Localized spin systems : from BEC to Luttinger liquids

T. Giamarchi

http://dpmc.unige.ch/gr_giamarchi/

UNIVERSITÉ DE GENÈVE

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FONDO NAZIONALE SVIZZERO
SWISS NATIONAL SCIENCE FOUNDATION

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D. Poilblanc ,
S Capponi (Toulouse)
A. Laüchli (Dresden)
B. Normand

Quantum magnetism

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- Highly non trivial physics and phases

Itinerant quantum systems

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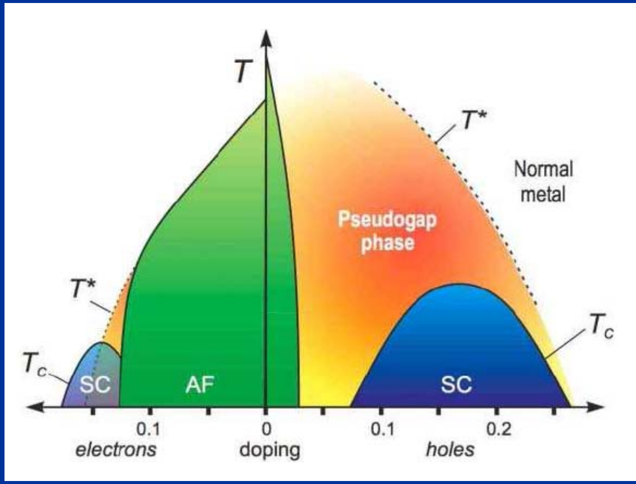

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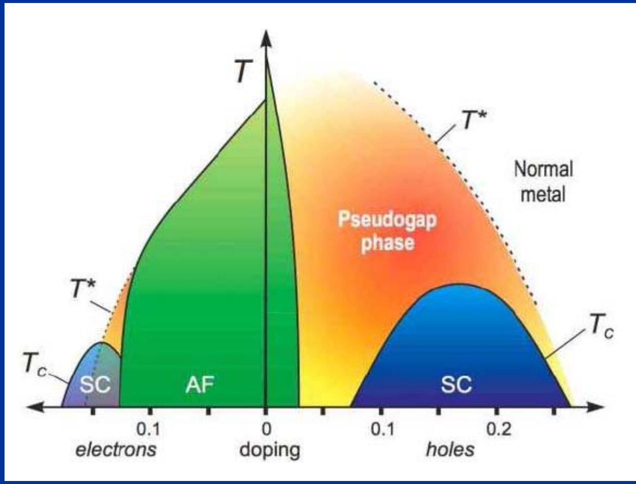

