PSM2010
INTERNATIONAL SYMPOSIUM ON PHYSICS OF NEW QUANTUM PHASES IN SUPERCLEAN MATERIALS
MARCH 9-12, 2010 YOKOHAMA, JAPAN

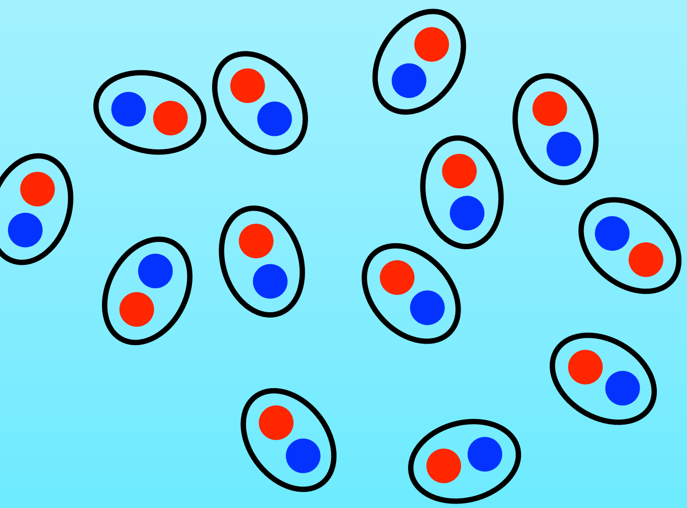
March 10, 2010

Quantum/Spin Liquids, Geometrical phases & Edge States

"Zoo of insulators & Edge states"

Yasuhiro Hatsugai

Institute of Physics
University of Tsukuba
JAPAN



$$i\gamma = \int dA = \int \langle\psi|d\psi\rangle$$

University of Tsukuba
筑波大学

Collaborators

Project :
"Topological order in quantum/spin liquids"

I. Maruyama : Osaka Univ.

M. Arikawa : Univ. of Tsukuba

S. Tanaya : Univ. of Tsukuba

Are insulators boring ??

- Metal is useful. copper, silver, gold: good conductors

  Lots of applications

- Metal is simple (if free)

  unstable against for perturbation (without some protection or fine tuning)

  "high energy" effective theory ?

with interaction: complicated
Anomalous metals, etc Critical : RG
Spin analogue (Gapless spin liquid) is tricky.

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- Insulators : Gapped
  - Band insulators
  - Superconductors
  - Integer & Fractional Quantum Hall States
  - Integer spin chains (Haldane)
  - Dimer Models (Shastry-Sutherland)
  - Valence bond solid (VBS) states
  - Half filled Kondo Lattice
  - Spin Hall insulators
  - Kitaev model & string net

Energy gap above the ground state

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Quantum/spin liquids (gapped)

Are insulators boring ??

- ☆ Insulators : Non metal, gapped

Gapped: Nothing in the gap : cf. Nambu-Goldstone boson

No low lying excitations

No Response against small perturbation

??

? ? ?

~~gapless modes:
acoustic phonons
zero sounds
spin waves~~

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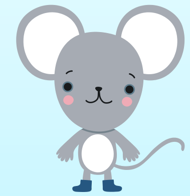
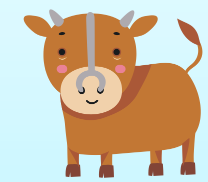

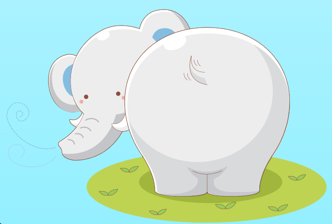

Zoo

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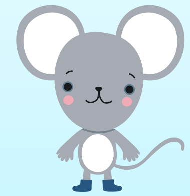

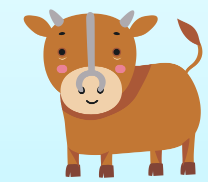

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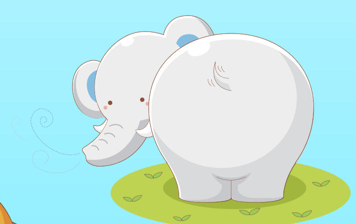

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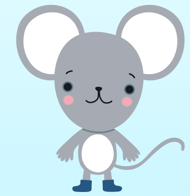

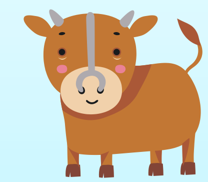

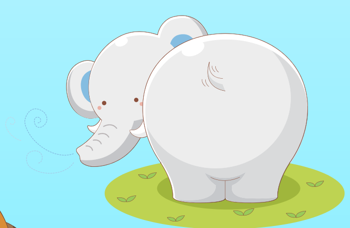

Topological Order

X.G.Wen '89

Zoo

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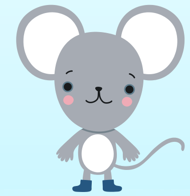

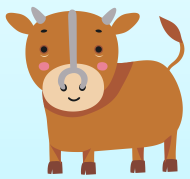

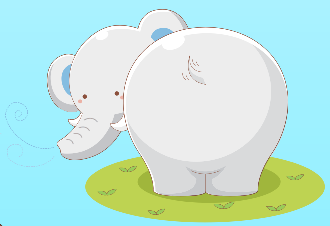

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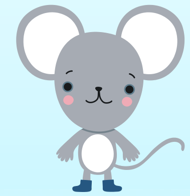

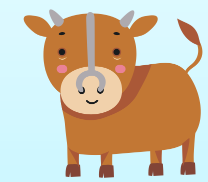

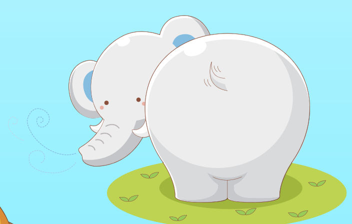

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☑ Berry connection:geometrical phase

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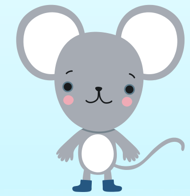
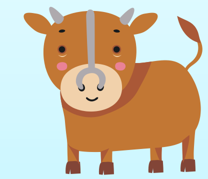

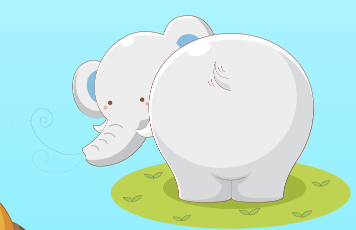

Topological Order
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Zoo

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- ☐ Topological order
- ☑ Berry connection:geometrical phase
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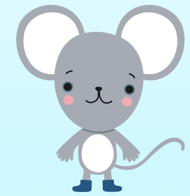

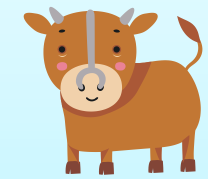

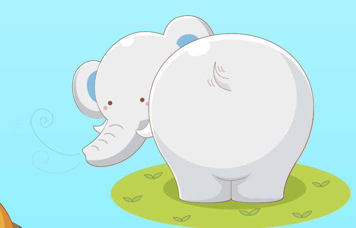

Topological Order

X.G.Wen '89

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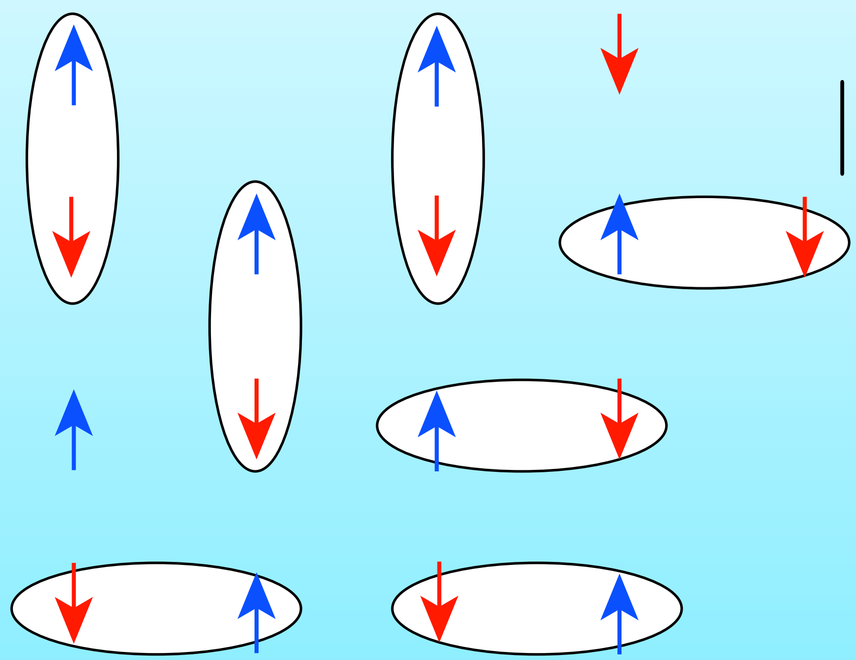
Local quantum object to characterize spin liquid



$$|\text{Singlet Pair}_{12}\rangle = \frac{1}{\sqrt{2}}(|\uparrow_1\downarrow_2\rangle - |\downarrow_1\uparrow_2\rangle)$$

$$|G\rangle = \sum_{J=\text{Dimer Covering}} c_J \otimes_{ij} |\text{Singlet Pair}_{ij}\rangle$$

Anderson

Local Singlet Pairs :

(Basic Objects)

