Odd frequency pairing in superconducting heterostructures

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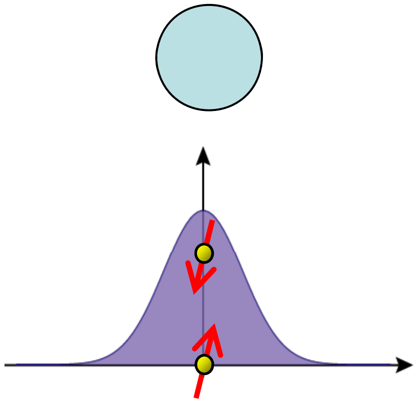
（1）What is odd-frequency pairing

（2）Normal metal / Superconductor junctions

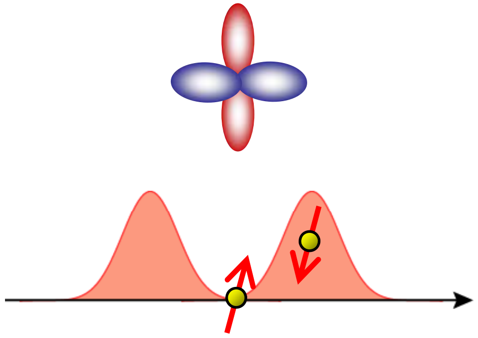
（3）Vortices in superconductors

Conventional Classification of Symmetry of Cooper pair

Spin-singlet Cooper pair → **Even Parity**

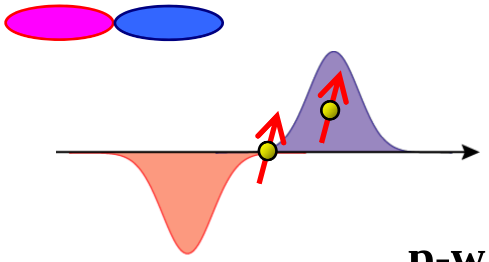


s-wave **BCS**



d-wave **Cuprate**

Spin-triplet Cooper pair → **Odd Parity**



p-wave **^{3}He** **Sr_2RuO_4**

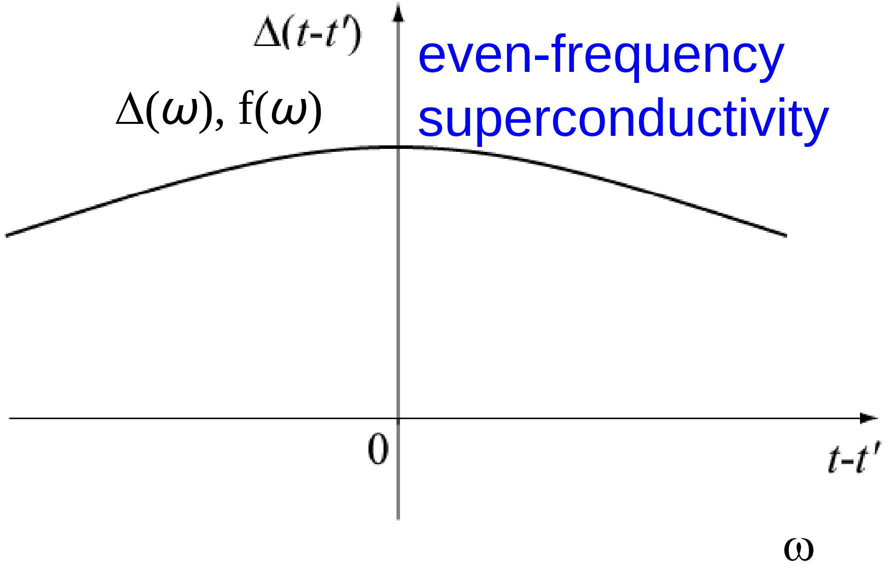
Odd-frequency pairing

Fermi-Dirac statistics

Symmetry of pair wave functions:

$$\mathbf{k} \otimes \sigma \otimes \omega = \mathrm{odd}$$

Momentum* x *Spin* x *Frequency

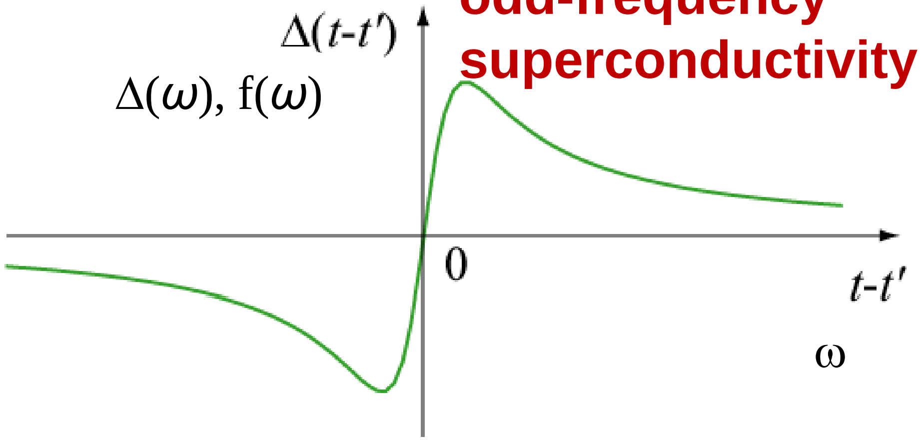


even-frequency superconductivity

Berezinskii
(1974):

Spin-triplet s-wave

Balatsky&Abrahams
(1992):
Spin-singlet p-wave



odd-frequency superconductivity

ω, Matsubara frequency

Odd-frequency pairing state

(1)Odd-frequency pairing (pair potential, gap function) in uniform bulk system (odd-frequency superconductor)

Uniform (bulk) system:

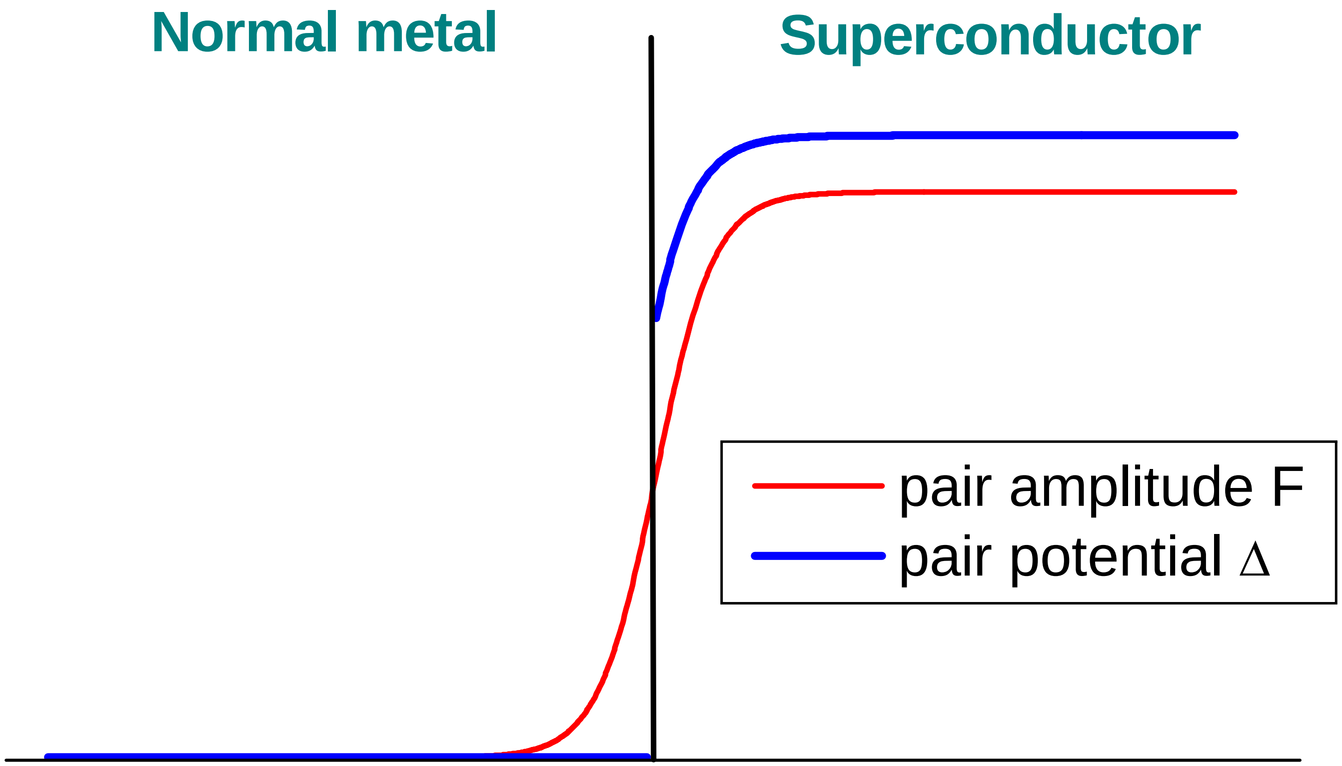
$$\Delta(\boldsymbol{k}, i\omega_n) = -T\sum_{\boldsymbol{k}',m} V(\boldsymbol{k}-\boldsymbol{k}', i\omega_n - i\omega_m) F(\boldsymbol{k}', i\omega_m)$$