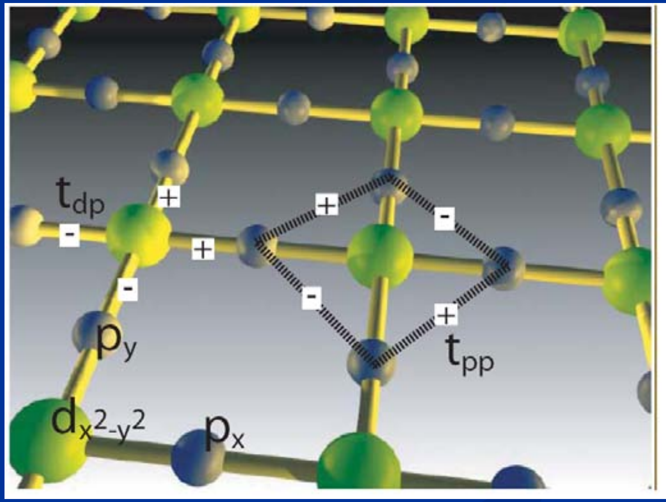

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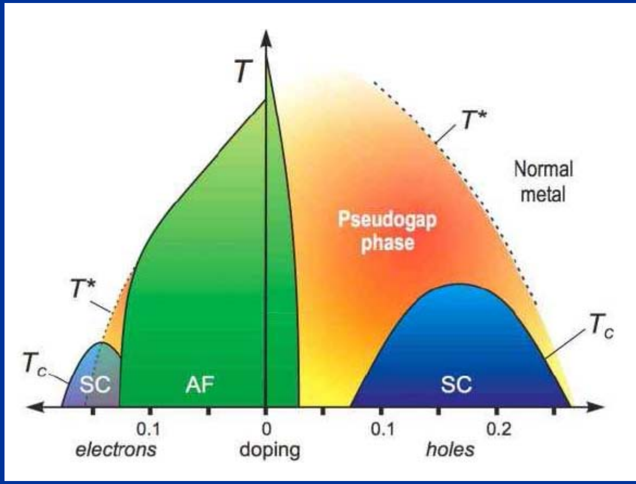

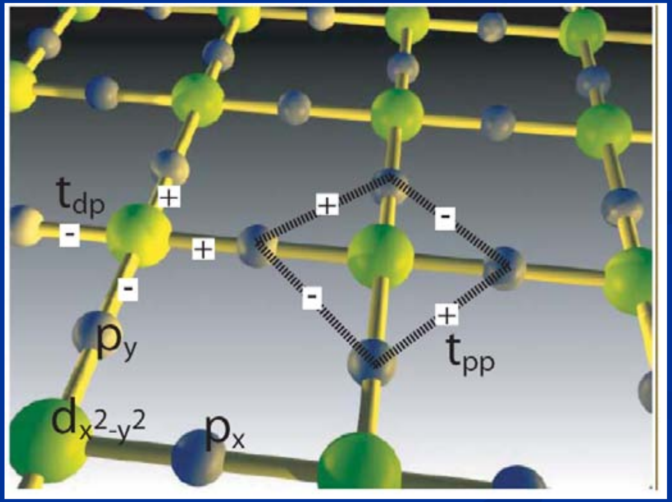

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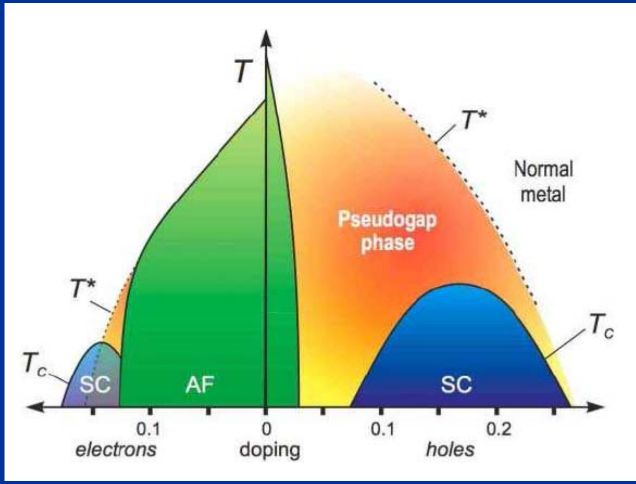

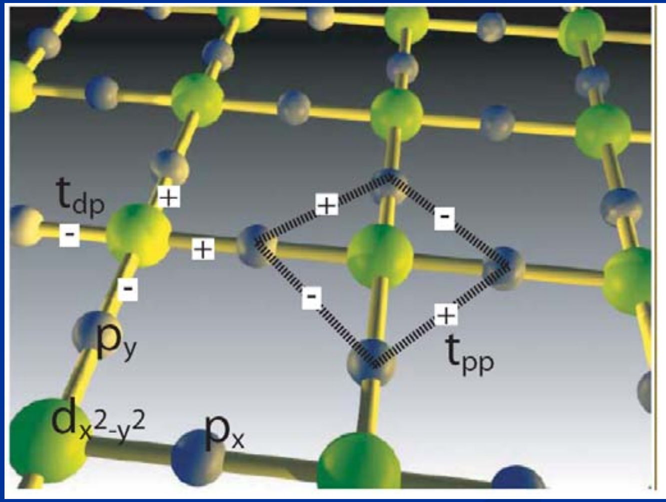

- Physics: artefact of the approximations used ?
- Use localised spin systems as controlled realizations