Singlet : quantum order parameter

DO NOT NEED ANY symmetry breaking

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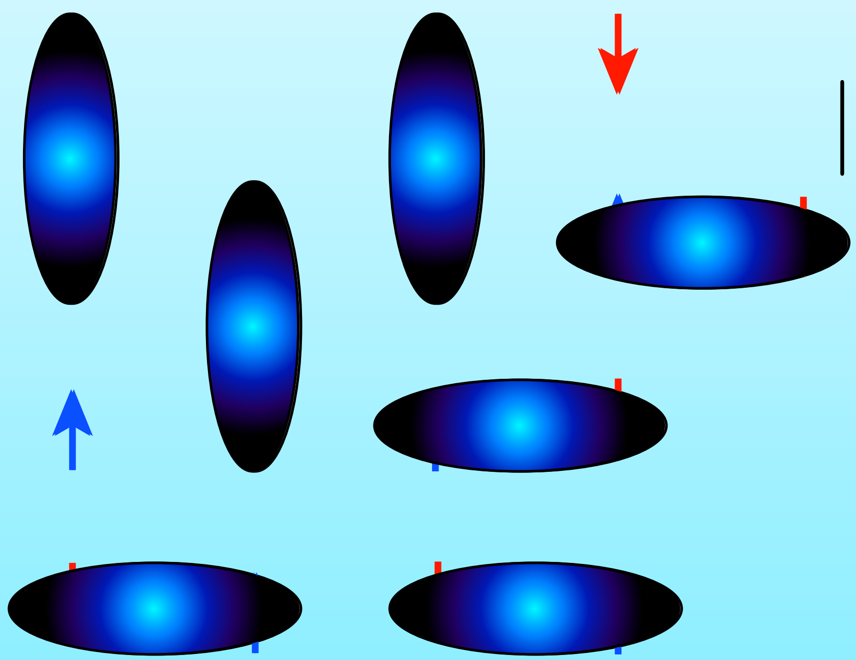

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Order formation

Landu-Ginzburg-Wilson scenario

Symmetry breaking & Long range order

local order parameter

LGW to Quantum Scenario

Landu-Ginzburg-Wilson scenario

Symmetry breaking & Long range order

local order parameter

Quantum Order parameter

Quantum Objects to construct the state

Use of Quantum interference

Do not need symmetry breaking

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LGW to Quantum Scenario

L

Quantum/Spin liquids: Collection of weakly coupled quantum local objects

Shastry-Sutherland '81

Quantum Order parameter

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LGW to Quantum Scenario

L

Quantum/Spin liquids: Collection of weakly coupled quantum local objects

Shastry-Sutherland '81

Topological quantities for quantum order parameters of quantum objects

Use of Quantum interference

Do not need symmetry breaking

Singlet

Classical to Quantum (for characterization)

- "Classical" Observables — Unitary invariant
  - Charge density, Spin density,... $\mathcal{O} = n_\uparrow \pm n_\downarrow, \cdots$

$$\langle \mathcal{O} \rangle_G = \langle G|\mathcal{O}|G\rangle = \langle G'|\mathcal{O}|G'\rangle = \langle \mathcal{O}\rangle_{G'}$$ charge, spin, ...

$$|G'\rangle = |G\rangle e^{i\phi}$$

- "Quantum" Observables ! — depend on the phase of the state
  - Quantum Interferences: $\langle G_1|G_2\rangle = \langle G_1'|G_2'\rangle e^{i(\phi_1-\phi_2)}$
    - Aharonov-Bohm Effects $|G_i\rangle = |G_i'\rangle e^{i\phi_i}$
    - Berry phases

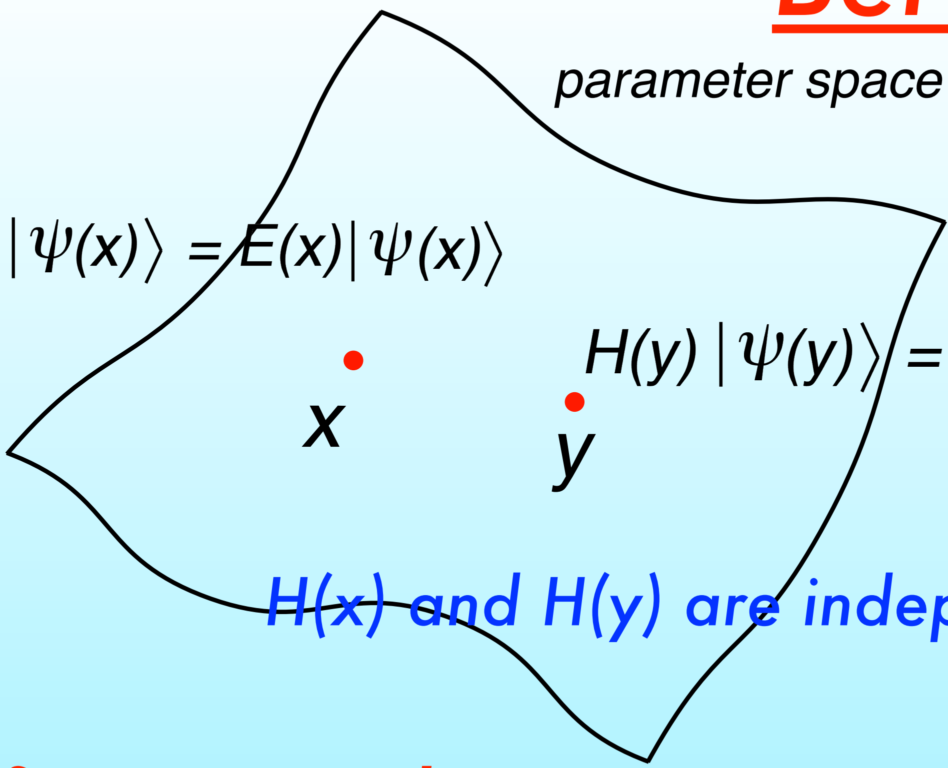
$$\langle G|G+dG\rangle = 1 + \langle G|dG\rangle$$

$A = \langle G|dG\rangle$:Berry Connection

$i\gamma = \int A$:Berry Phase

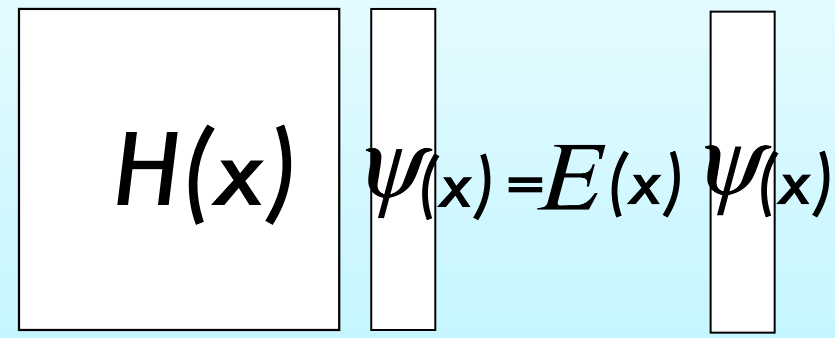
Use Quantum observables for the characterization

Berry phase



Fiber Bundle

Eigenvectors (space) with Parameters



(Abelian)

Information between nearby states

Berry connection : $A_\psi = \langle\psi|d\psi\rangle = \langle\psi|\frac{d}{dx}\psi\rangle dx.$

Gauge Transformation

$$|\psi(x)\rangle = |\psi'(x)\rangle e^{i\Omega(x)}$$

$$A_\psi = A'_\psi + id\Omega = A'_\psi + i\frac{d\Omega}{dx}dx$$

gauge potential

Geometrical quantities

$$i\gamma_C(A_\psi) = \int_C A_\psi \quad \text{: Berry phase}$$

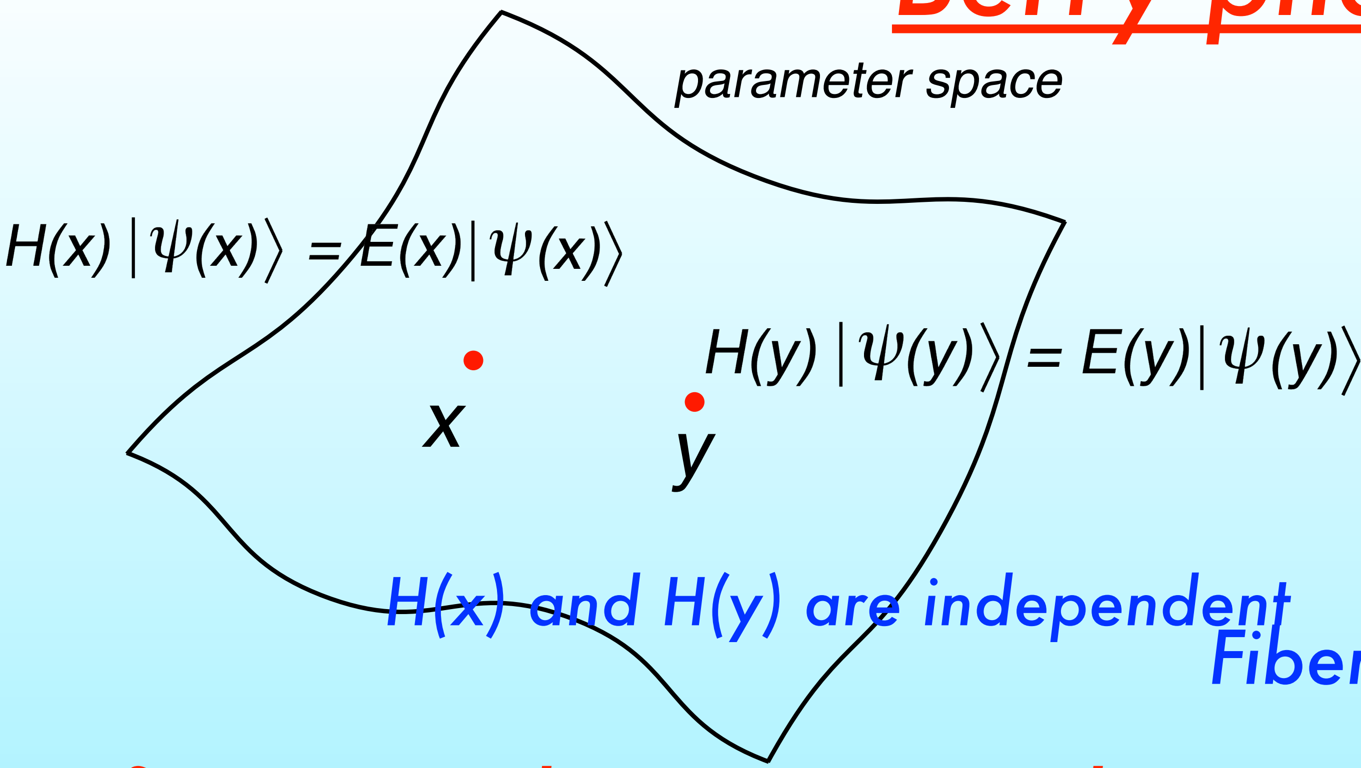
$$\gamma_C(A_\psi) = \gamma_C(A_{\psi'}) + \int_C d\Omega \leftarrow 2\pi\times(\text{integer}) \text{ if } e^{i\Omega} \text{ is single valued}$$

$$\gamma_C(A_\psi) \equiv \gamma_C(A_{\psi'}) \mod 2\pi$$

only well-defined in mod 2π

Berry phase

Eigenvectors (space) with Parameters



Fiber Bundle

$$H(x)\,\psi(x) = E(x)\,\psi(x)$$

(Abelian)