Pair potential
Energy Gap

electron-interaction

Pair amplitude

(2)Odd-frequency pairing state (pair amplitude) in superconducting junctions



Weak coupling BCS: $\Delta = \lambda T \sum_m F(i\omega_m)$

Pair amplitude (pair correlation)

$$F_{\alpha,\beta}(\boldsymbol{r}_1 t_1, \boldsymbol{r}_2 t_2) = -i \left\langle \mathsf{T} \psi_\alpha(\boldsymbol{r}_1 t_1) \psi_\beta(\boldsymbol{r}_2 t_2) \right\rangle$$

$$= -i\theta(t_1 - t_2) \left\langle \psi_\alpha(\boldsymbol{r}_1 t_1) \psi_\beta(\boldsymbol{r}_2 t_2) \right\rangle + i\theta(t_2 - t_1) \left\langle \psi_\beta(\boldsymbol{r}_2 t_2) \psi_\alpha(\boldsymbol{r}_1 t_1) \right\rangle$$

Exchange of two electrons

$$F_{\alpha,\beta}(\boldsymbol{r}_1 t_1, \boldsymbol{r}_2 t_2) = -F_{\beta,\alpha}(\boldsymbol{r}_2 t_2, \boldsymbol{r}_1 t_1)$$

Fermi-Dirac statistics

Pair amplitude

Exchange of time

Even-frequency pairing (conventional pairing)

$$F_{\alpha,\beta}(\boldsymbol{r}_1 t_1, \boldsymbol{r}_2 t_2) = F_{\alpha,\beta}(\boldsymbol{r}_1 t_2, \boldsymbol{r}_2 t_1)$$

Odd-frequency pairing

$$F_{\alpha,\beta}(\boldsymbol{r}_1 t_1, \boldsymbol{r}_2 t_2) = -F_{\alpha,\beta}(\boldsymbol{r}_1 t_2, \boldsymbol{r}_2 t_1)$$

Symmetry of the pair amplitude

+ symmetric, – anti-symmetric

	Frequency (time)	Spin	Orbital	Total
ESE	+(even)	– (singlet)	+(even)	–
ETO	+(even)	+ (triplet)	–(odd)	–
OTE	–(odd)	+ (triplet)	+(even)	–
OSO	–(odd)	– (singlet)	–(odd)	–

BCS
Cuprate

^{3}He
Sr_2RuO_4

ESE (Even-frequency spin-singlet even-parity)
ETO (Even-frequency spin-triplet odd-parity)
OTE (Odd-frequency spin-triplet even-parity) Berezinskii
OSO (Odd-frequency spin-singlet odd-parity) Balatsky, Abrahams

Previous studies of odd-frequency pairing

Bulk state (Pair potential, Gap function)

Berezinskii (1974)

Balatsky, Abrahams, Schrieffer, Scalapino(1992-1993)

Zachar, Kievelson, Emery (1996)

Coleman, Mirranda, Tsvelik (1997)

Vojta, Dagotto (1999)

Fuseya, Kohno, Miyake (2003)

Shigeta, Onari, Yada, Tanaka (2009)

Junction (No pair potential)

Induced odd-frequency pair amplitude in ferromagnet attached to spin-singlet s-wave superconductor

Bergeret, Efetov, Volkov, (2001)

- **Odd-frequency pairing state** is possible in inhomogeneous superconductors even for conventional even-frequency paring in the bulk

Broken spin rotation symmetry or spatial invariance symmetry can induce odd-frequency pairing state:

- **ferromagnet/superconducor junctions:**
Bergeret,Volkov&Efetov, 2001

- **non-uniform systems:**

Junctions: Tanaka&Golubov, 2007; Eschrig&Lofwander, 2007

Vortices: Yokoyama *et al.*, 2008; Tanuma *et al.*, 2009)

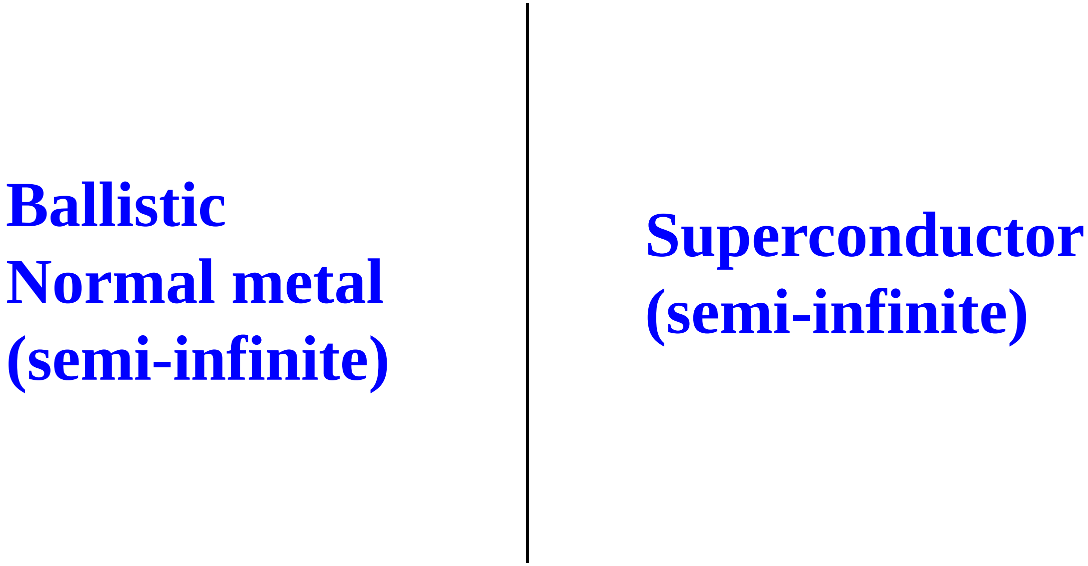
Contents

（１）What is odd-frequency pairing

(2) Ballistic normal metal junctions

(3）Vortices in superconductors

Ballistic junction



Ballistic
Normal metal
(semi-infinite)

Superconductor
(semi-infinite)