Dimer systems

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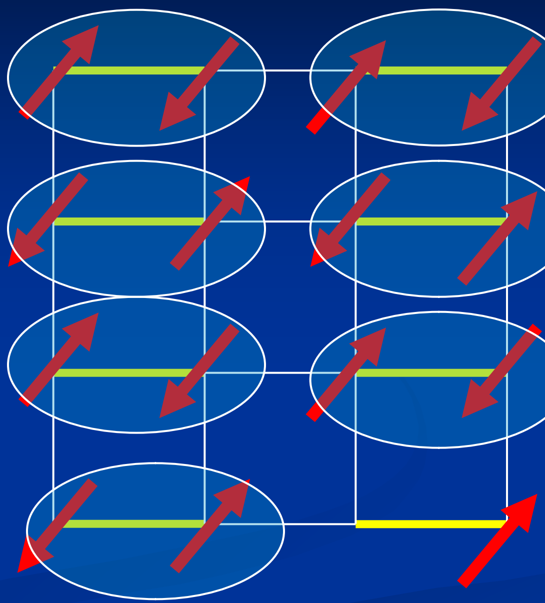
$J_r \gg J$



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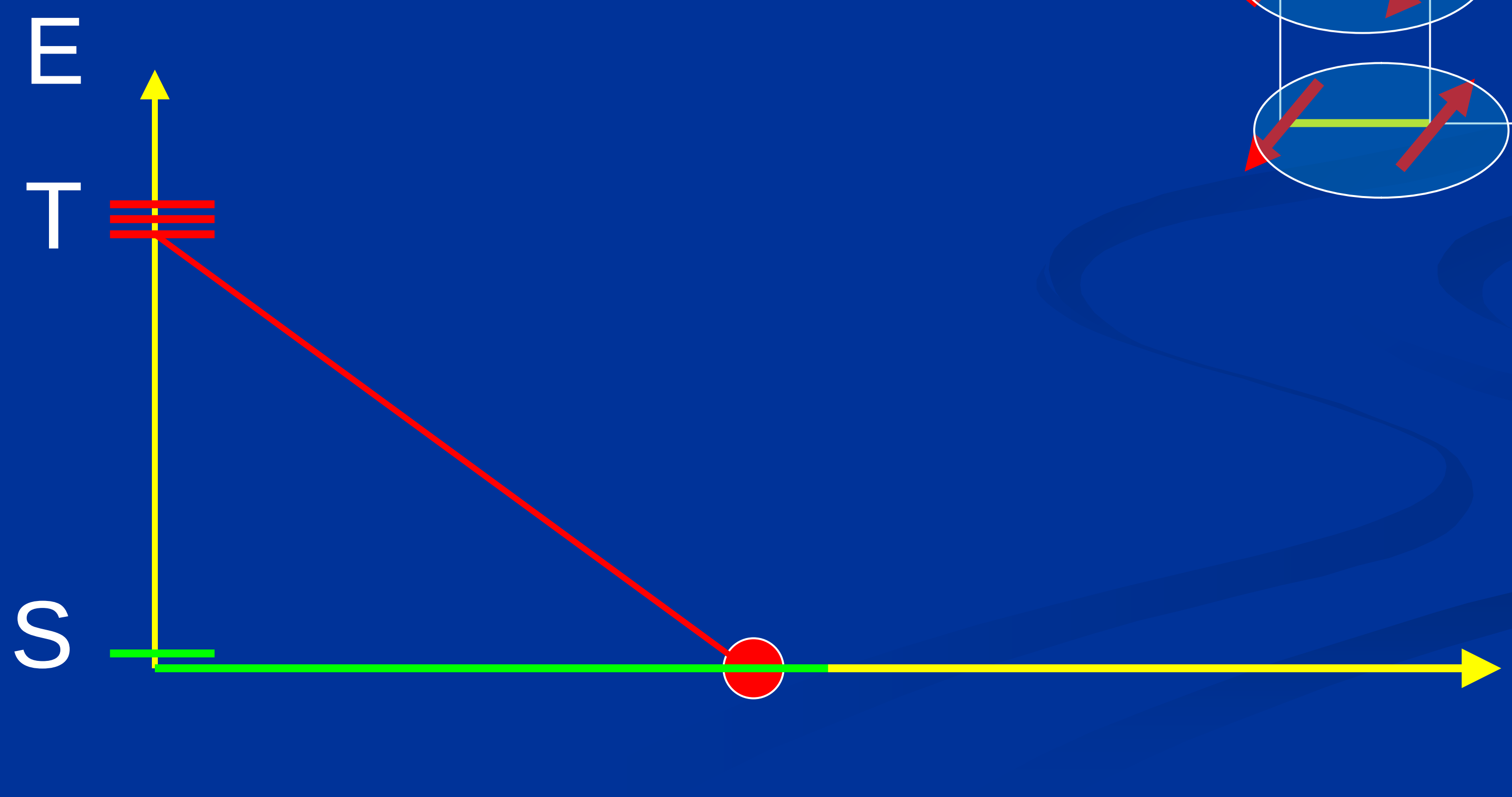

