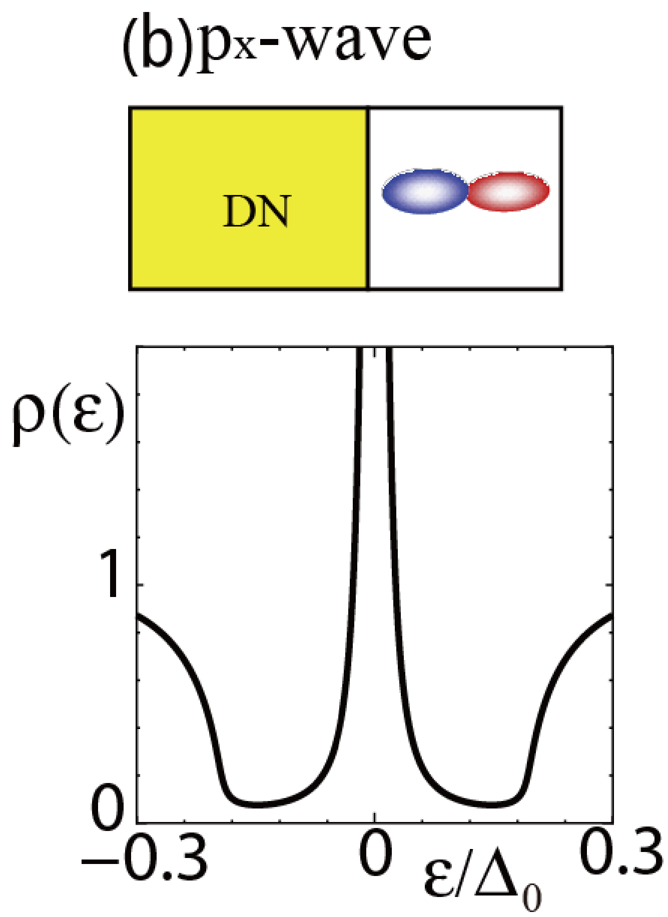
Odd frequency spin triplet s-wave (OTE) pair is induced in DN

Y.Tanaka, A.A.Golubov, Phys.Rev.Lett. **98**, 037003 (2007)

Density of states in DN

Tanaka, Kashiwaya PRB 70 012507 (2004)