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Anti-Unitary Invariant State and Z_2 Berry Phase

$\Theta_N^2 = 1$

- Anti-Unitary Symmetry $[H(x), \Theta] = 0$
- Invariant State $^\exists\varphi, \quad |\Psi^\Theta\rangle = \Theta|\Psi\rangle = |\Psi\rangle e^{i\varphi}$
  - ex. Unique Eigen State $\simeq |\Psi\rangle$ Gauge Equivalent (Different Gauge)
- To be compatible with the ambiguity, the Berry Phases have to be quantized as

$\mathbb{Z}_2$ Berry phase

$$\gamma_C(A^\Psi) = \begin{cases} 0 \\ \pi \end{cases} \mod 2\pi$$

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$i^2 = j^2 = k^2 = ijk = -1$

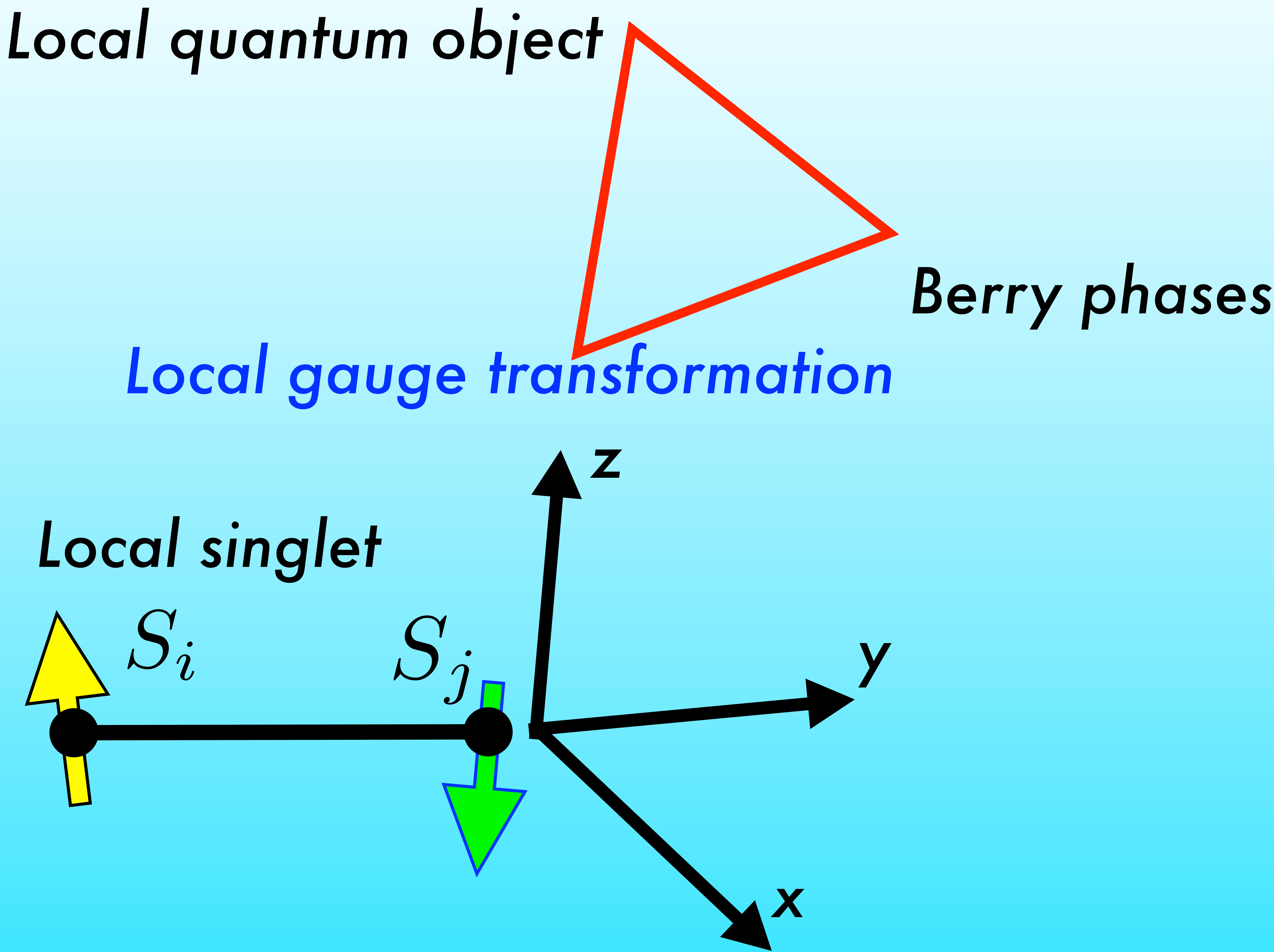
Also quaternionic generalization with Kramers degeneracy $\Theta_N^2 = -1$

$$\gamma_3 = -\frac{1}{8\pi^2}\int_{S^3} \omega_3$$

$$\omega_3 = \mathrm{Tr}\,(AdA + \frac{2}{3}A^3)$$

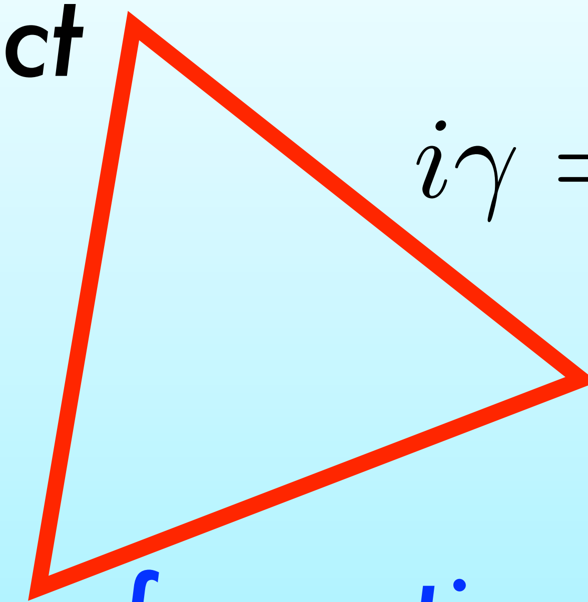
YH '09

How to characterize the local quantum object



How to characterize the local quantum object

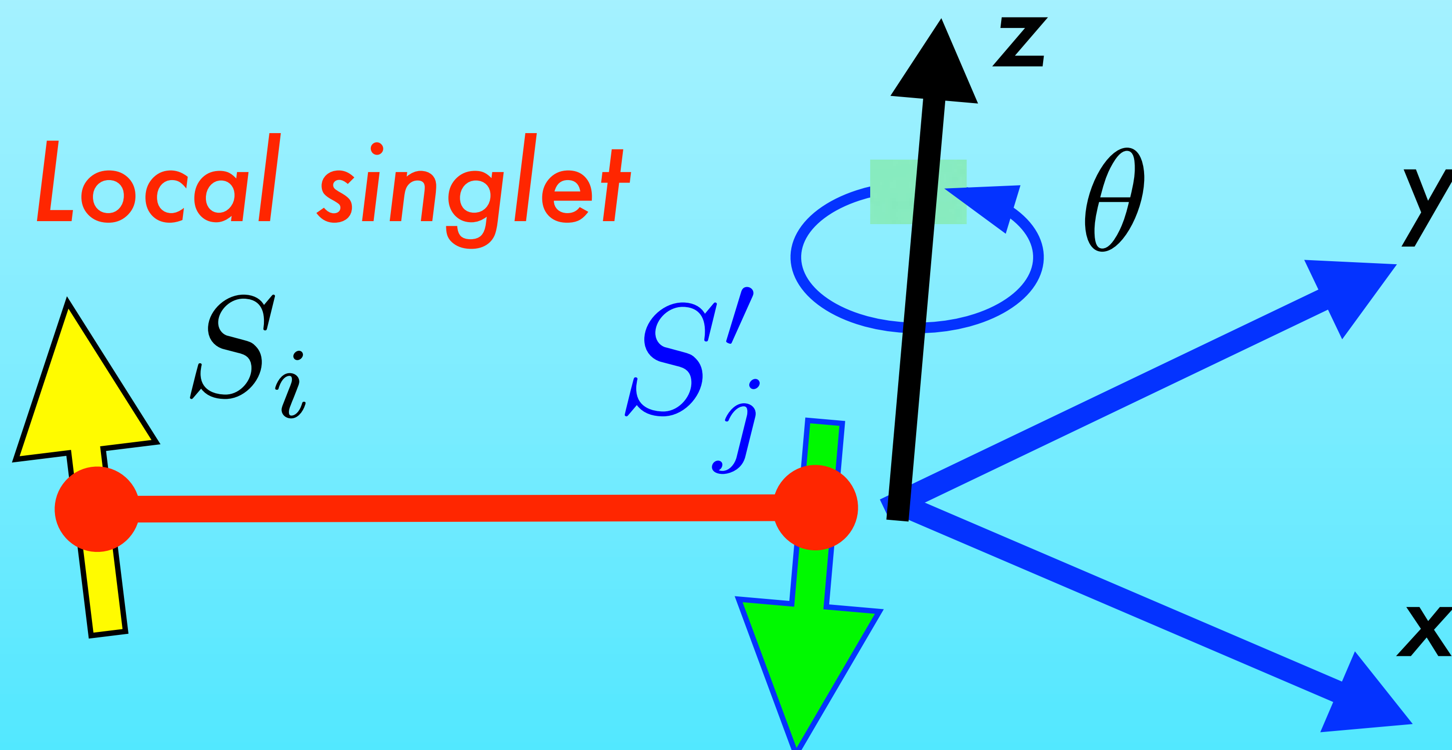
Local quantum object



$$i\gamma = \int dA = \int \langle \psi | d\psi \rangle$$

Berry phases

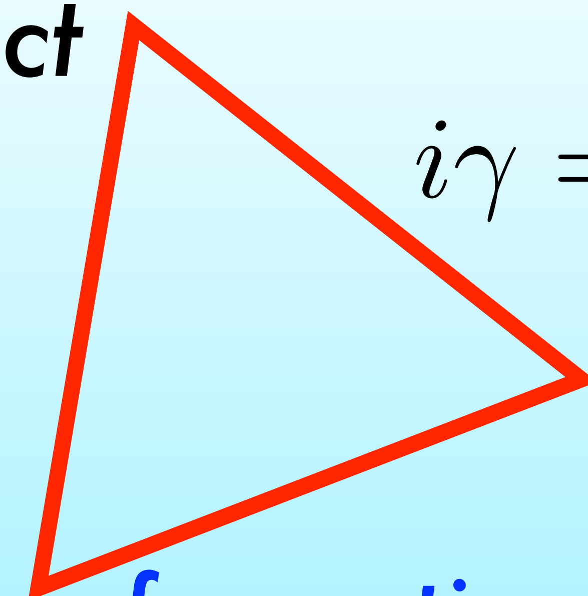
Local gauge transformation



$$\boldsymbol{S}_i \cdot \boldsymbol{S}_j \rightarrow \boldsymbol{S}_i \cdot \boldsymbol{S}'_j = \frac{1}{2}(e^{-i\theta} S_{i+} S_{j-} + e^{+i\theta} S_{i-} S_{j+}) + S_{iz} S_{jz}$$