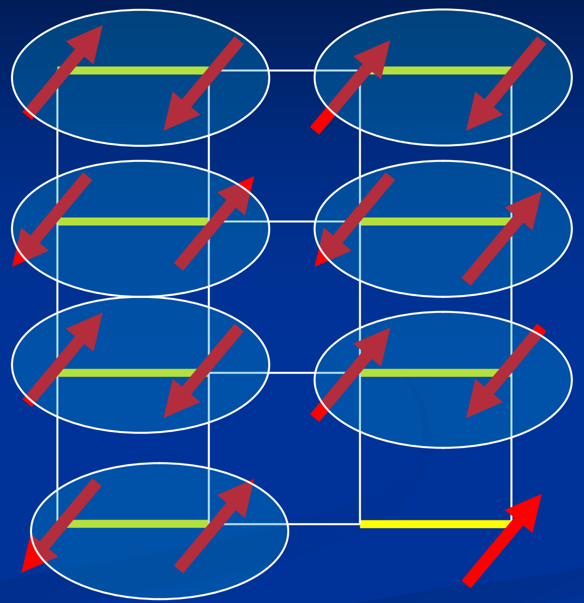

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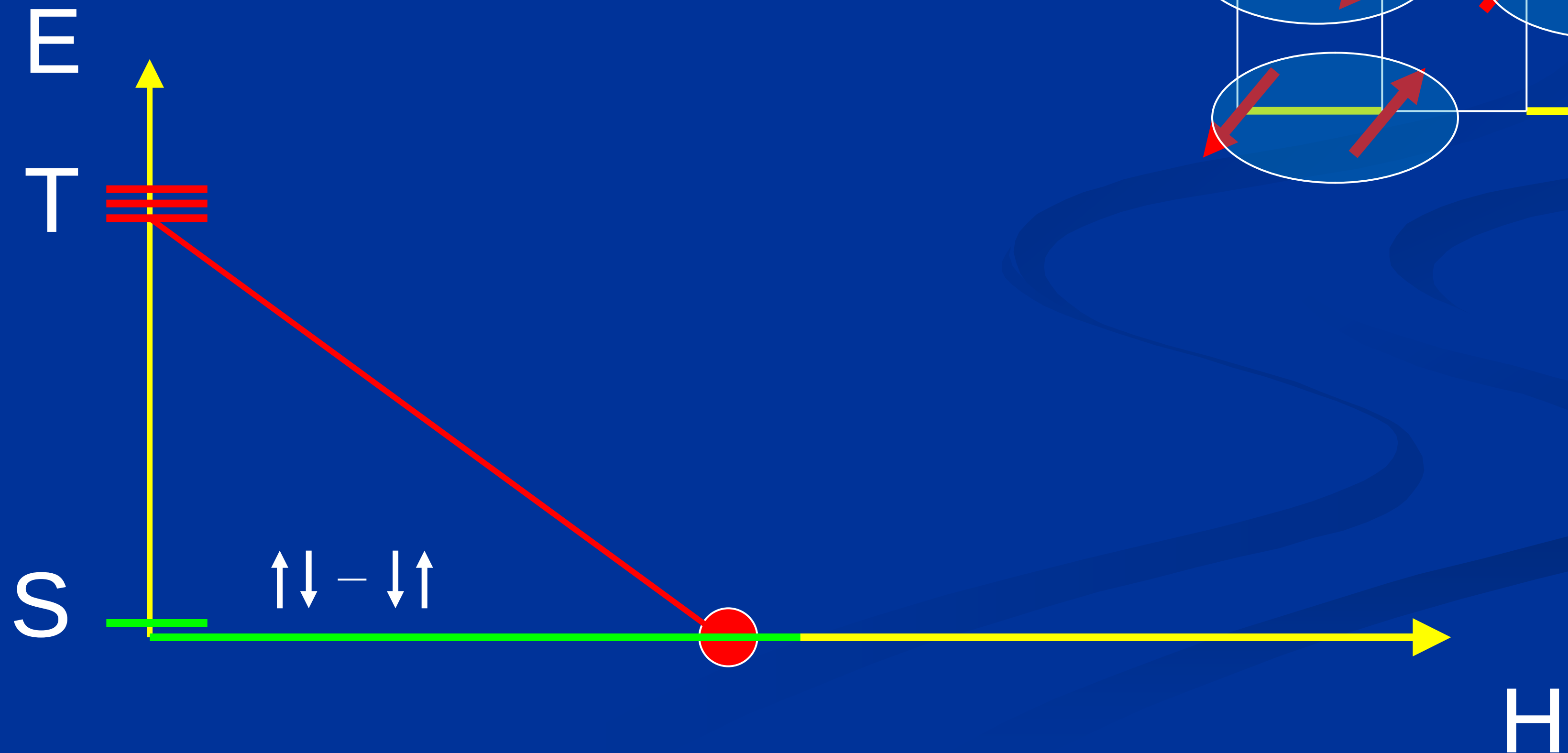