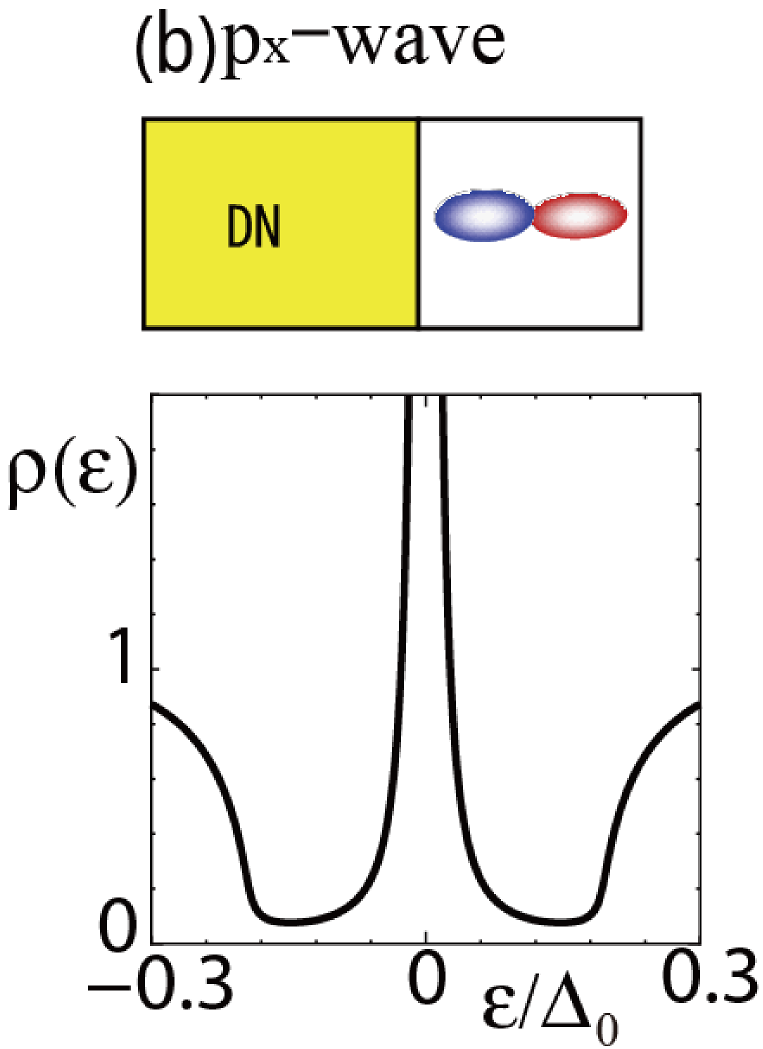


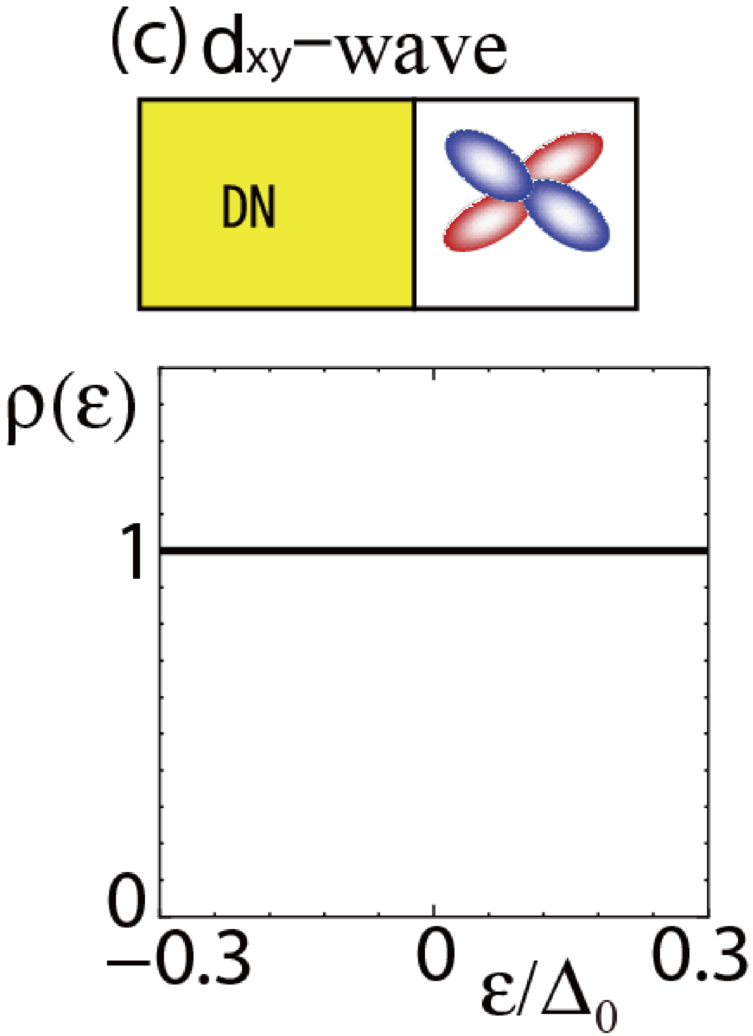


Conventional proximity effect with Even-frequency Cooper pair in DN

Unconventional proximity effect with Odd-frequency Cooper pair in DN

Peak(dip) width, Thouless energy $E_{Th} = D/L^2$





Unconventional proximity effect

No proximity effect

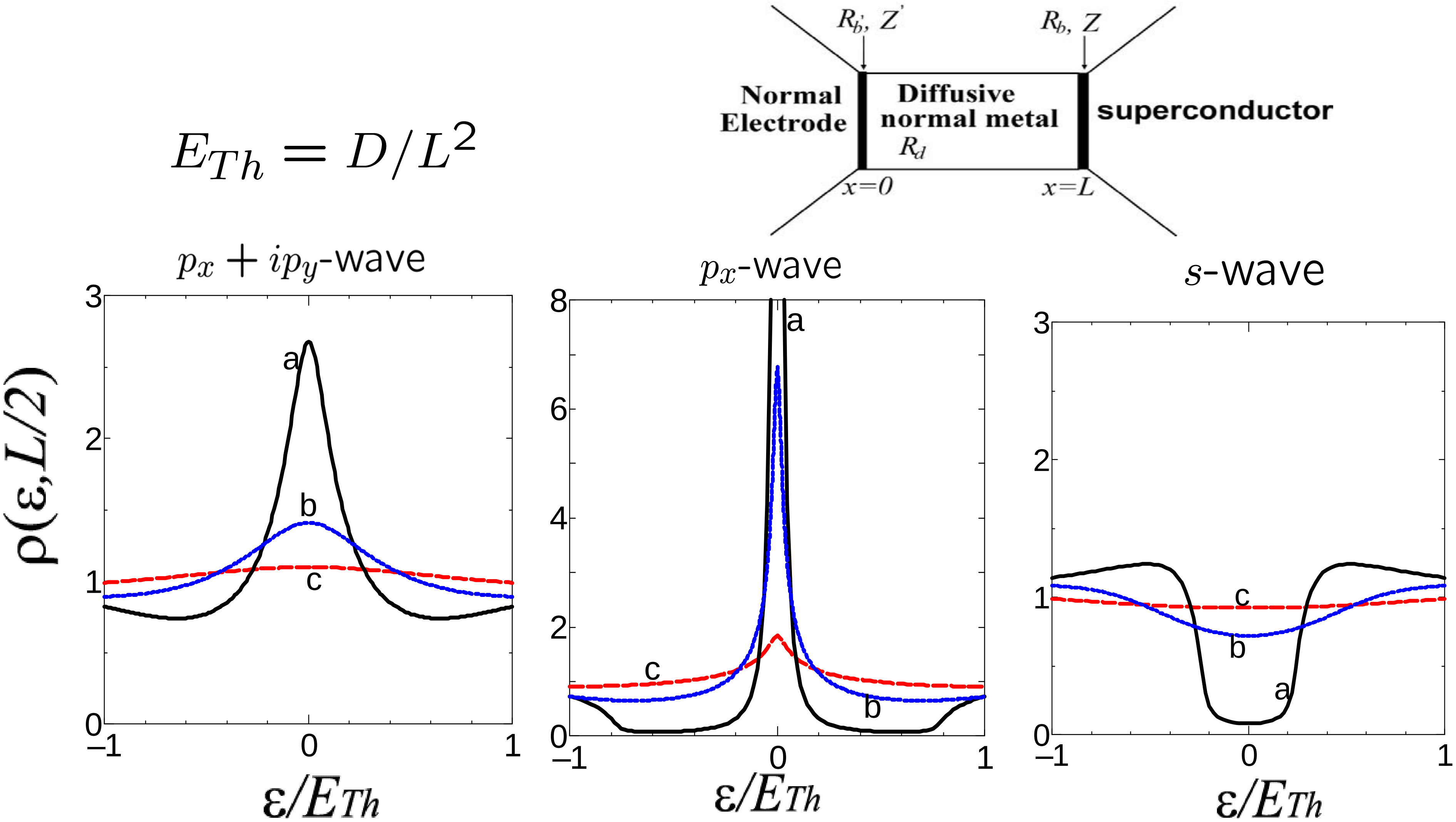
Odd-frequency pairing at the interface includes s-wave component

Odd-frequency pairing at the interface: Odd-parity (can not enter)

Peak width, Thouless energy $E_{Th} = D/L^2$

Local density of state in DN

$$E_{Th} = D/L^2$$

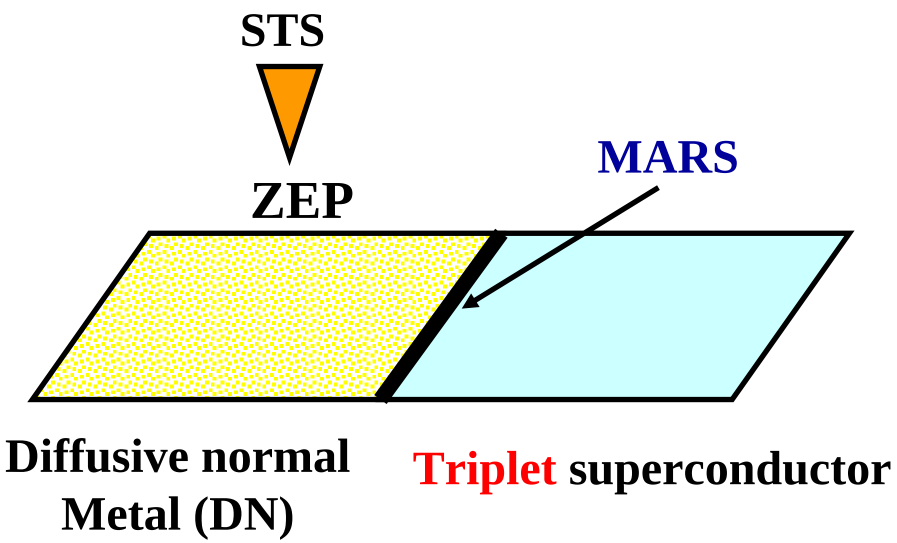


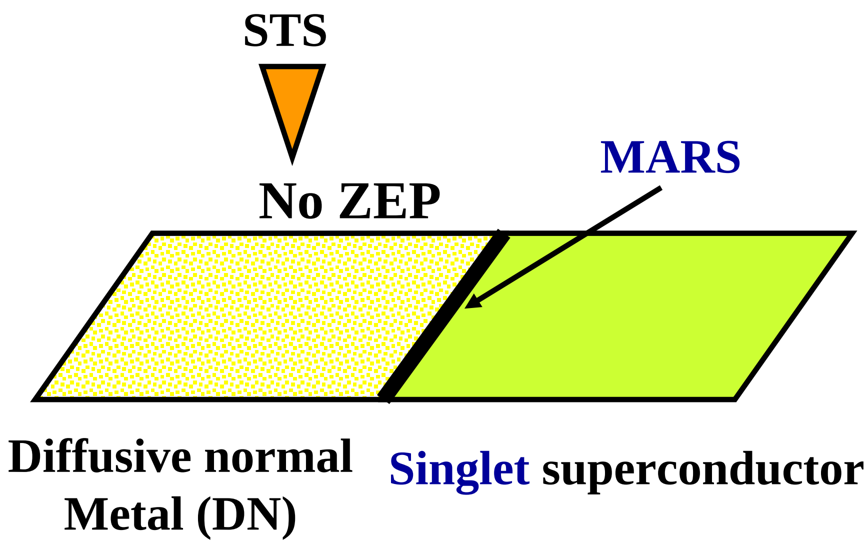
$R_d/R_{b'} = 0.01$ $\quad$ $R_d/R_{b'} = 1$ $\quad$ $R_d/R_{b'} = 100$

How to detect triplet superconductor

MARS (Mid gap Andreev resonance state) can penetrate into DN by proximity effect only for triplet superconductor junctions

Y. Tanaka & S. Kashiwaya, PRB 70, 012507 (2004)





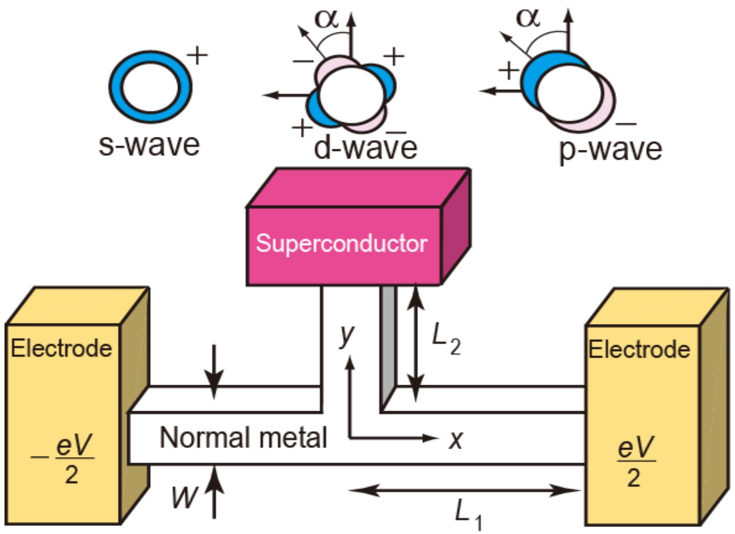
LDOS in DN has a zero energy peak

LDOS in DN does not have a zero energy peak

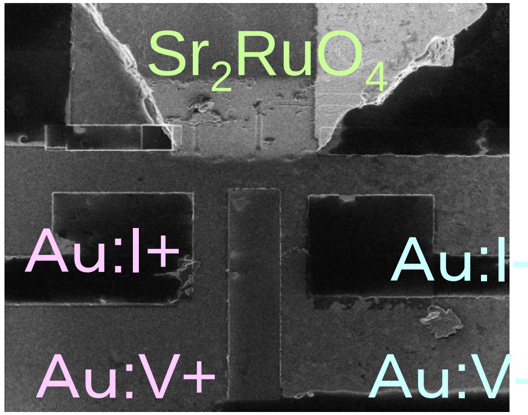
Proximity effect via odd-frequency pairing

Theoretical prediction to detect odd-frequency paring amplitude

Asano Tanaka Golubov Kashiwaya, PRL 99, 067005 (2007).