Y. Tanaka, A. Golubov, S. Kashiwaya, and M. Ueda
Phys. Rev. Lett. 99 037005 (2007)

M. Eschrig, T. Lofwander, Th. Champel, J.C. Cuevas and G. Schon
J. Low Temp. Phys 147 457(2007)

Eilenberger equation

(explicitly denote direction of motion)

$$\mp i v_{Fx} \partial_x f_{1\pm} = 2\omega_n f_{2\pm} - 2\bar{\Delta}_\pm(x) g_\pm$$

$$\mp i v_{Fx} \partial_x g_\pm = 2\bar{\Delta}_\pm(x) f_{1\pm},$$

$$\mp i v_{Fx} \partial_x f_{2\pm} = -2\omega_n f_{1\pm},$$

Pair potential	$\bar{\Delta}_\pm(x)$
Quasiparticle function	$g_\pm$
Pair amplitudes	$f_{1\pm},\ f_{2\pm}$

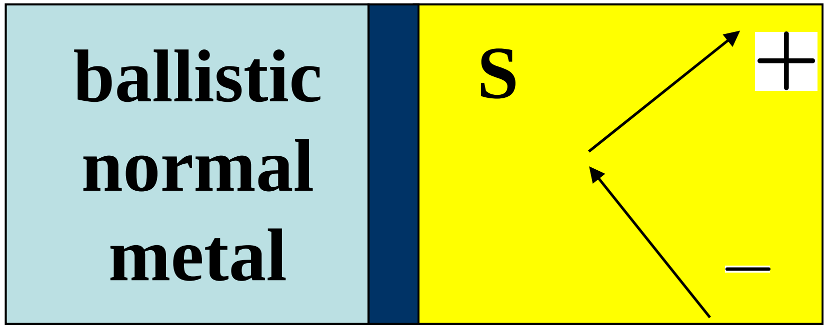
$$f_{1\pm}^2 + f_{2\pm}^2 + g_\pm^2 = 1,$$

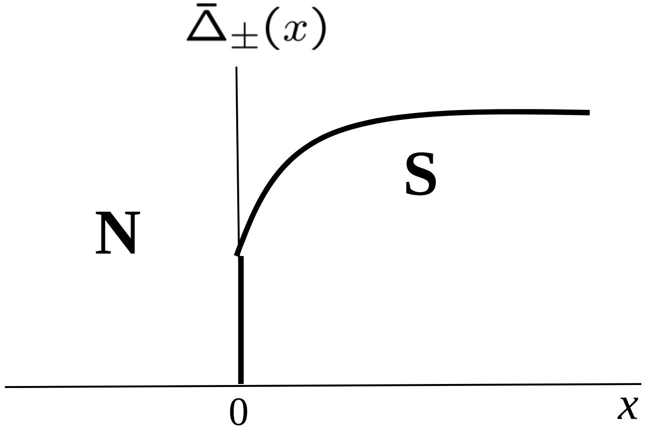
$$\bar{\Delta}_\pm(x) = \Delta(x)\Phi_\pm(\theta).$$