How to characterize the local quantum object

Local quantum object



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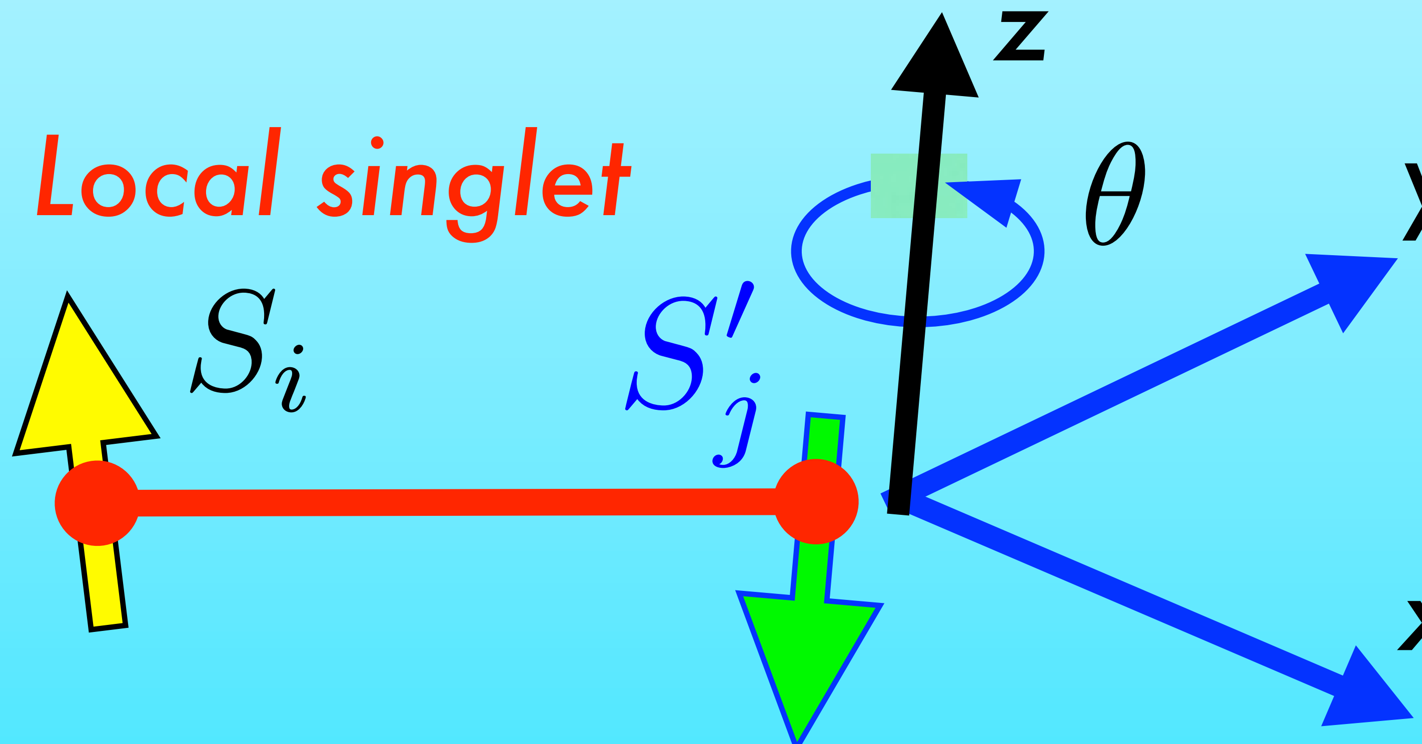


$$|\psi(\theta)\rangle = U(\theta)|\psi(0)\rangle$$

$$U(\theta) = e^{i(S-S_z)\theta}$$

$$A = \langle \psi | d\psi \rangle = S d\theta$$

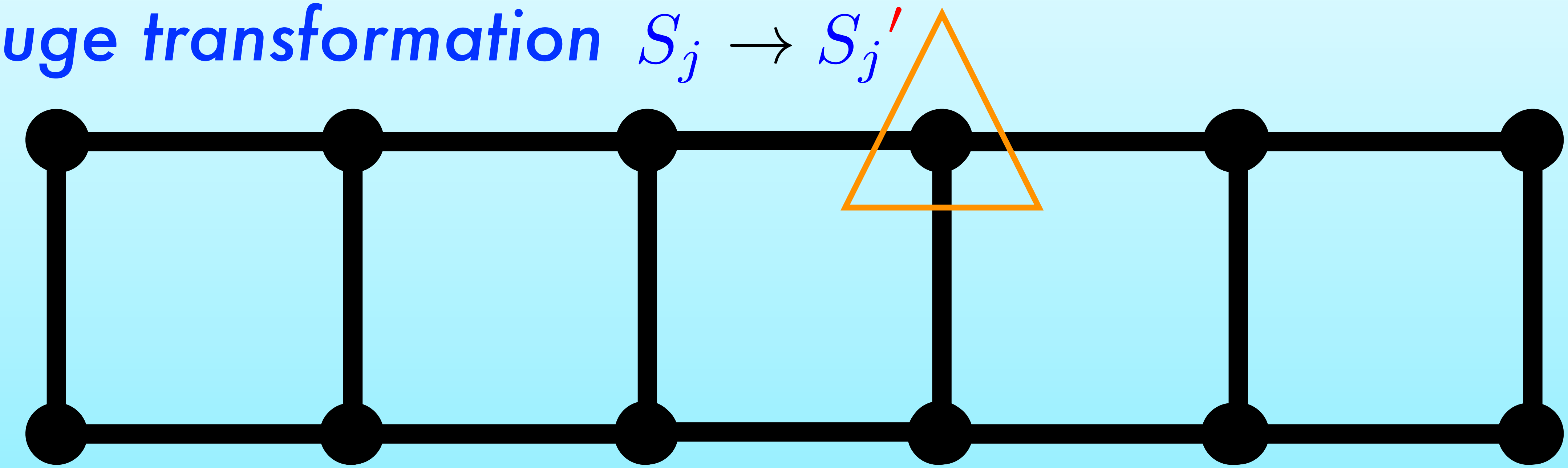
$$\gamma = 2\pi S = \pi$$

$$S = 1/2$$

$$\boldsymbol{S}_i \cdot \boldsymbol{S}_j \to \boldsymbol{S}_i \cdot \boldsymbol{S}'_j = \frac{1}{2}(e^{-i\theta} S_{i+} S_{j-} + e^{+i\theta} S_{i-} S_{j+}) + S_{iz} S_{jz}$$

Local object in a many spin system

Gauge transformation $S_j \to S_j{}'$



Local object in a many spin system

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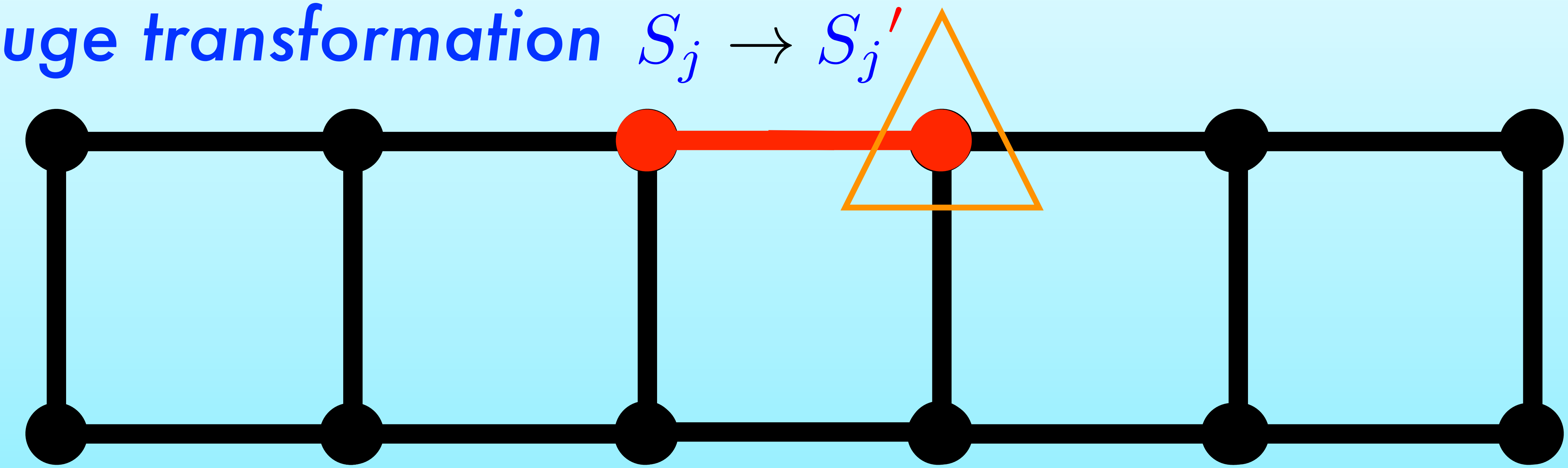