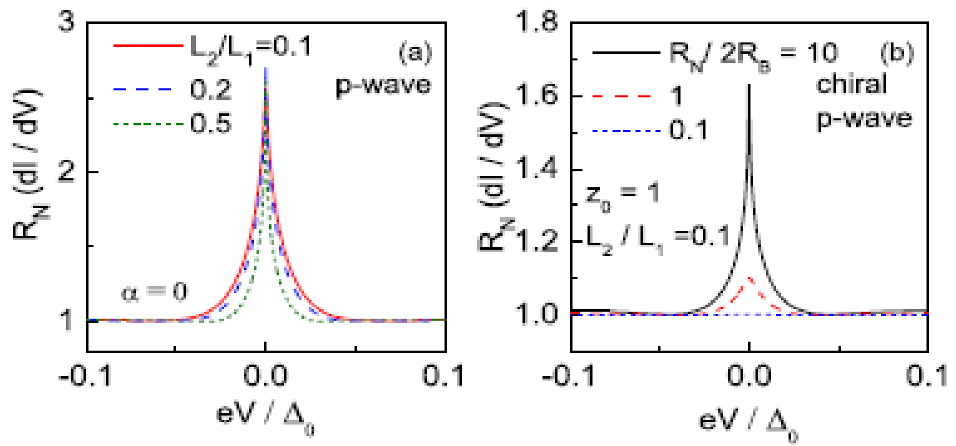
OTE proximity



Kashiwaya, Maeno 2007

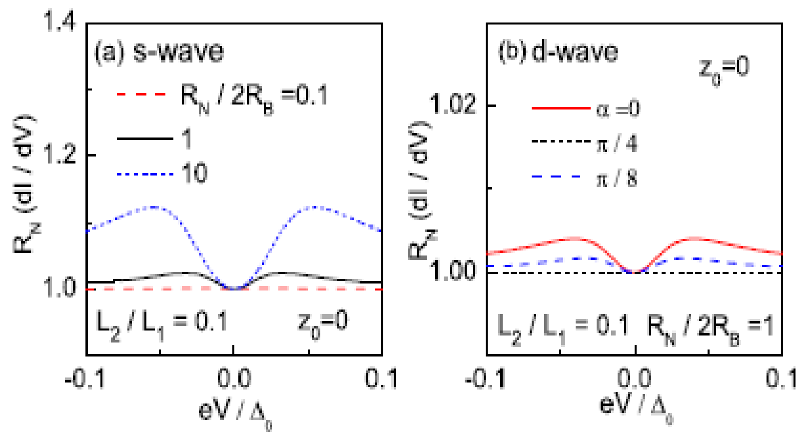
OTE (Odd-frequency spin-triplet even-parity)

ESE (Even-frequency spin-singlet even-parity)



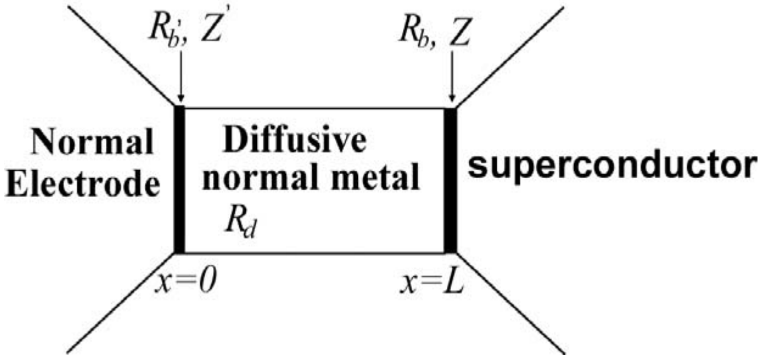
Zero energy peak

ESE proximity (conventional)



No Zero energy peak

Meissner effect



$$\hat{R}_N(x) = \sin\theta\hat{\tau}_2 + \cos\theta\hat{\tau}_3$$

$$j(x) = \pi e^2 N(0) DT \sum_{\omega_n} \mathrm{Trace}[\hat{\tau}_3 \hat{R}_N(x)[\hat{\tau}_3, \hat{R}_N(x)]] A(x)$$

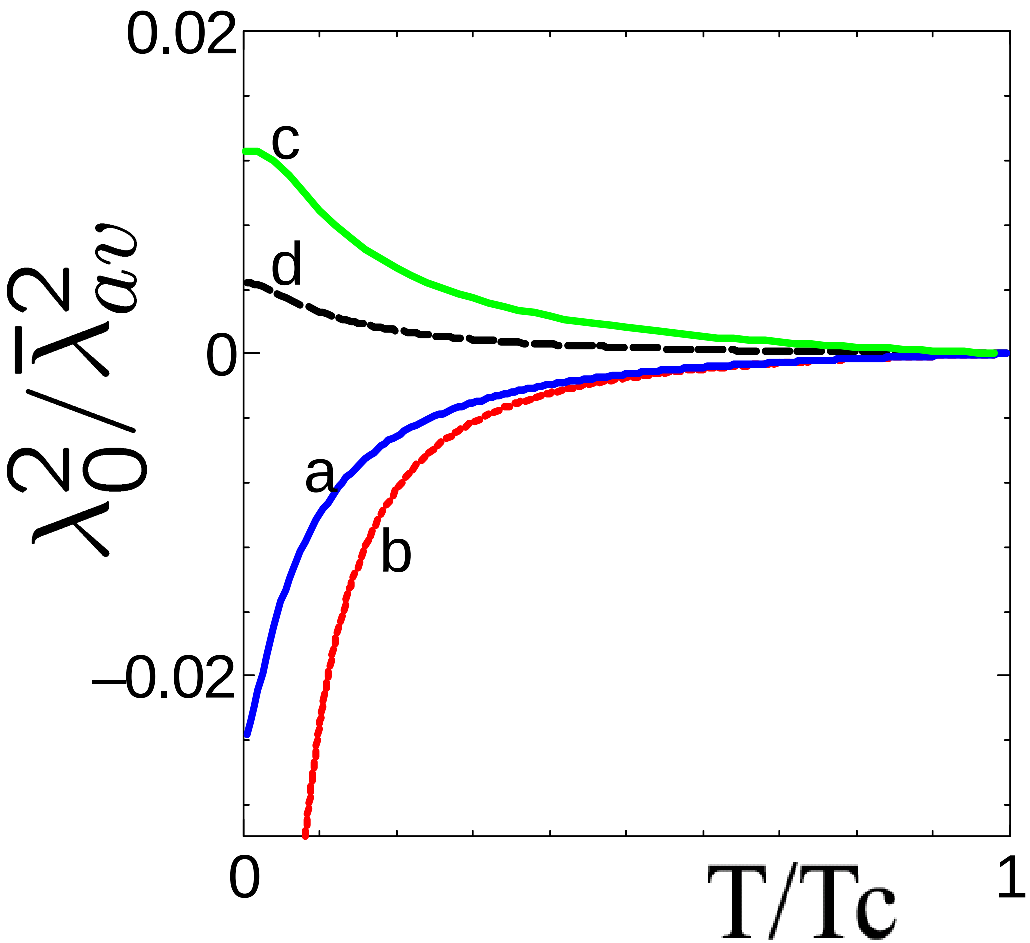
$$H(x) \sim \exp(-x/\lambda(x))$$

$$\frac{1}{\lambda^2(x)} = \frac{T\sum_{\omega_n} \sin^2\theta(\omega_n)}{\lambda_0^2}, \quad \lambda_0^{-2} = 32\pi^2 e^2 N(0) DT_C$$

$$\bar{\lambda}_{av}^2 = L / \int_0^L \frac{dx}{\lambda^2(x)}$$

Narikiyo and Fukuyama, J. Phys. Soc. Jpn. 58, 4557 (1989)
Belzig Bruder PRB 53 5727 (1996)