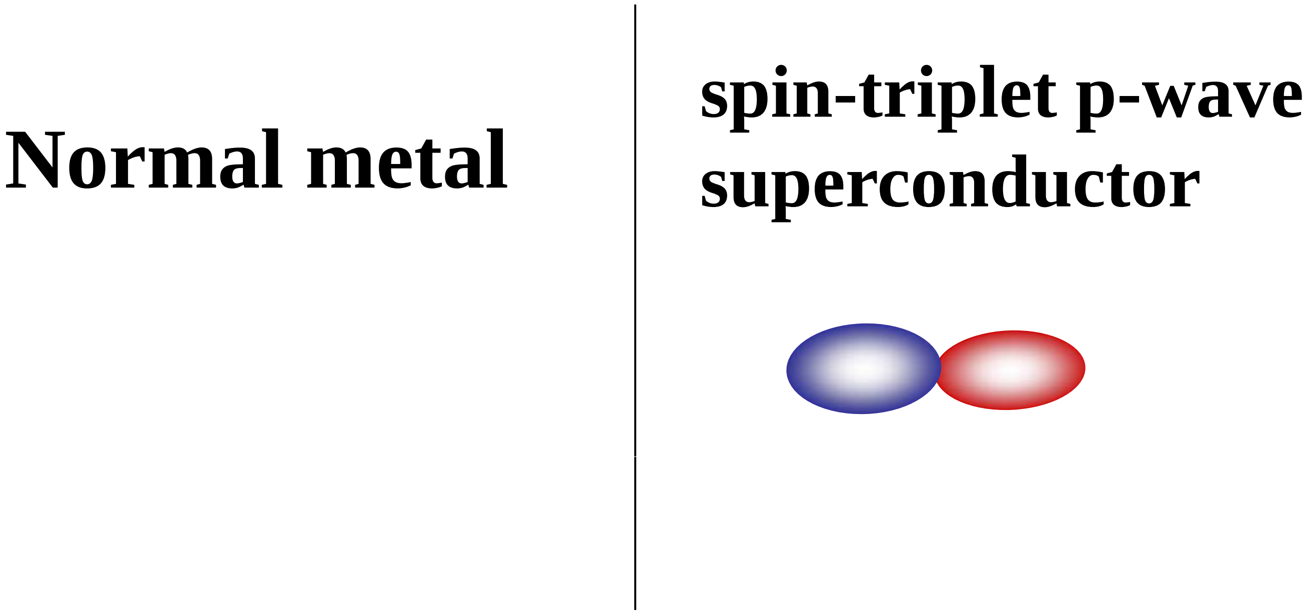
$\Phi_\pm(\theta)$ Form factor

Bulk state

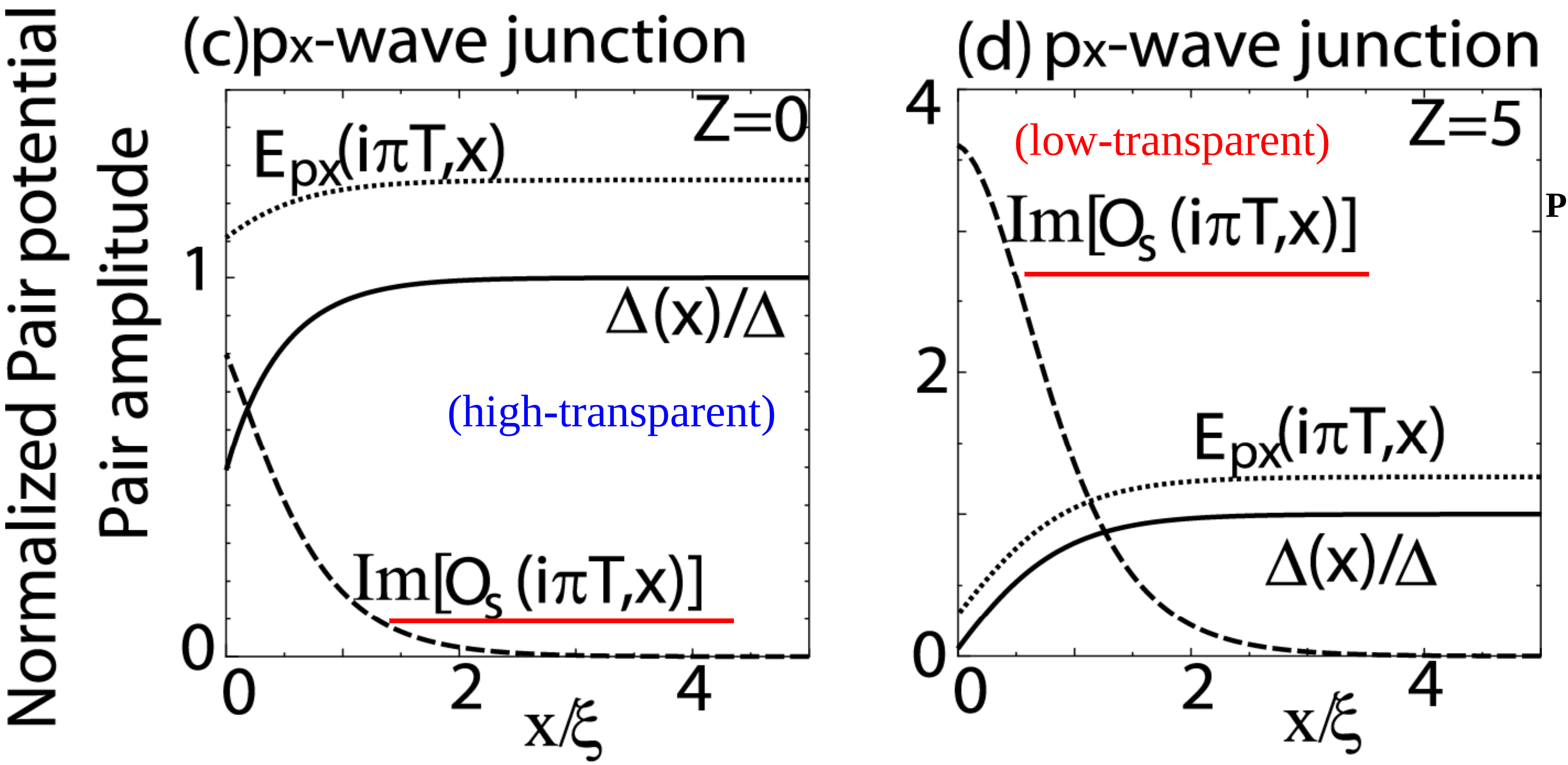
Only $f_{2\pm}$






Normal metal
spin-triplet p-wave superconductor

Symmetry of the bulk pair potential is ETO



$\Delta(x)$

Pair potential

$E_{px}(i\omega_n, x)$ **p_x-wave component of ETO pair amplitude**

$O_s(i\omega_n, x)$ **s-wave component of OTE pair amplitude**

ETO (Even-frequency spin-triplet odd-parity)
OTE (Odd-frequency spin-triplet even-parity) **Y. Tanaka, et al PRL 99 037005 (2007)**

Underlying physics

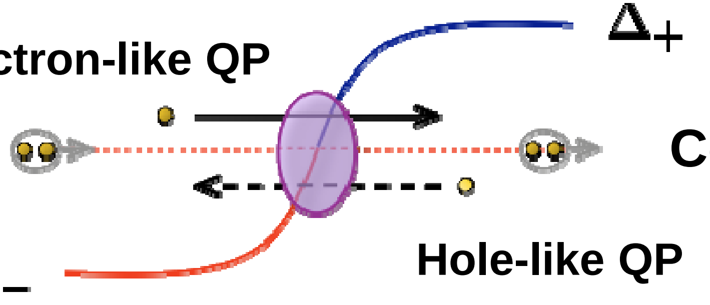
Near the interface, even and odd-parity pairing states (pair amplitude) can mix due to the **breakdown of the translational symmetry**.

⇩ **Fermi-Dirac statistics**

The interface-induced state (pair amplitude) should be odd in frequency where the bulk pair potential has an even -frequency component since there is no spin flip at the interface.

Andreev bound states in inhomogeneous systems are manifestations of odd-frequency pairing amplitude