If the fundamental quantum object is a link,

Local object in a many spin system

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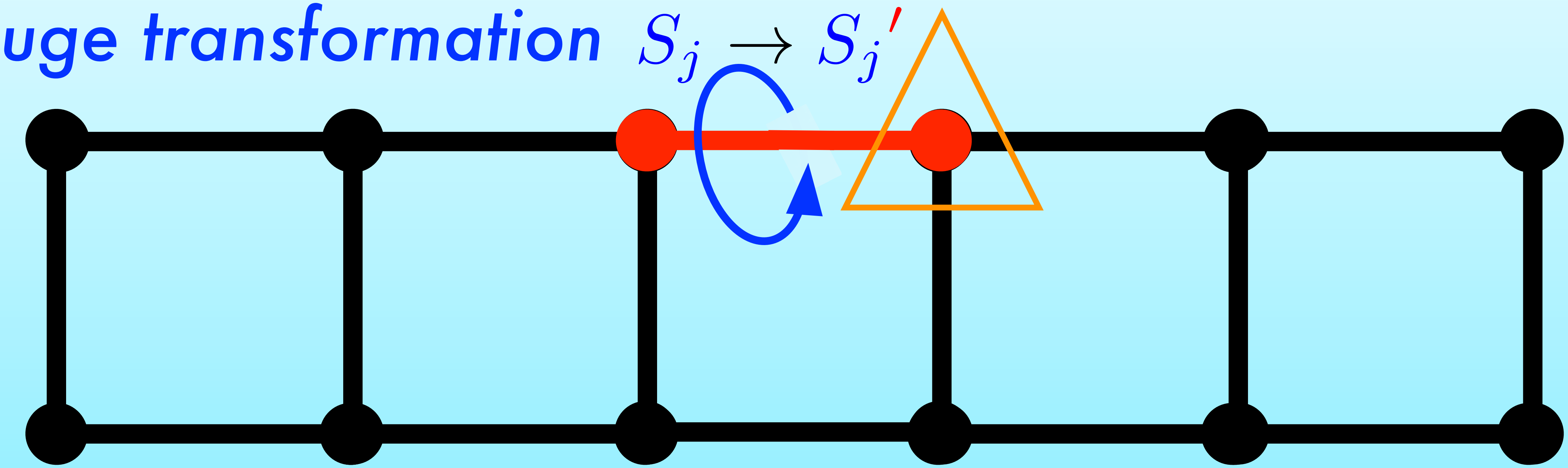


If the fundamental quantum object is a link,

modify the exchange only at the link

to define the Berry phase

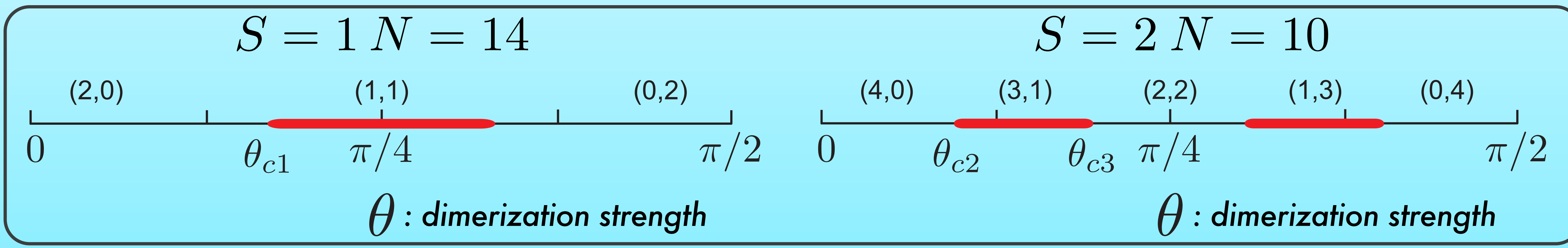
Topological Classification of Gapped Spin Chains

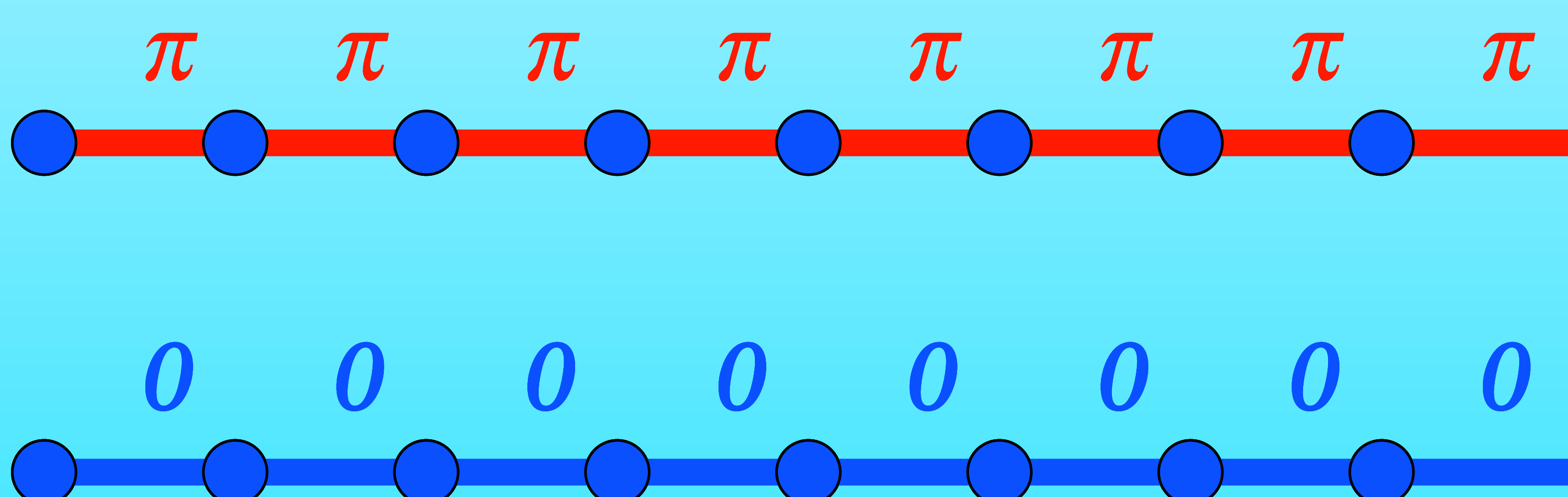
T.Hirano, H.Katsura &YH, Phys.Rev.B77 094431'08

- S=1,2 dimerized Heisenberg model

$$H = \sum_{i=1}^{N/2} (J_1 \boldsymbol{S}_{2i} \cdot \boldsymbol{S}_{2i+1} + J_2 \boldsymbol{S}_{2i+1} \cdot \boldsymbol{S}_{2i+2}) \qquad J_1 = \cos\theta, J_2 = \sin\theta$$