Temperature dependence of averaged value of local penetration depth



$\bar{\lambda}_{av}$

a purely imaginary number for spin-triplet junctions

a: $p_x + ip_y$-wave

b: p_x-wave

c: s-wave

d: $d_{x^2-y^2} + id_{xy}$-wave

Tanaka, et al, PRB **72**, 140503R 2005

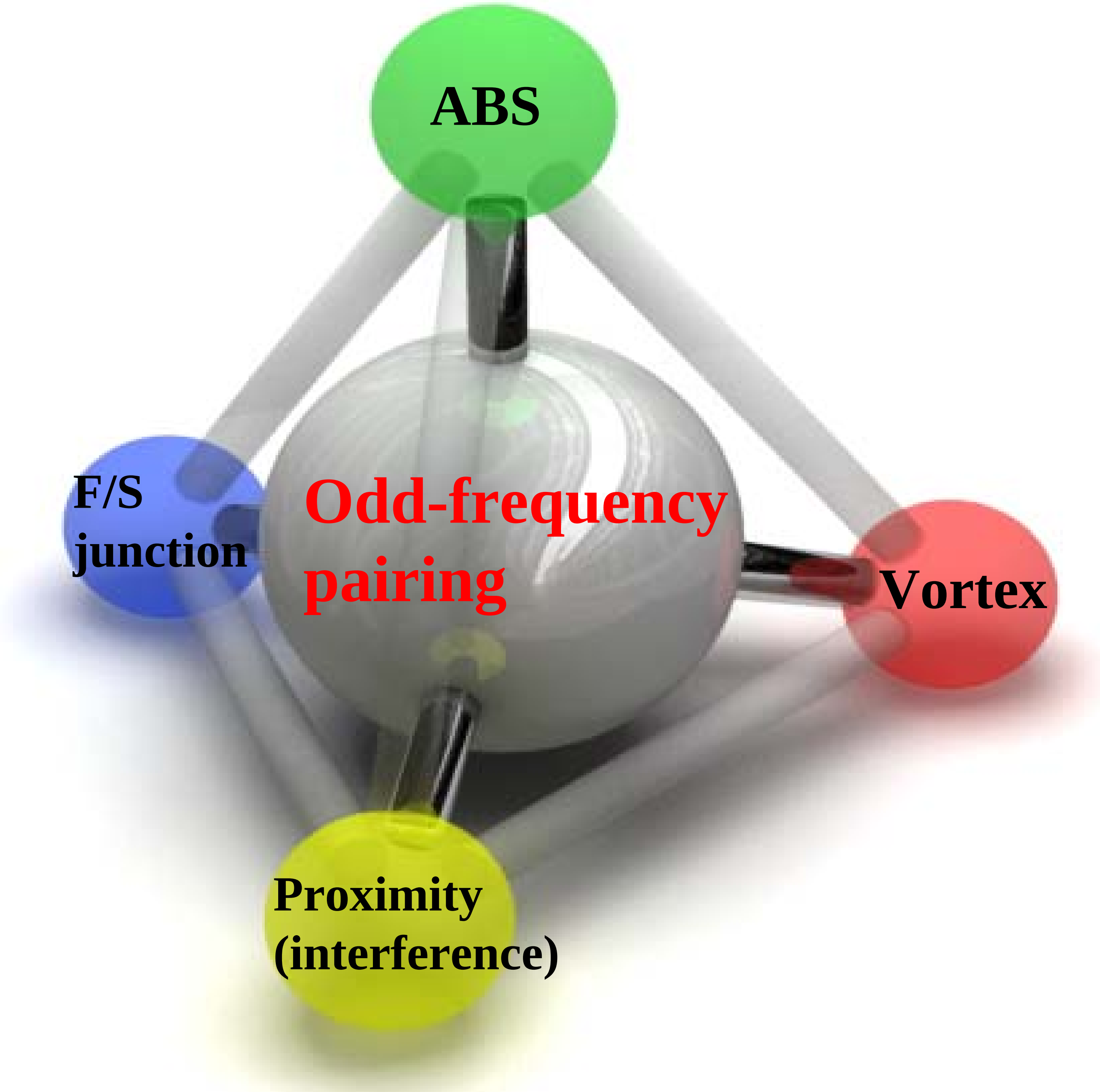
Summary

(1)For spin-triplet superconductor / diffusive normal metal (DN) junctions, pure odd-frequency pairing is possible in the diffusive normal metal.

(2)We can expect anomalous proximity effect with enhanced zero energy density of states.

(3)Sr_2RuO_4 junction is very interesting.

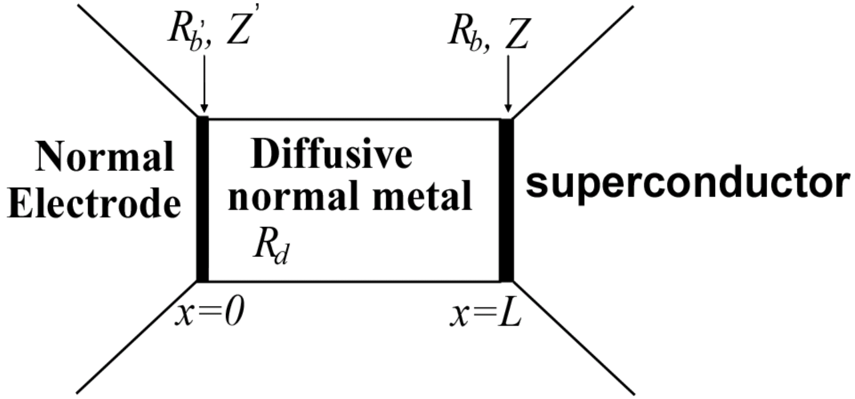
Phys. Rev. Lett. 98 037003 (2007)
Phys. Rev. Lett. 99 067005 (2007).

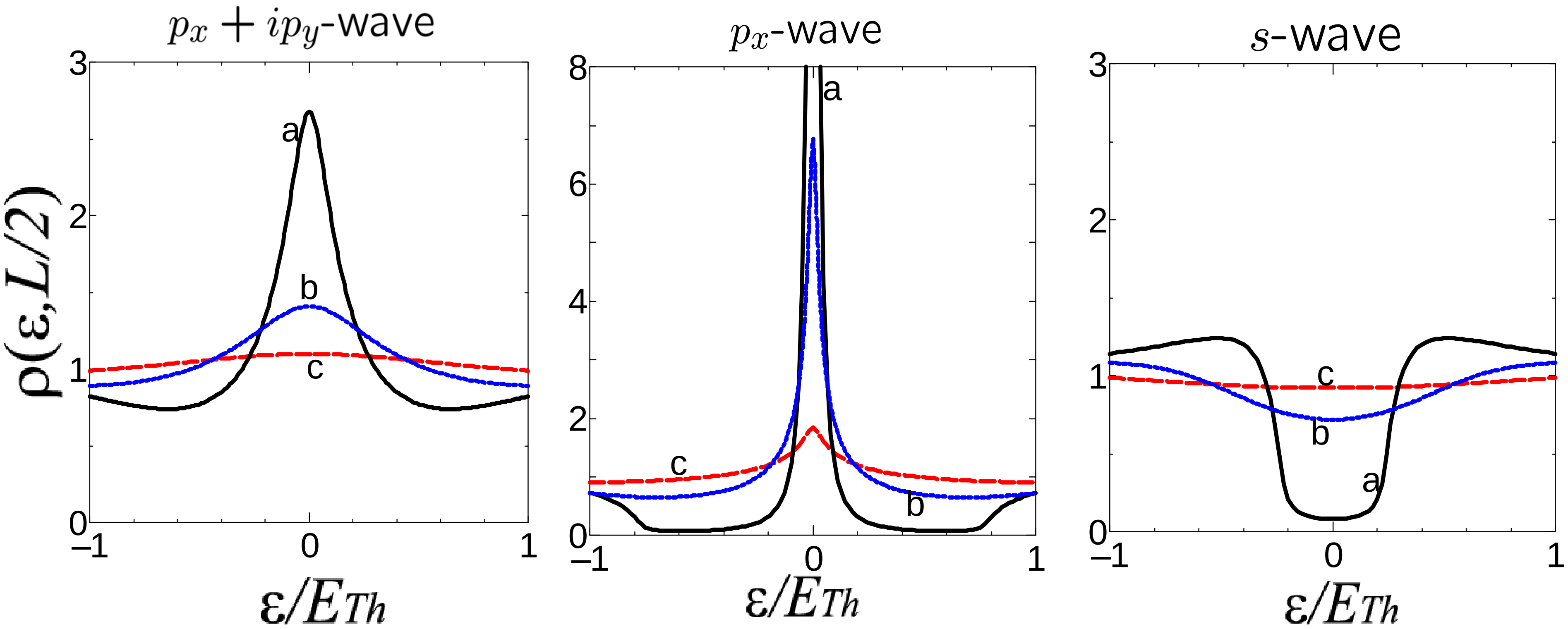

ABS
F/S
junction
Odd-frequency
pairing
Vortex
Proximity
(interference)