Andreev bound states



Positive pair potential

Δ_+

Electron-like QP

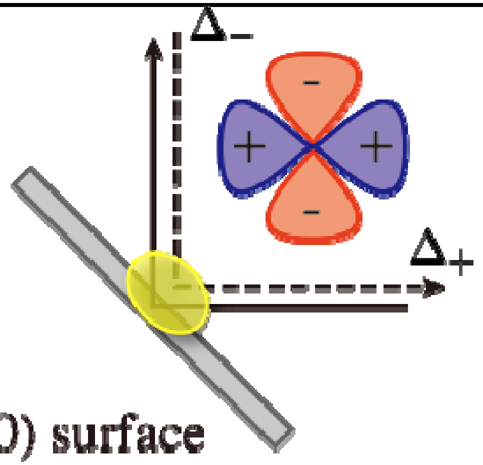
Cooper pair

Hole-like QP

Δ_-

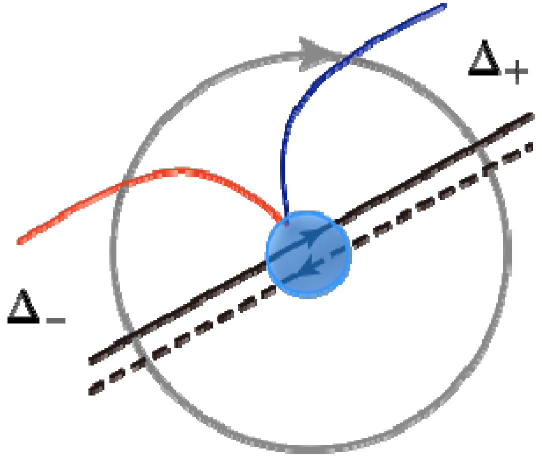
Negative pair potential

Surface: Tanaka *et al*, 2007



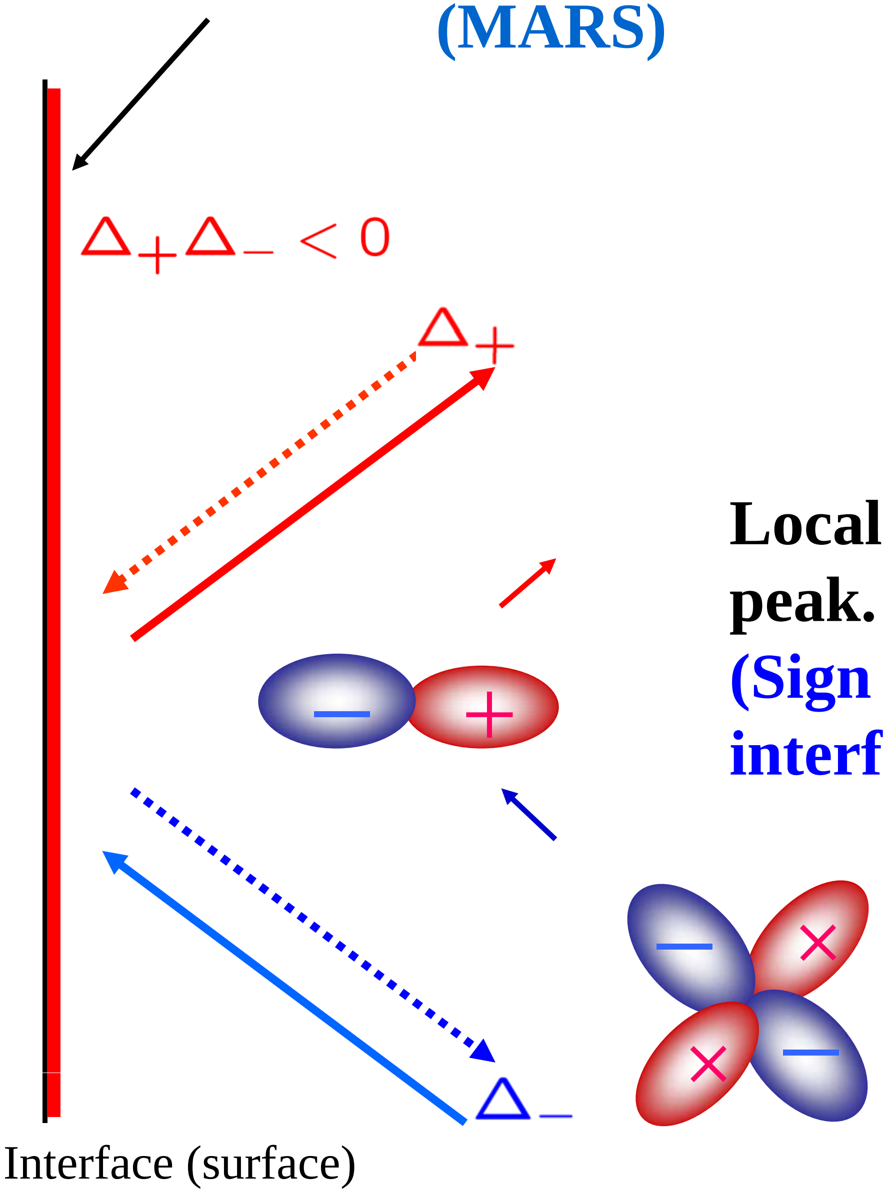
Scattering direction of QP

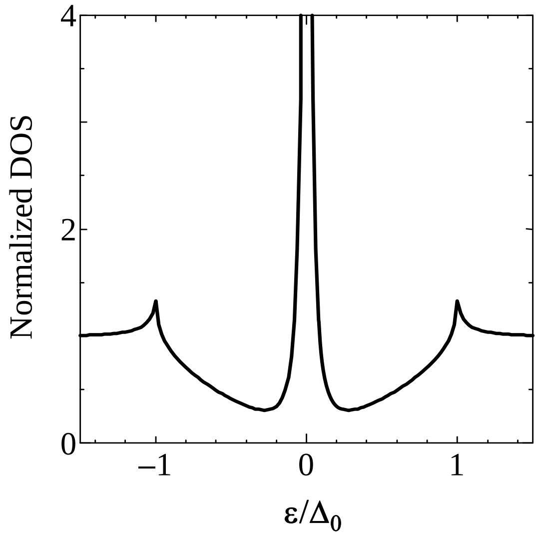
Vortex : Tanuma *et al*, 2009



Phase change due to a vortex

Mid gap Andreev resonant (bound) state (MARS)





Local density of state has a zero energy peak.

(Sign change of the pair potential at the interface)

Tanaka Kashiwaya PRL 74 3451 (1995),
Rep. Prog. Phys. 63 1641 (2000)
Buchholz(1981) Hara Nagai(1986)
Hu(1994) Matsumoto Shiba(1995)
Ohashi Takada(1995)
Hatsugai and Ryu (2002)

Superconducting Materials where MARS is observed

$YBa_2CuO_{7-\delta}$ (Geerk, Kashiwaya, Iguchi, Greene, Yeh,Wei..)

$Bi_2Sr_2CaCu_2O_y$ (Ng, Suzuki, Greene….)

$La_{2-x}Sr_xCuO_4$ (Iguchi)

$La_{2-x}Ce_xCuO_4$ (Cheska)

$Pr_{2-x}Ce_xCuO_4$ (R.L.Greene)

Sr_2RuO_4 (Mao, Meno, Kawamura,Laube)

κ–$(BEDT\text{-}TTF)_2X$, $X=Cu[N(CN)_2]Br$ (Ichimura)

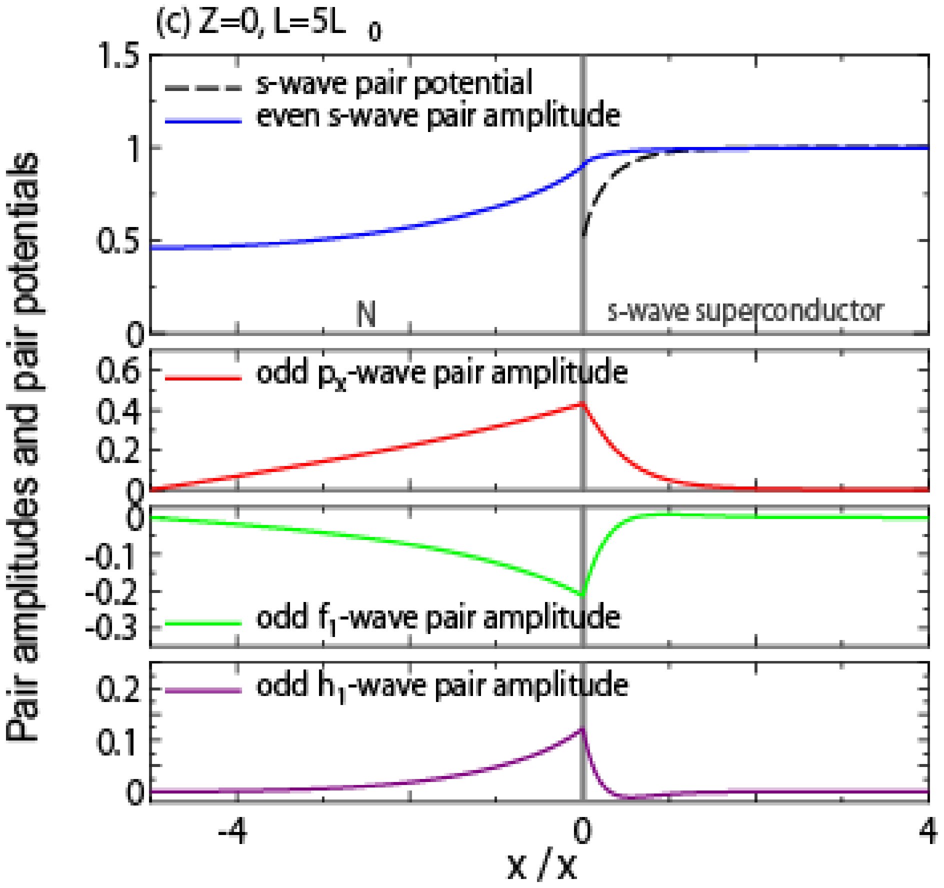
UBe_{13} (Ott)

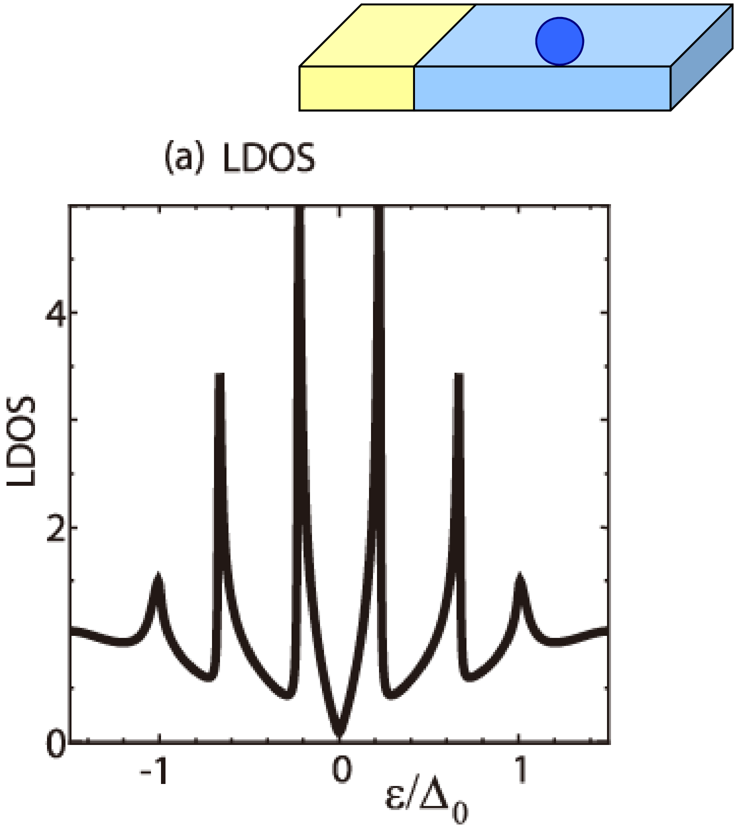
$CeCoIn_5$ (Wei Greene)