Z_2 Berry phase

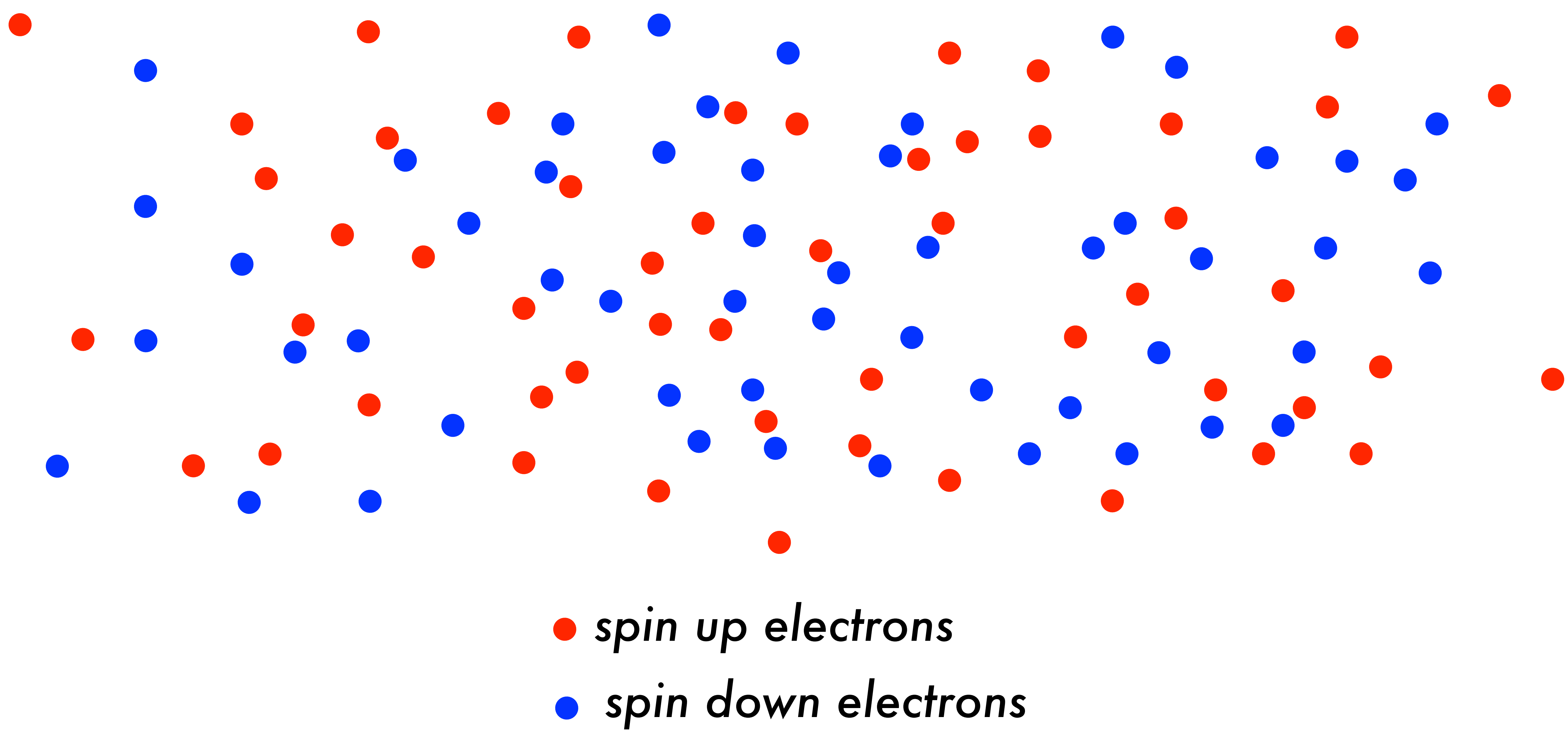




recursive transitions with dimerization

Topological Quantum Phase Transitions with translation invariance

BEC-BCS crossover as a local quantum phase transition

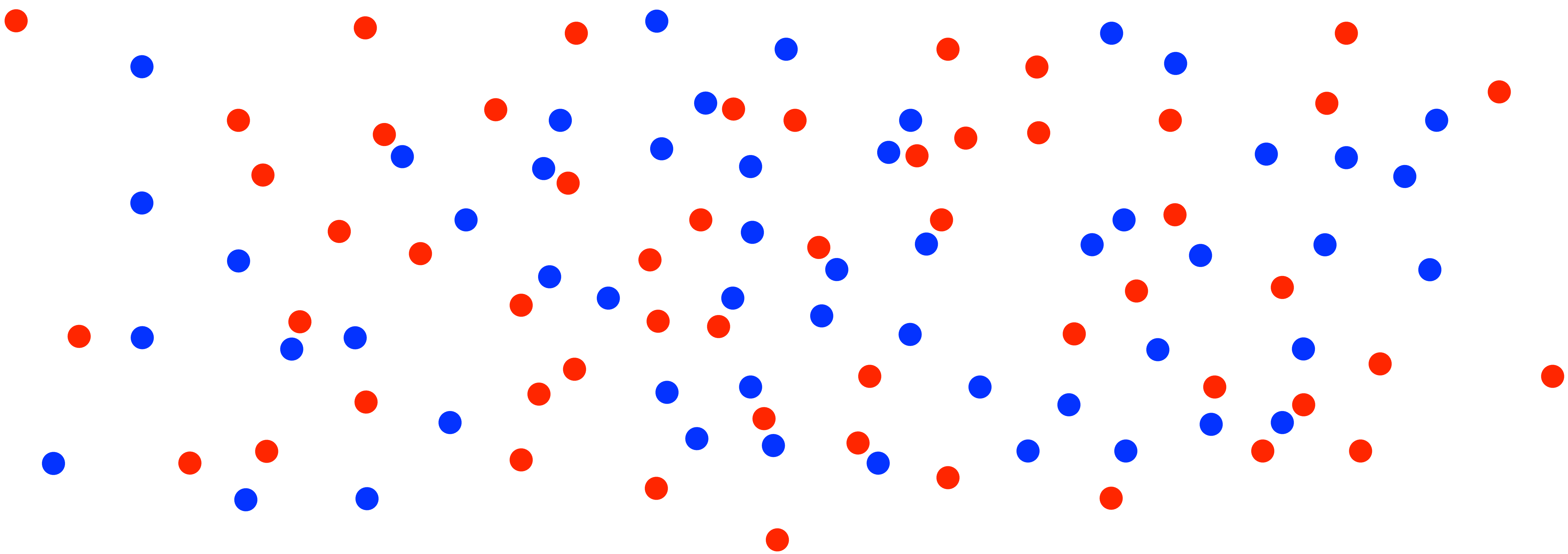


M. Arikawa, I. Maruyama and Y. Hatsugai, arXiv:1003.4735

PSM2010, Yokohama, March 10, 2010

BEC-BCS crossover as a local quantum phase transition

Switching on attractive interaction among particles

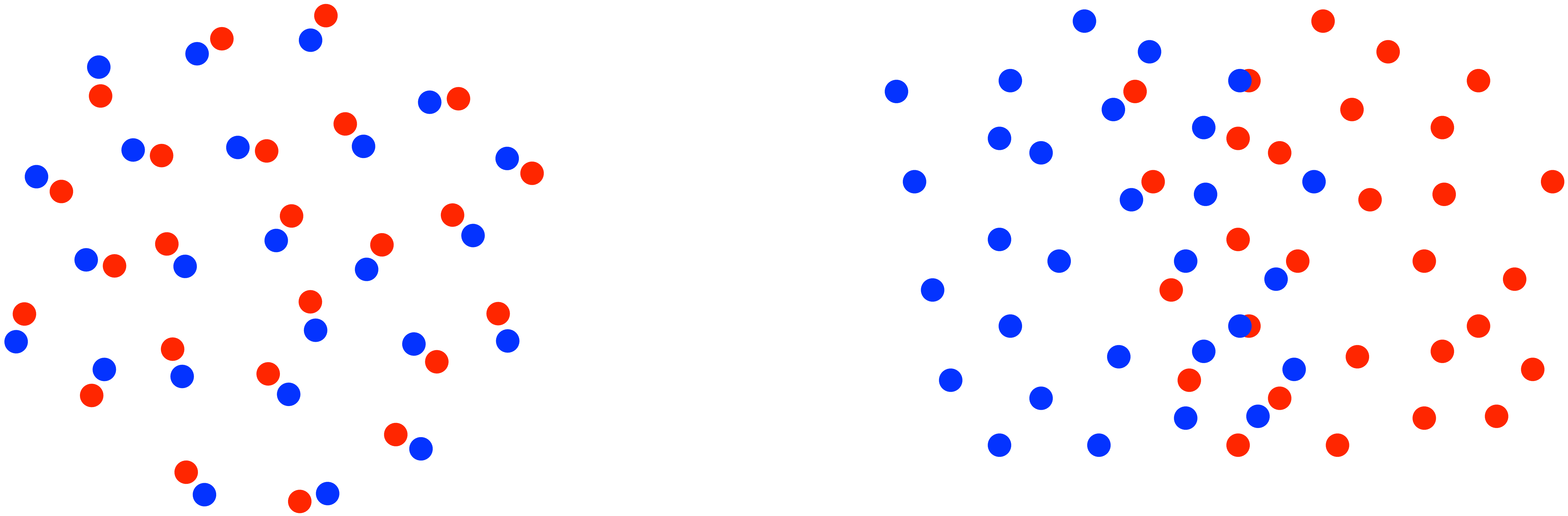


● spin up electrons

● spin down electrons

M. Arikawa, I. Maruyama and Y. Hatsugai, arXiv:1003.4735