Local density of state in DN

$Z = 1,\ R_d/R_b = 1$

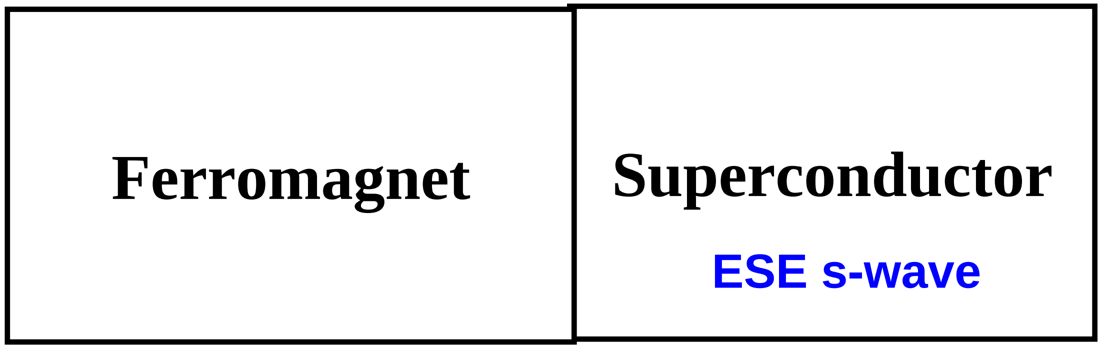
$Z' = 1,\ E_{Th} = 0.25\Delta_0$





$R_d/R_{b'} = 0.01$ $R_d/R_{b'} = 1$ $R_d/R_{b'} = 100$

Ferromagnet (metal)/superconductor junctions



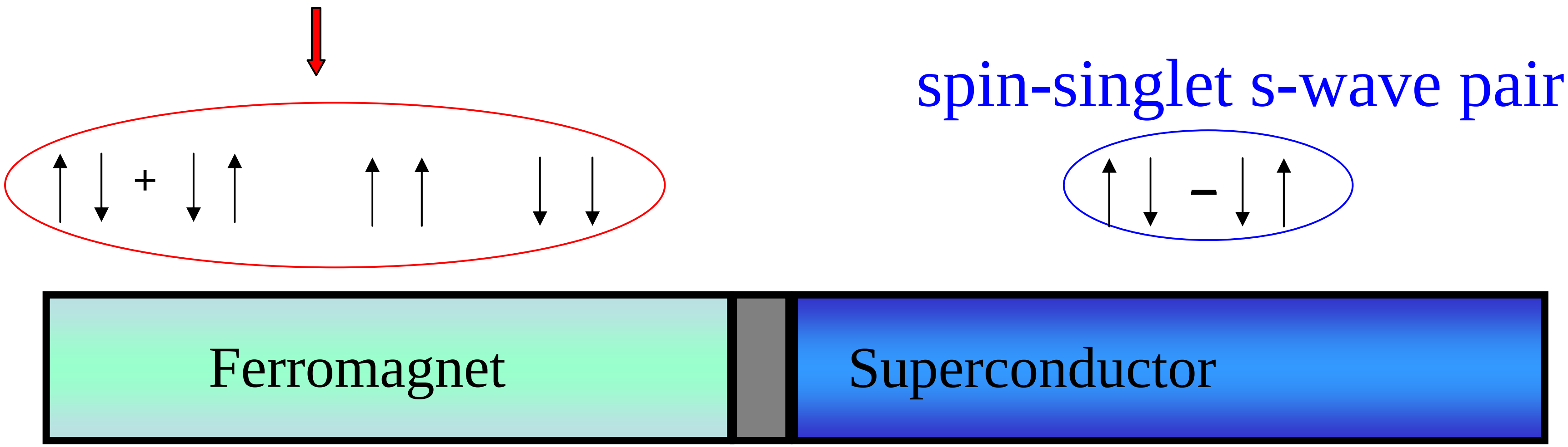
All four kinds of pairing is possible in ferromagnet
(Eschrig, 2007)

(1) **Generation of OSO pairing by broken inversion (translational) symmetry**
(2) **Generation of OTE pairing by broken time reversal symmetry**
(3) **Generation of ETO pairing both in the presence of broken inversion (translational) symmetry and broken time reversal symmetry**

ESE : Even-frequency spin-Singlet Even-parity
ETO : Even-frequency spin-Triplet Odd-parity
OSO : Odd-frequency spin-Singlet Odd-parity
OTE : Odd-frequency spin-Triplet Even-parity

Odd-frequency Pair amplitude not pair potential) is generated in ferromagnet junctions

Odd frequency spin-triplet s-wave pair

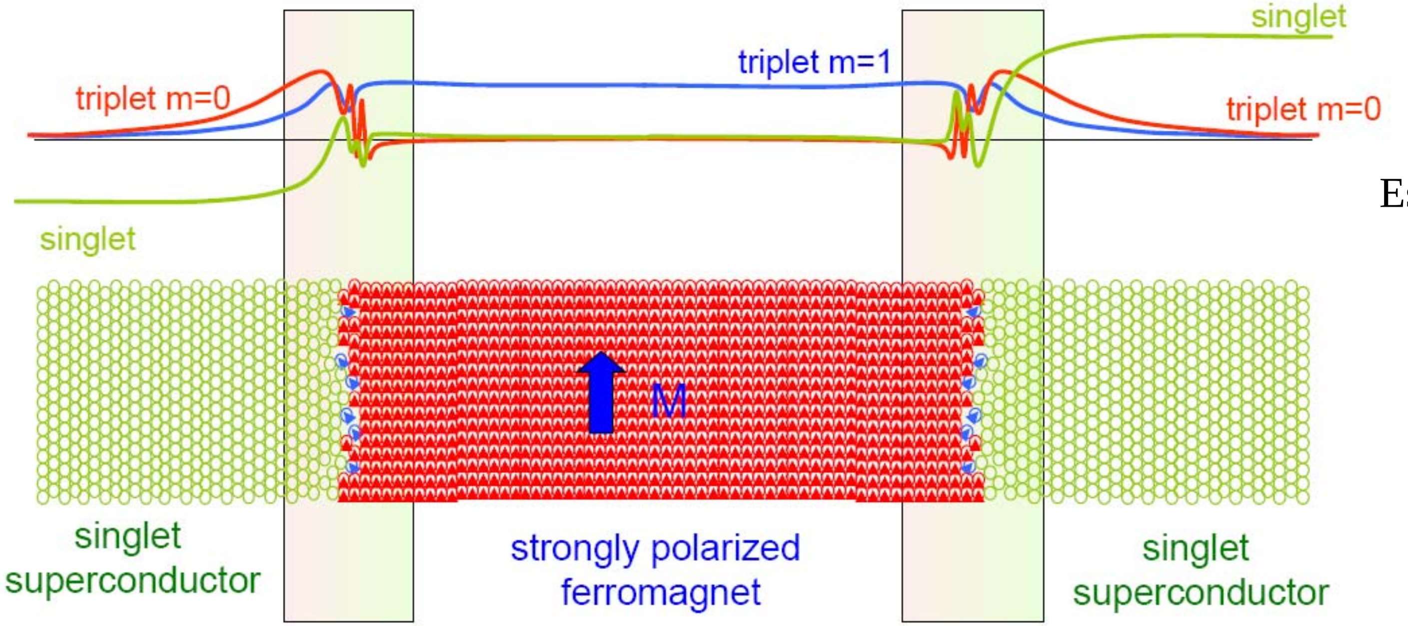


Bergeret, Efetov, Volkov, (2001)