$PrOs_4Sb_{12}$ (Wei)

Superfluid ^3He (Okuda, Nomura, Higashitani, Nagai)

Odd-frequency pairing state in N/S junctions (N finite length)





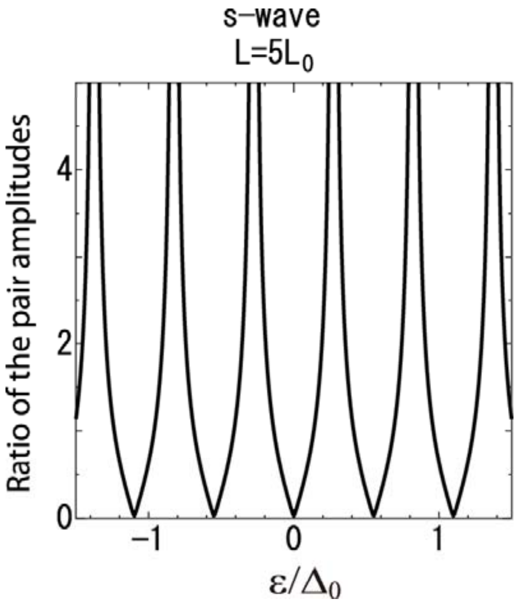
Bounds state are formed in the normal metal

Y. Tanaka, Y. Tanuma and A.A.Golubov, Phys. Rev. B **76**, 054522 (2007)

Ratio of the pair amplitude in the N region (odd/even)

$$\frac{\left|f_{1+}^{(N)}(\varepsilon,\theta)\right|}{\left|f_{2+}^{(N)}(\varepsilon,\theta)\right|} = \left|\tan\left(\frac{2\varepsilon}{v_{Fx}}(L+x)\right)\right|.$$

At some energy, odd-frequency component can exceed over even frequency one.



$$\theta = 0$$
$$x = 0$$

$f_{1+}^{(N)}(\varepsilon,\theta)$

Odd-frequency pairing

$f_{2+}^{(N)}(\varepsilon,\theta)$

Even-frequency pairing

Hidden odd-frequency component in the s-wave superconductor junctions

Ratio of the pair amplitude at the N/S interface and the bound state level

$\Delta_0 \gg \varepsilon$ **Bound states condition (Z=0)**

（McMillan Thomas Rowell)

$$\varepsilon_n = \frac{\pi v_{Fx}}{2L}(n + 1/2), \quad n = 0, 1, 2, \ldots$$

$$\frac{\left| f_{1+}^{(N)}(\varepsilon,\theta) \right|}{\left| f_{2+}^{(N)}(\varepsilon,\theta) \right|} = \left| \tan\left(\pi/2 + \pi n \right) \right| = \infty.$$

$f_{1+}^{(N)}(\varepsilon,\theta)$

Odd-frequency pairing

$f_{2+}^{(N)}(\varepsilon,\theta)$

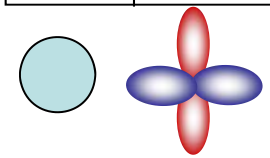
Even-frequency pairing

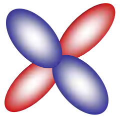
Bound states are due to the generation of the odd-frequency Cooper pair amplitude

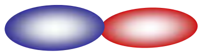
Y. Tanaka, Y. Tanuma and A.A. Golubov, PRB 76 054522 (2007)

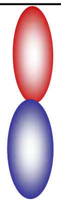
Symmetry of the Cooper pair (No spin flip)

	Bulk state	Sign change **(MARS)**	Interface-induced symmetry (subdominant component)
(1)	ESE (s,d_{x2-y2} -wave)	No	**ESE** + (OSO)
(2)	ESE (d_{xy}-wave)	Yes	**OSO** +(ESE)
(3)	ETO (p_x-wave)	Yes	**OTE** + (ETO)
(4)	ETO (p_y-wave)	No	**ETO** + (OTE)

(1) 

(2) 

(3) 

(4) 

- **ESE (Even-frequency spin-singlet even-parity)**
- **ETO (Even-frequency spin-triplet odd-parity)**
- **OTE (Odd-frequency spin-triplet even-parity)**
- **OSO (Odd-frequency spin-singlet odd-parity)**

Phys. Rev. Lett. 99 037005 (2007)

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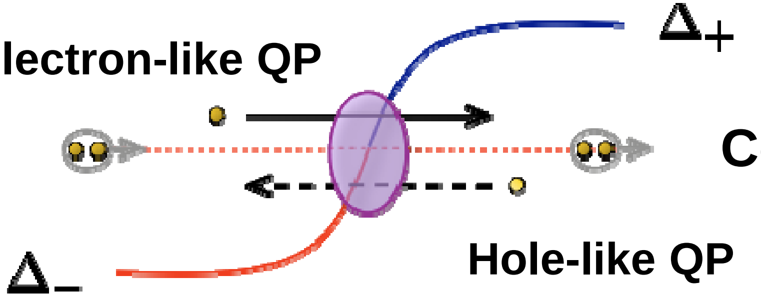
Andreev bound states in inhomogeneous systems are manifestations of odd-frequency pairing amplitude