BEC-BCS crossover as a local quantum phase transition



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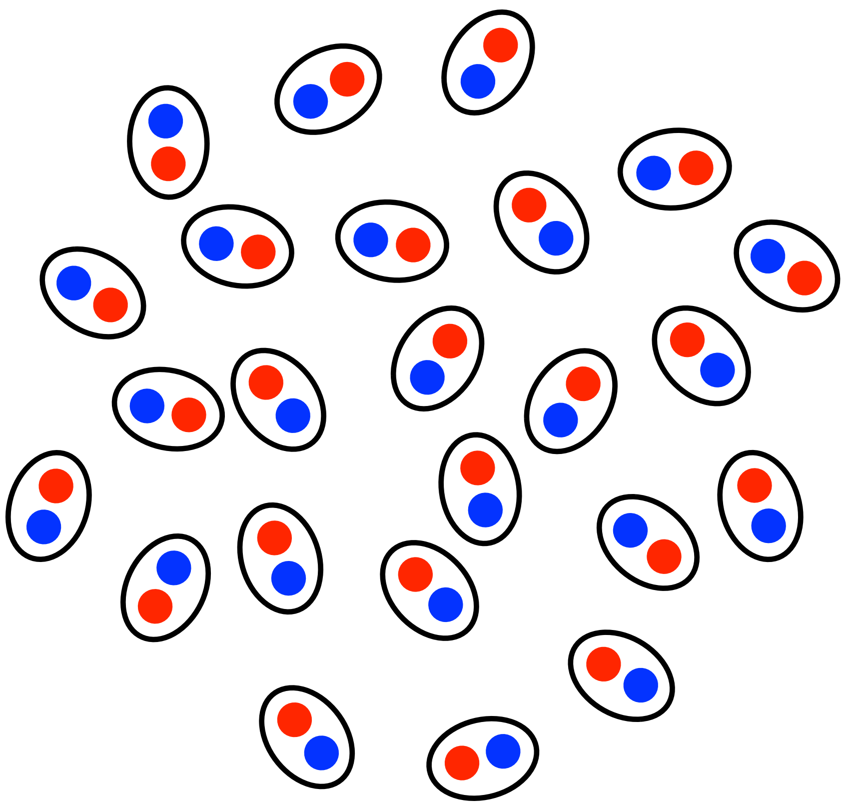
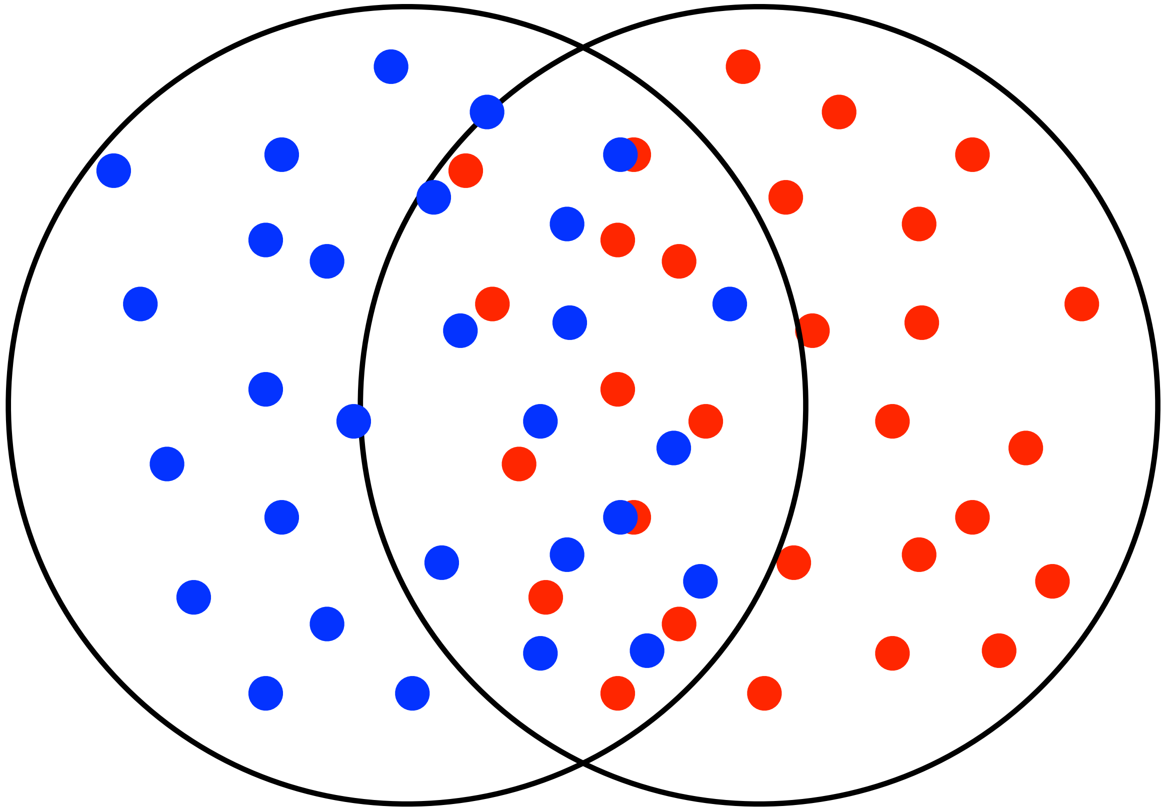
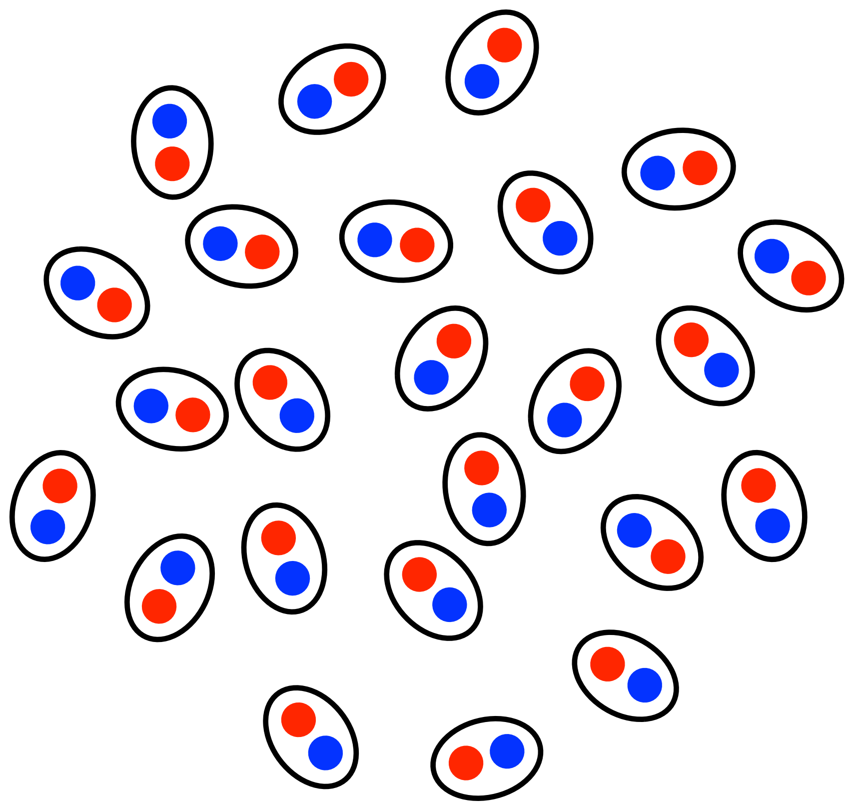
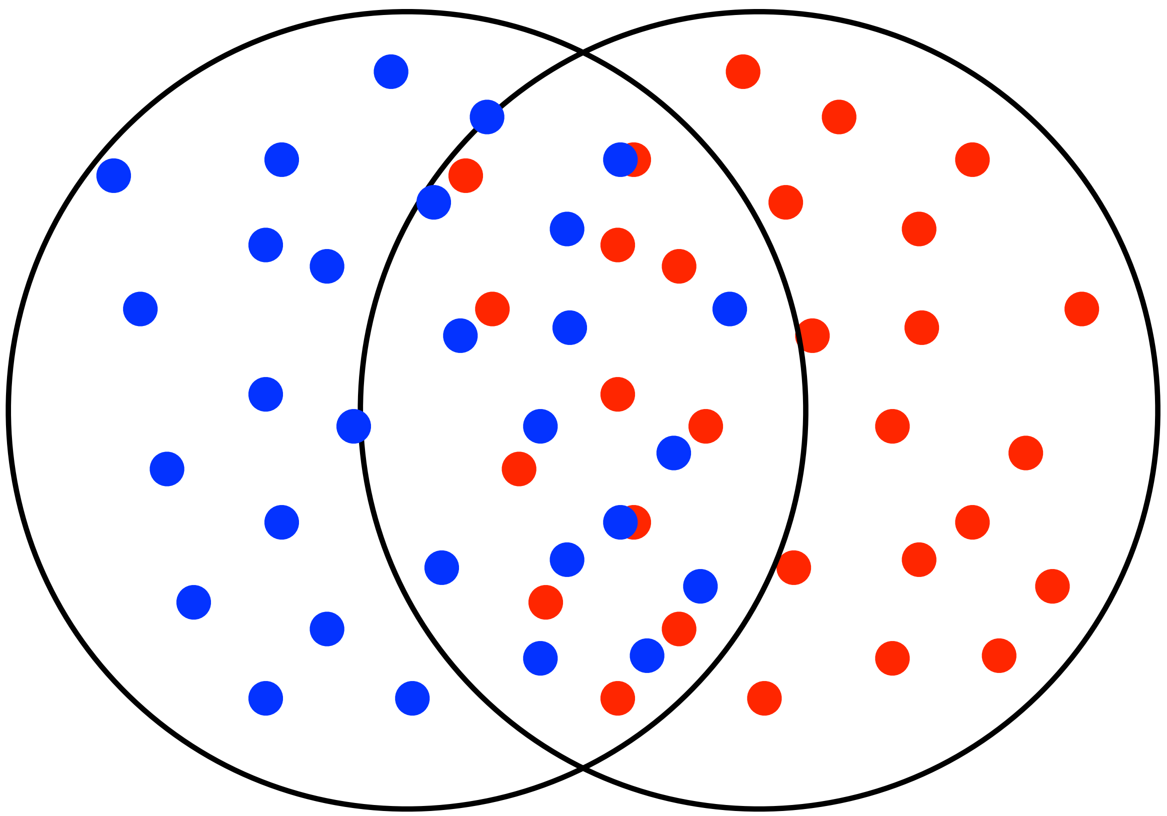




BEC-BCS crossover as a local quantum phase transition

BEC : strong coupling



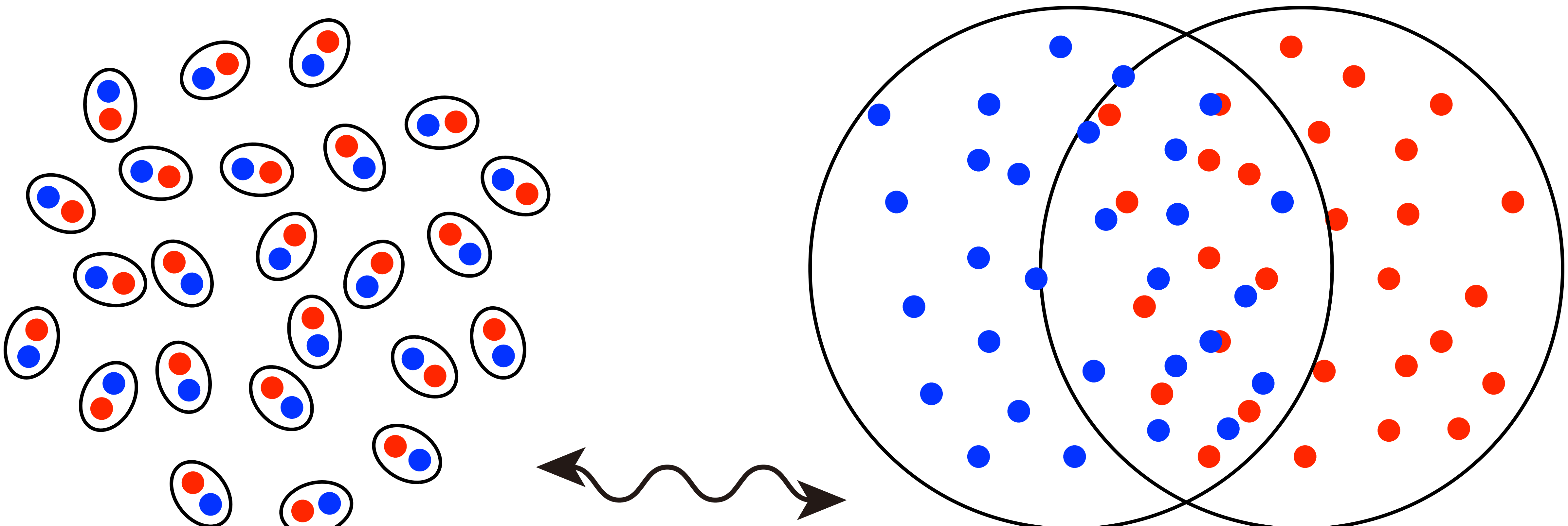


Making bosons in real space then condense

BEC-BCS crossover as a local quantum phase transition

BEC : strong coupling

BCS : weak coupling



Crossover

adiabatically connected

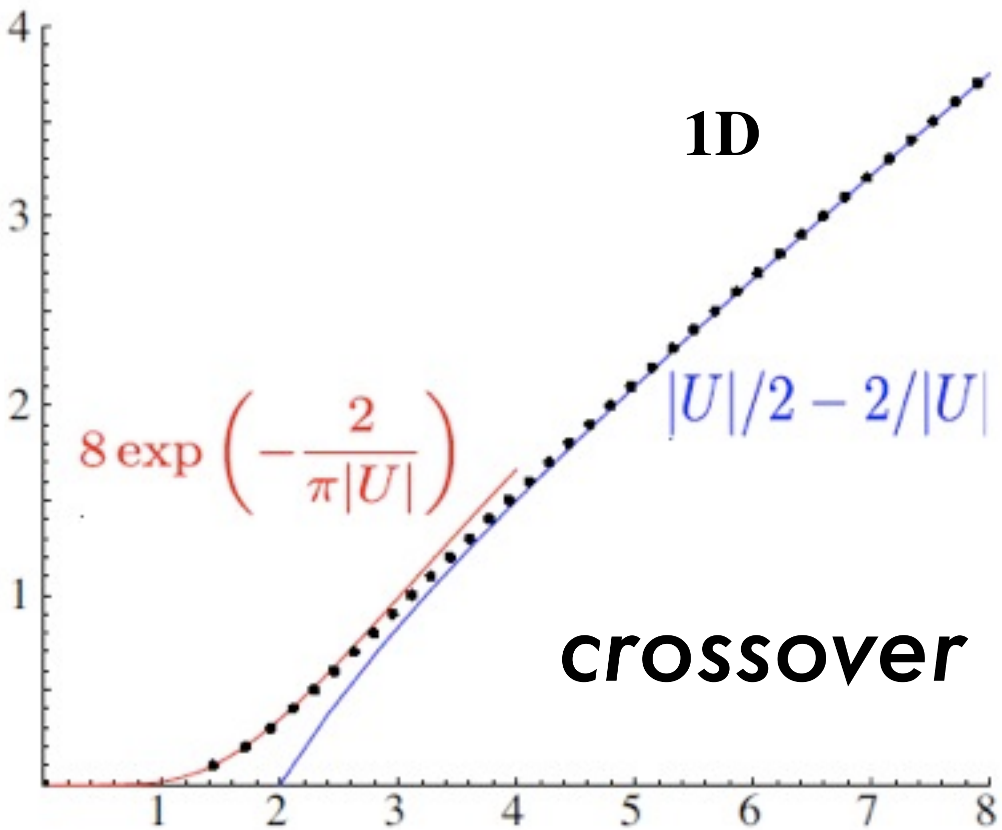
Making bosons in real space then condense