Eschrig, Buzdin,Golubov, Kadigrobov,Fominov, Radovic…

Generation of the odd-frequency pair amplitude in ferromagnet

Josephson current through half metal



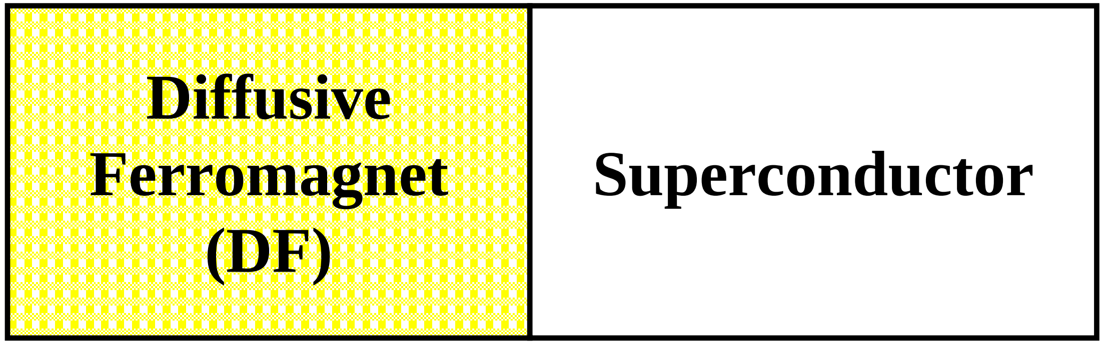
Eschrig (2008)

(1) Spin precession,   triplet pairing with m=0 is generated from singlet pairing

(2) Spin rotation,   triplet pairing with m=1 is generated

(3) even-frequency triplet or odd-frequency triplet

Ferromagnet (metal)/superconductor junctions



Only s-wave pairing state is possible in DF

(1) Weak spin-polarized ferromagnet

T. Yokoyama, Y. Tanaka, and A.A. Golubov PRB 75 134510 (2007)

(only spin precession)

(2) Fully spin-polarized ferromagnet

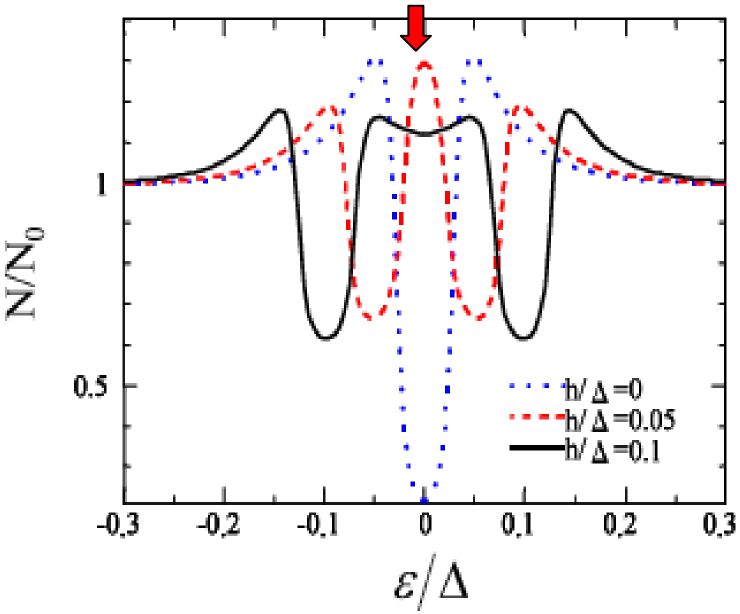
Y.Asano, Y.Tanaka and A.A. Golubov PRL 98, 107002 (2007)

Purely odd-frequency equal spin-triplet pairing is possible

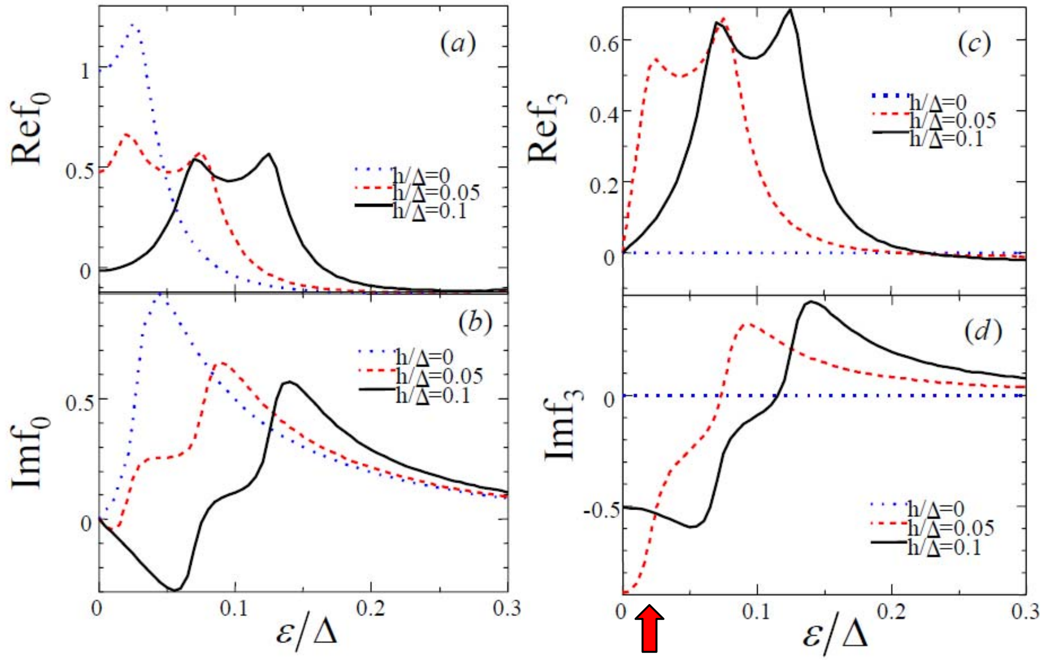
(spin precession & rotation)

Pair amplitude

LDOS (x=0)

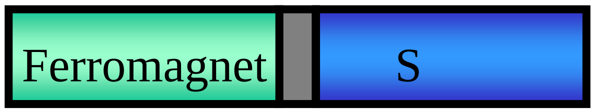


ESE (x=0) OTE (x=0)



$Z = 3 \quad Z' = 3 \quad R_d / R_b = 1 \quad R_d / R'_b = 0.1 \quad E_{Th} / \Delta = 0.1$

LDOS at ε=0 is enhanced, when the magnitude of the OTE pair amplitude is enhanced.

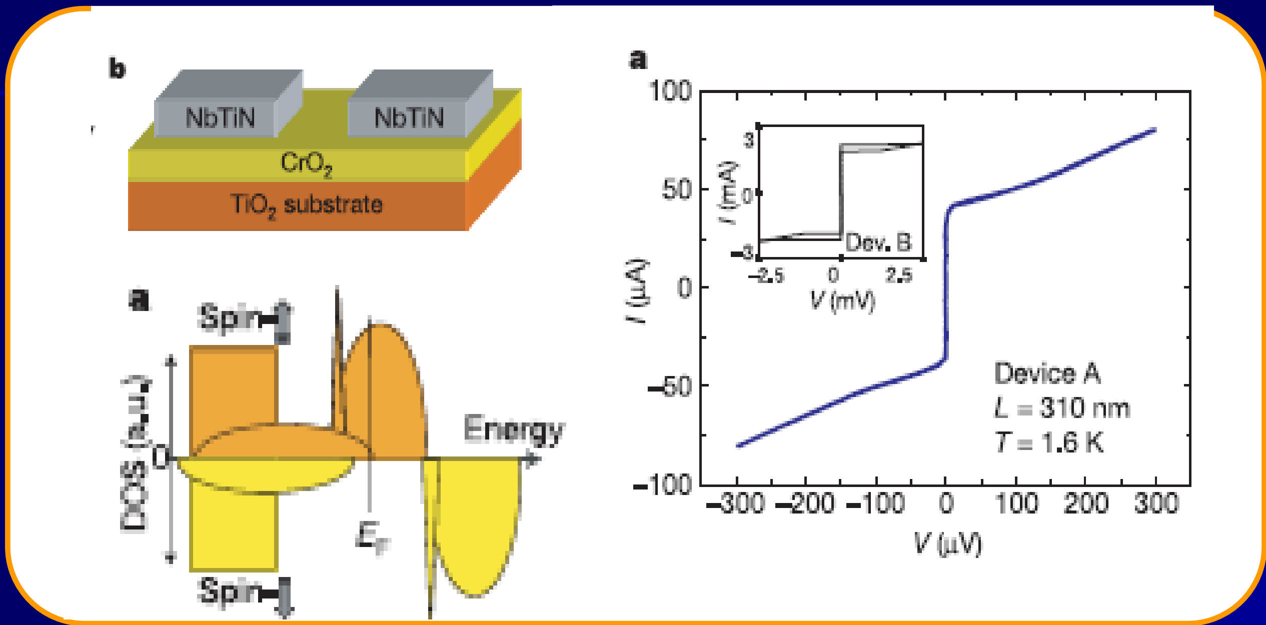


ESE (Even-frequency spin-singlet even-parity)
OTE (Odd-frequency spin-triplet even-parity)