Andreev bound states



Positive pair potential

Δ_+

Electron-like QP

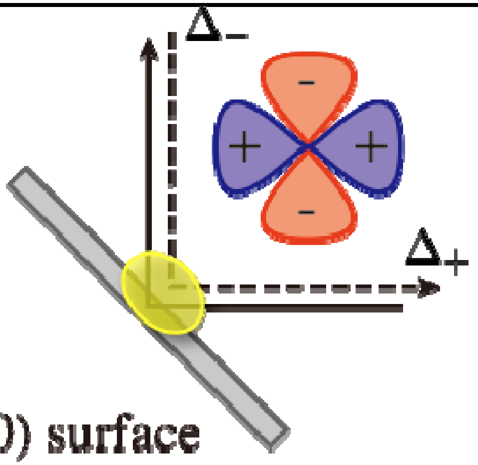
Cooper pair

Hole-like QP

Δ_-

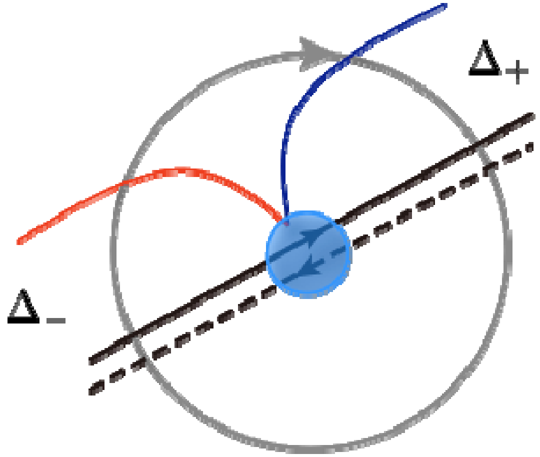
Negative pair potential

Surface: Tanaka *et al*, 2007



Scattering direction of QP

Vortex : Tanuma *et al*, 2009



Phase change due to a vortex

Symmetry of the Cooper pair in a vortex core

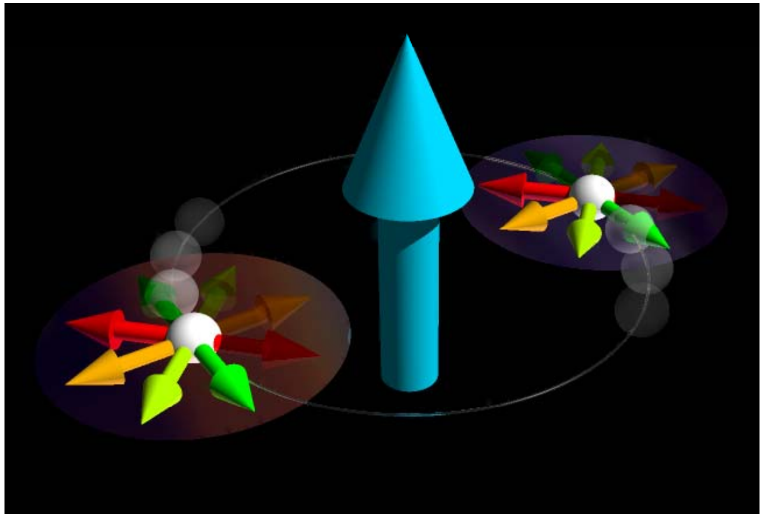
l; **angular momentum**	m; **vorticity**	bulk	Center of the vortex core
Even	Even	ESE (s-wave..)	ESE
Even	Odd	ESE (s-wave..)	**OSO**
Odd	Even	ETO (chiral p-wave)	ETO
Odd	Odd	ETO (chiral p-wave)	**OTE**

$$\Delta(\mathbf{r}) = \Delta_0 \exp(il\varphi)\tanh\left(\frac{\sqrt{x^2+y^2}}{\xi}\right)\left(\frac{x+iy}{\sqrt{x^2+y^2}}\right)^m$$

ESE (Even-frequency spin-singlet even-parity)
ETO (Even-frequency spin-triplet odd-parity)
OTE (Odd-frequency spin-triplet even-parity)
OSO (Odd-frequency spin-singlet odd-parity

Yokoyama *et al.*, Physical Review B, Vol. 78, 012508, 2008

Vortex core spectroscopy of chiral p-wave superconductors



Sr_2RuO_4

Maeno (1994)

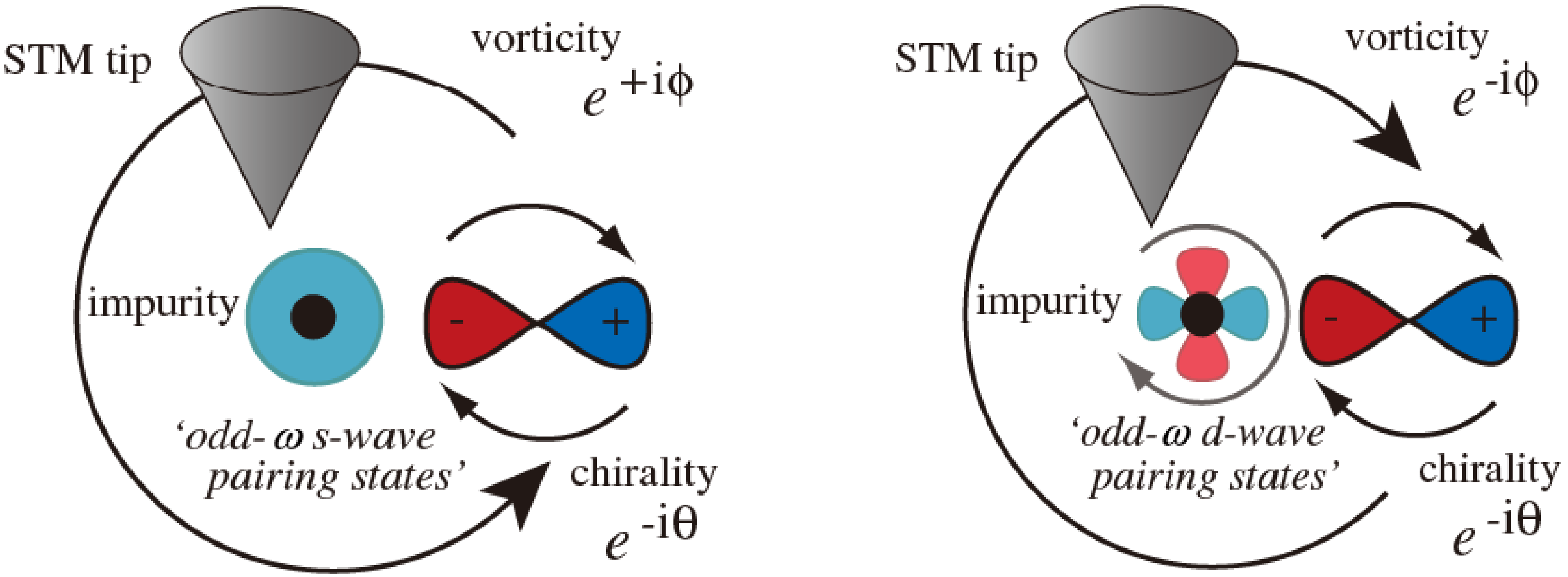
Y. Tanuma, N. Hayashi, Y. Tanaka, A. A. Golubov

Phys. Rev. Lett. 102, 117003 (2009)

Chirality and vorticity: Y. Kato and N. Hayashi (2000, 2001,2002)

Focus on the impurity scattering effect (Born limit)

Difference of the angular momentum of the odd-frequency pair at the core center



(a) Antiparallel chiral p-wave votex

(b) Parallel chiral p-wave votex

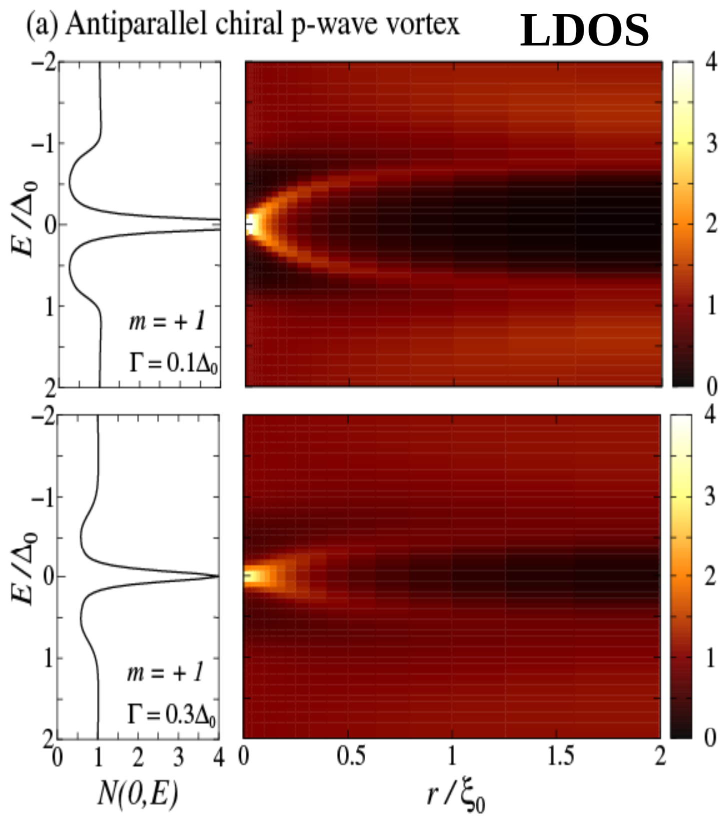
Angular momentum at the center of core; $l+m$