Cooper pairing in momentum space

BCS Model at half filling

$$H = -t \sum_{\sigma,i,j} c_{i\sigma}^{\dagger} c_{j\sigma} - |U| \sum_{ij} \Delta_{ij} c_{i\uparrow} c_{i\downarrow}$$

Arikawa-Maruyama-YH, 2010



crossover : gapped always

weak coupling (BCS)

strong coupling (BEC)

M. Arikawa, I. Maruyama and Y. Hatsugai, arXiv:1003.4735

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Arikawa-Maruyama-YH, 2010

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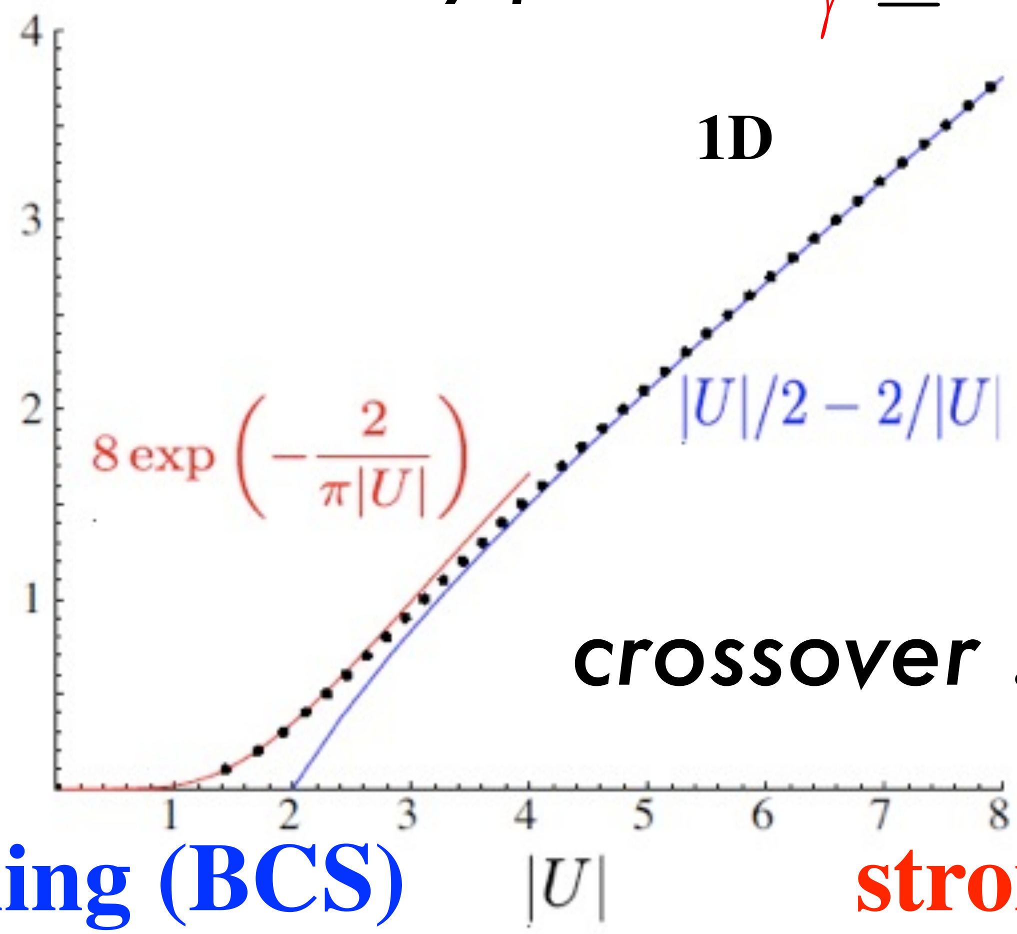
modify only at special (local) order parameter $\Delta_{ij} \to \Delta_{ij} e^{i\theta}$ $(\theta : 0 \to 2\pi)$

to calculate the Berry phase $\gamma = -i \int_0^{2\pi} d\theta \, \langle \psi | \partial_\theta \psi \rangle$

$|\psi\rangle$ manybody state



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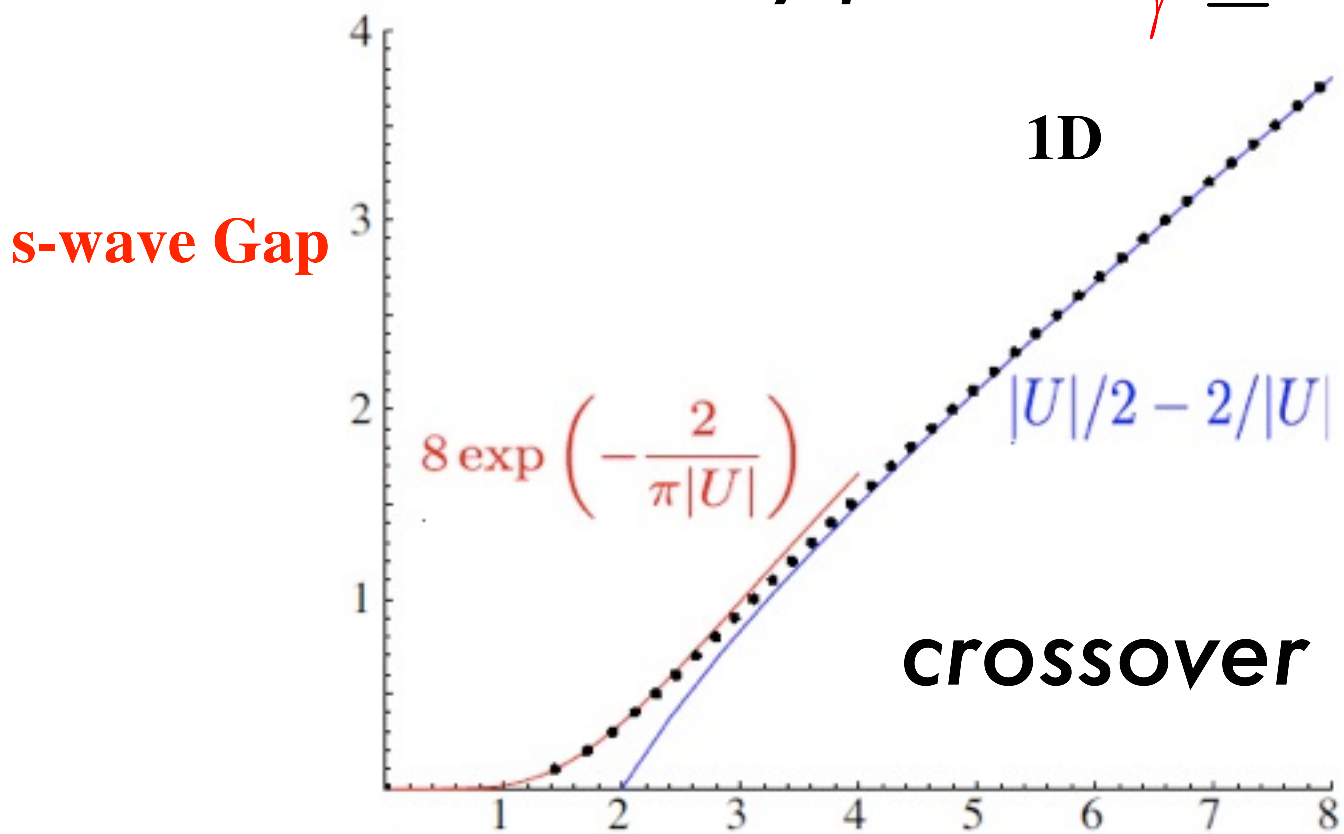

crossover : gapped always

weak coupling (BCS) $|U|$ **strong coupling (BEC)**

$\gamma = 0$ $\gamma = \pi$

$|U_C|/t = 2/\sqrt{3} = 1.15, \; 1.25, \; 1.6$

1D 2D 3D

M. Arikawa, I. Maruyama and Y. Hatsugai, arXiv:1003.4735

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Arikawa-Maruyama-YH, 2010

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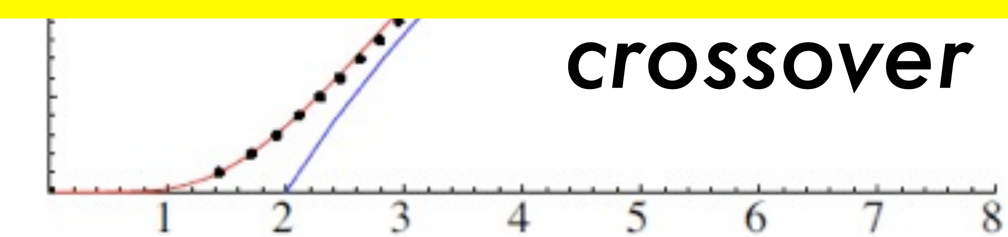
Crossover of the bulk
by Quantum Phase transition with local gauge twist



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March 10, 2010

Summary

- Quantum/Spin liquid as a zoo of insulators
- Berry phase as a quantum interference for the characterization

Singlets & BEC-BCS crossover