T. Yokoyama, Y. Tanaka, and A.A. Golubov PRB 75 134510 (2007)

Josephson current in S/HM/S

Half metal (HM) : CrO_2 Keizer et.al., Nature ('06)



Spin active interface Bergeret et. al., PRL('01), Kadigrobov et. al., Europhys Lett.('01)

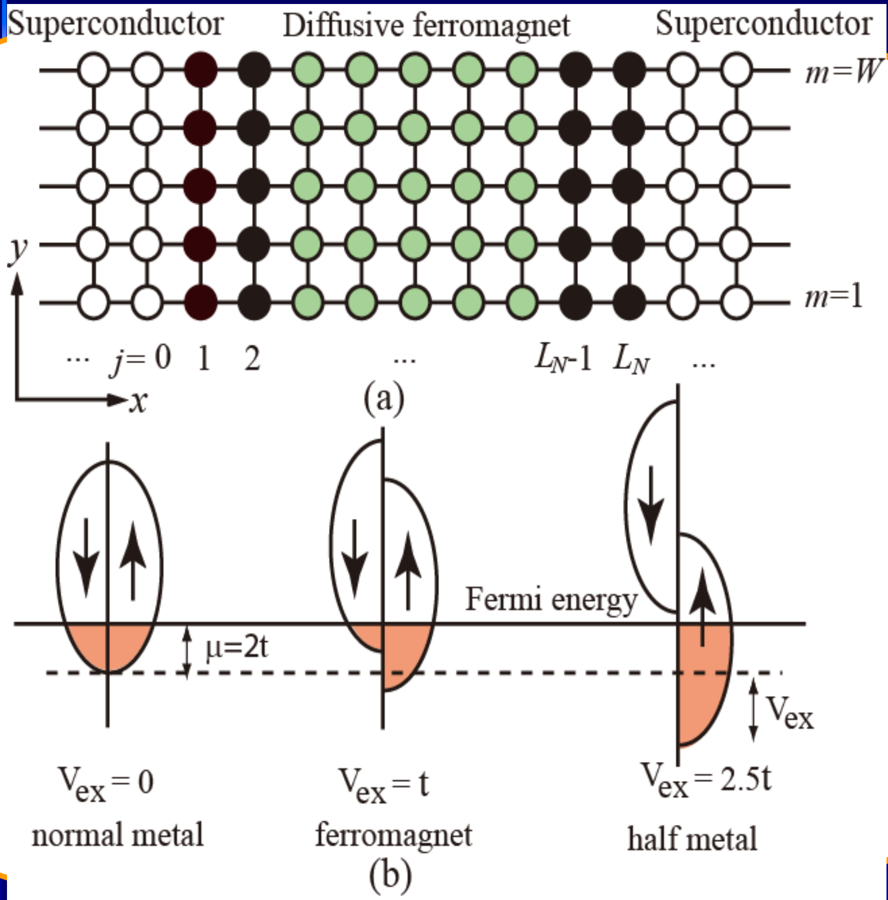
Theory in the clean limit Eschrig et. al., PRL(03)

Theory in the diffusive limit Aasno Tanaka Golubov, PRL('07)

Theory in general case Eschrig, Lofwander Nature Physics(08)

Lattice model （numerical)

Furusaki, Physica B(‘92),
Asano, PRB(‘01)



Advantages

SNS, SFS, S/HM/S

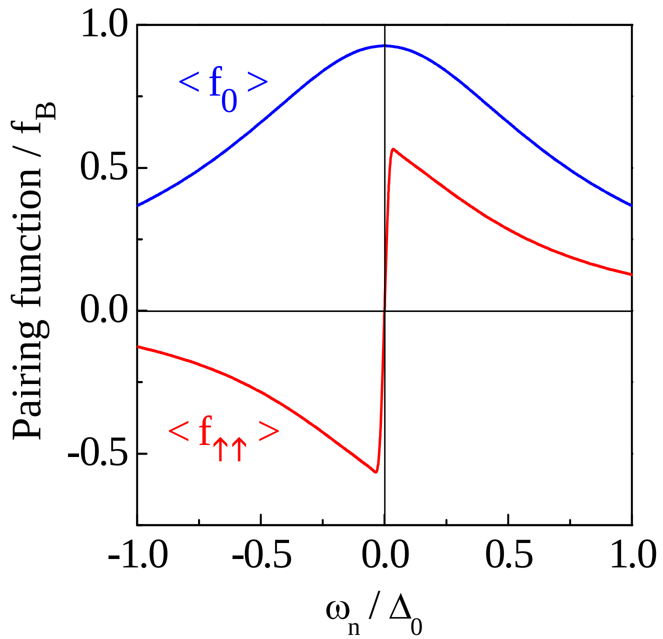
Parameters

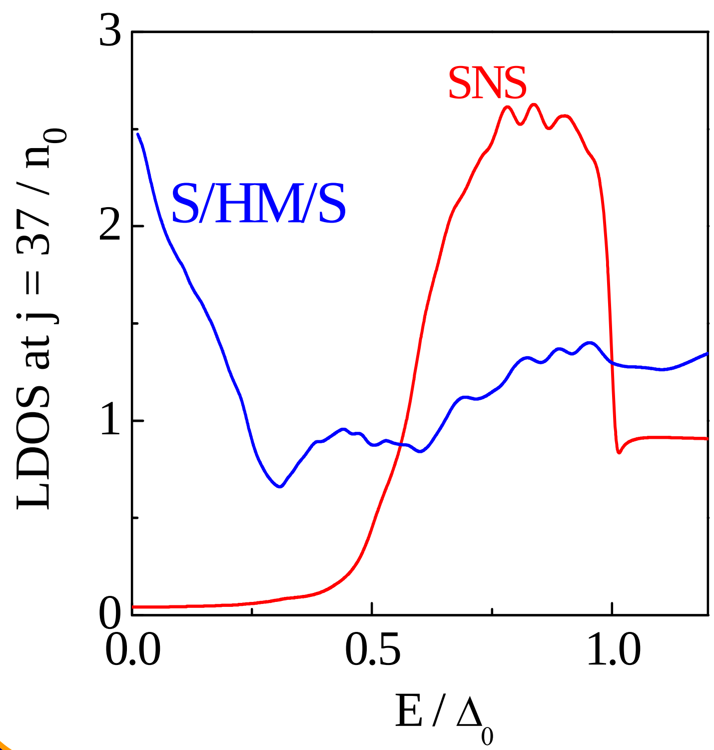
V_{ex} : exchange

V_S : spin-flip (interface)

Y.Asano, Y.Tanaka and A.A. Golubov PRL 98, 107002 (2007)
Eschrig Lofwander Nature Physics(2008)
Braude Nazarov PRL 98 07003 (2007)
Takahashi Hikino et al. PRL 99 057003(2007)

Pair amplitude and LDOS





$< f_0 >$ Even-frequency spin-singlet s-wave (ESE) $V_{ex}=0$ $V_S=0$ S/N/S

$< f_{\uparrow\uparrow} >$ Odd-frequency equal-spin-triplet s-wave (OTE) in S/HF/S