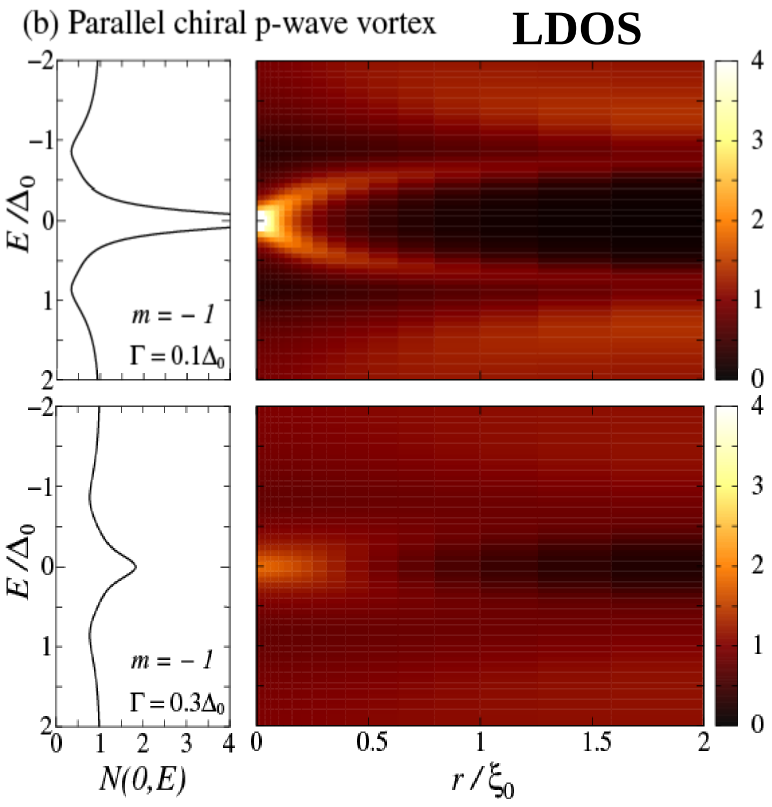
l: angular momentum **m: vorticity**

Tanuma, Hayashi, Tanaka Golubov Phys. Rev. Lett. 102, 117003 (2009).

Chirality and vorticity: Y. Kato and N. Hayashi (2000, 2001,2002)

Impurity effect (Born approximation)





Γ, impurity scattering strength

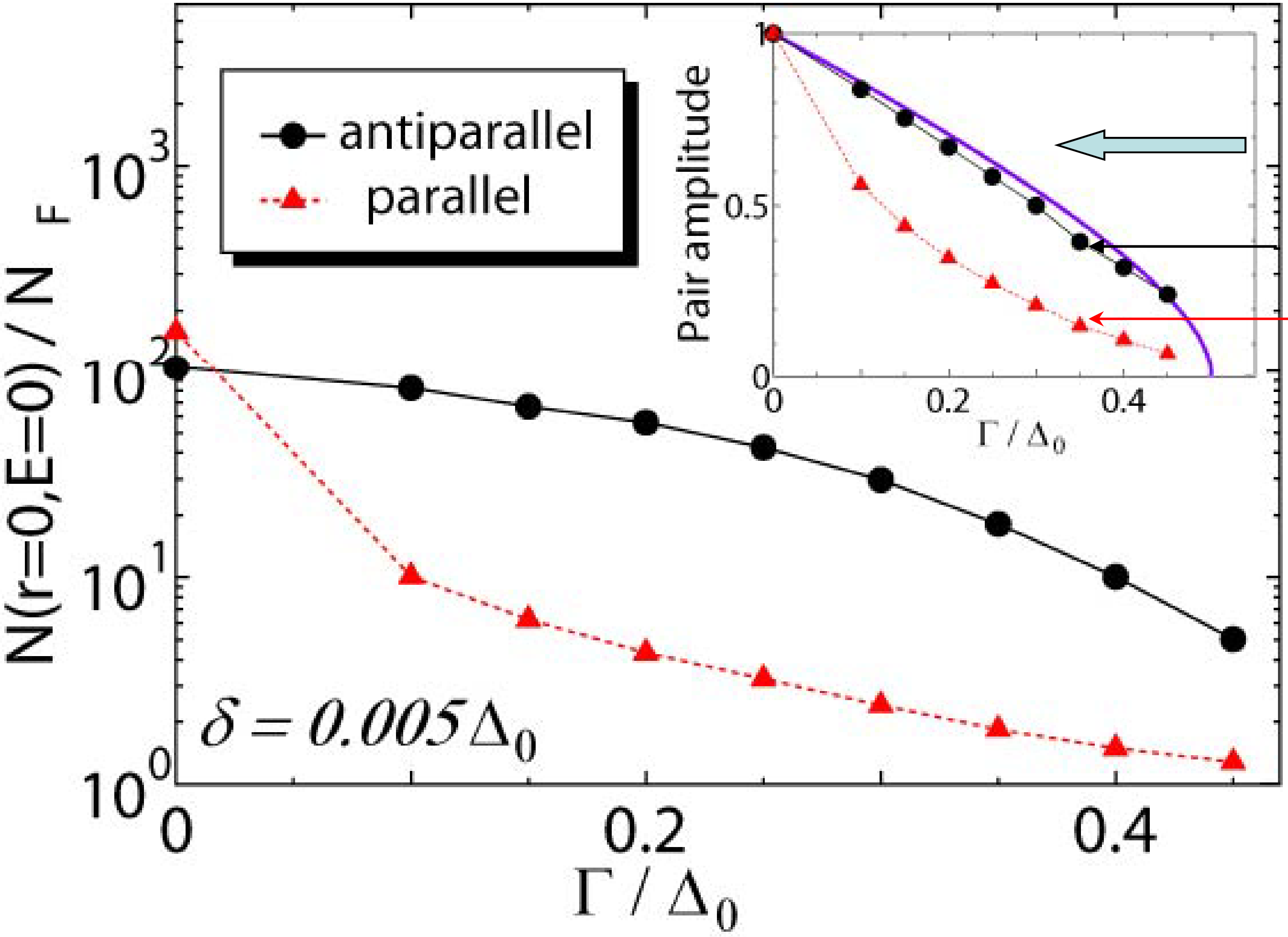
Odd-frequency s-wave

Odd-frequency d-wave

Odd frequency s-wave state; more robust against the impurity scattering **Tanuma, Hayashi, Tanaka Golubov, PRL 102 117003 (2009).**

Chirality and vorticity: Y. Kato and N. Hayashi (2000, 2001,2002)

Impurity scattering



Abrikosov Gor’kov plot

Odd-s-wave pair

Odd d-wave pair

Chirality and vorticity: Y. Kato and N. Hayashi (2000, 2001,2002)

Chiral domain and vortex

Parallel vortex	**Antiparallel vortex**	**Parallel vortex**	**Antiparallel vortex**
●	●	●	●
Odd d-wave	**Odd s-wave**	**Odd d-wave**	**Odd s-wave**
$p_x + ip_y$	$p_x - ip_y$	$p_x + ip_y$	$p_x - ip_y$
Weak ZEP	**Strong ZEP**	**Weak ZEP**	**Strong ZEP**

We can detect the presence of chiral domain by vortex spectroscopy via odd-frequency Cooper pair.

Phys. Rev. Lett. 102 117003 (2009).

Summary (vortices)

(1)If we consider Abrikosov vortex (*m*=1), only the odd-frequency Cooper pair is possible at the center of the vortex core.

Physical Review B, Vol. 78, 012508, 2008

(2)Vortex core spectroscopy in chiral p-wave superconductor in the presence of impurity enables us to identify the presence of chirality and the odd-frequency pairing.

Phys. Rev. Lett. 102, 117003 (2009).

Summary

(1) Ubiquitous presence of the odd-frequency pairs in inhomogeneous systems.

(2) Low energy Andreev bound states can be expressed in terms of odd-frequency pairing (proximity effect and vortices).

(3) Odd-frequency Cooper pairing is realized at the center of a vortex core -> allows to identify the presence of chirality