Y.Asano, Y.Tanaka and A.A. Golubov PRL 98, 107002 (2007)

Anomalous Josephson effect between odd-frequency superconductor/ even frequency superconductor junctions

Y. Tanaka, A. Golubov, S. Kashiwaya, and M. Ueda

Phys. Rev. Lett. 99 037005 (2007)

Josephson couplings between even-frequency superconductor and odd-frequency one

	bulk state	sign change	interface state
(1)	ESE (s or $d_{x^2-y^2}$-wave)	No	ESE
(2)	ESE (d_{xy}-wave)	Yes	OSO
(3)	ETO (p_y-wave)	No	ETO
(4)	ETO (p_x-wave)	Yes	OTE
(5)	OSO (p_y-wave)	No	OSO
(6)	OSO (p_x-wave)	Yes	ESE
(7)	OTE (s or $d_{x^2-y^2}$-wave)	No	OTE
(8)	OTE (d_{xy}-wave)	Yes	ETO

1. **(1) and (6)**
2. **(2) and (5)**
3. **(3) and (8)**
4. **(4) and (7)**

Presence of the Lowest order Josephson coupling

PRL 99 037005 (2007)

Previous theory

Abrahams, Balatsky, Scalapino, and Schrieffer
Phys. Rev. B 52, 1271 - 1278 (1995)

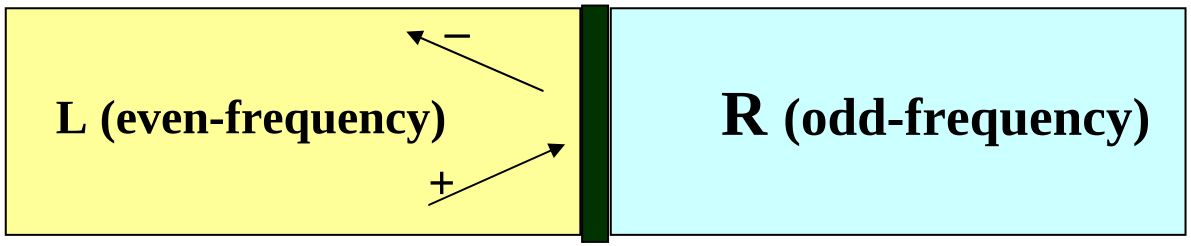
There is no lowest-order Josephson coupling between odd- and even-frequency superconductors.

Interface induced state is neglected!!

Josephson current

$$R_N I(\varphi) = \frac{\pi}{2e} \sum_{\sigma} k_B T \sum_{\omega} \{ < f_{1L+} f_{1R+} + f_{2L+} f_{2R+} > \sin\varphi + < f_{1L+} f_{2R+} - f_{2L+} f_{1R+} > \cos\varphi \}$$

(Lowest Order coupling)



φ

(Macroscopic phase difference between two superconductors)

f_{1L+} f_{1R+} **Interface state**

(1)L-side (Even-frequency superconductor)

f_{1L+} **Odd function of Matsubara**

f_{2L+} **Even function of Matsubara**

→ $\cos\varphi$

(2)R-side (odd-frequency superconductor)

f_{1R+} **Even function of Matsubara**

f_{2R+} **Odd function of Matsubara**

Anomalous current phase relation

PRL 99 037005 (2007)

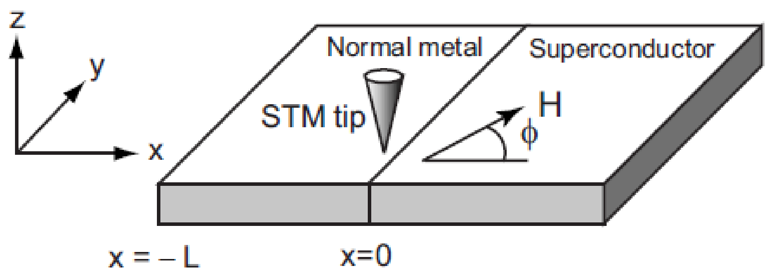
Summary (4)

(1)Ubiquitous presence of odd-frequency pairing in non-uniform superconducting systems.

(2)Bound state can be reinterpreted as a manifestation of odd-frequency pairing.

(3)Possible existence about odd-frequency energy gap function in Q1D system.

LDOS (magneto-tunneling spectroscopy) based on the Doppler effect



$$(A_x, A_y) = -\lambda H \exp(-z/\lambda)(\sin\phi, \cos\phi)$$

Shift of the quasiparticle energy

$$\varepsilon - H\Delta_0 \sin(\phi - \theta)/B_0$$

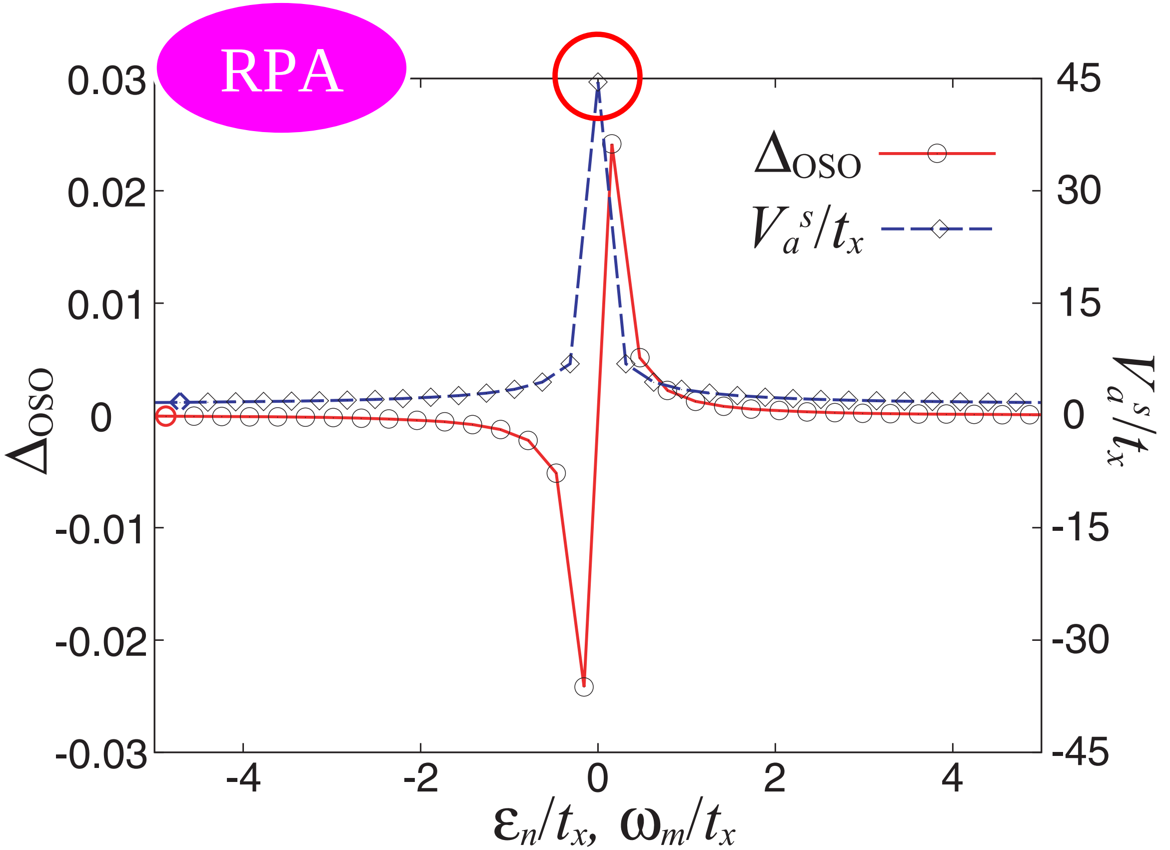
$$B_0 = h/(2e\pi^2\xi\lambda) \qquad \xi = \hbar v_F/\pi\Delta_0$$

$$B_0 \sim 0.02\text{Tesla} \qquad\qquad \xi \sim \lambda \sim 100\text{nm}$$

OSO frequency-dependence

Gap function $\Delta_{\mathrm{OSO}}(i\varepsilon_n, \pi/2, 0)$

Pairing interaction $V_a^s(i\omega_m, \pi, \pi/2)$



$t_y/t_x = t_2/t_x = 0.1,\ S_t = 0.95,\ T/t_x = 0.05$

Josephson current through half metal