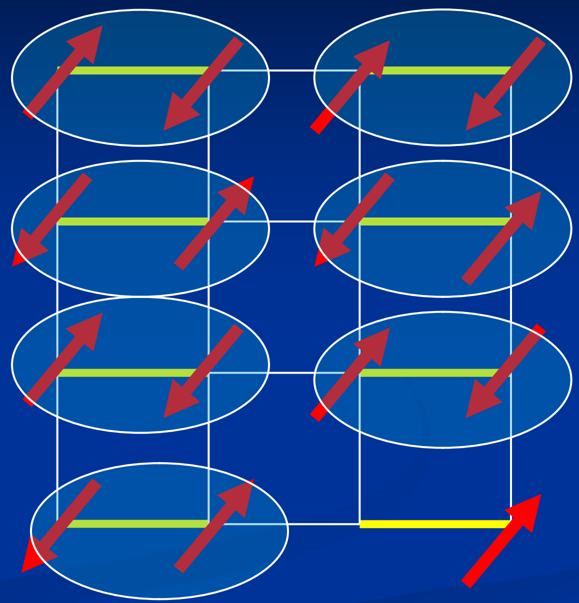

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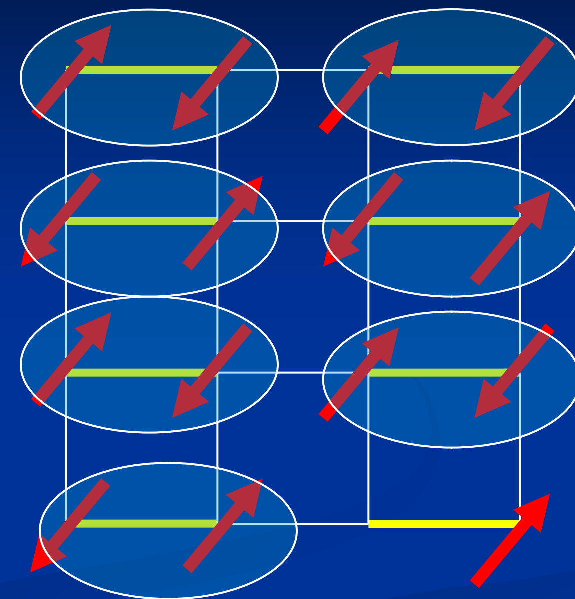

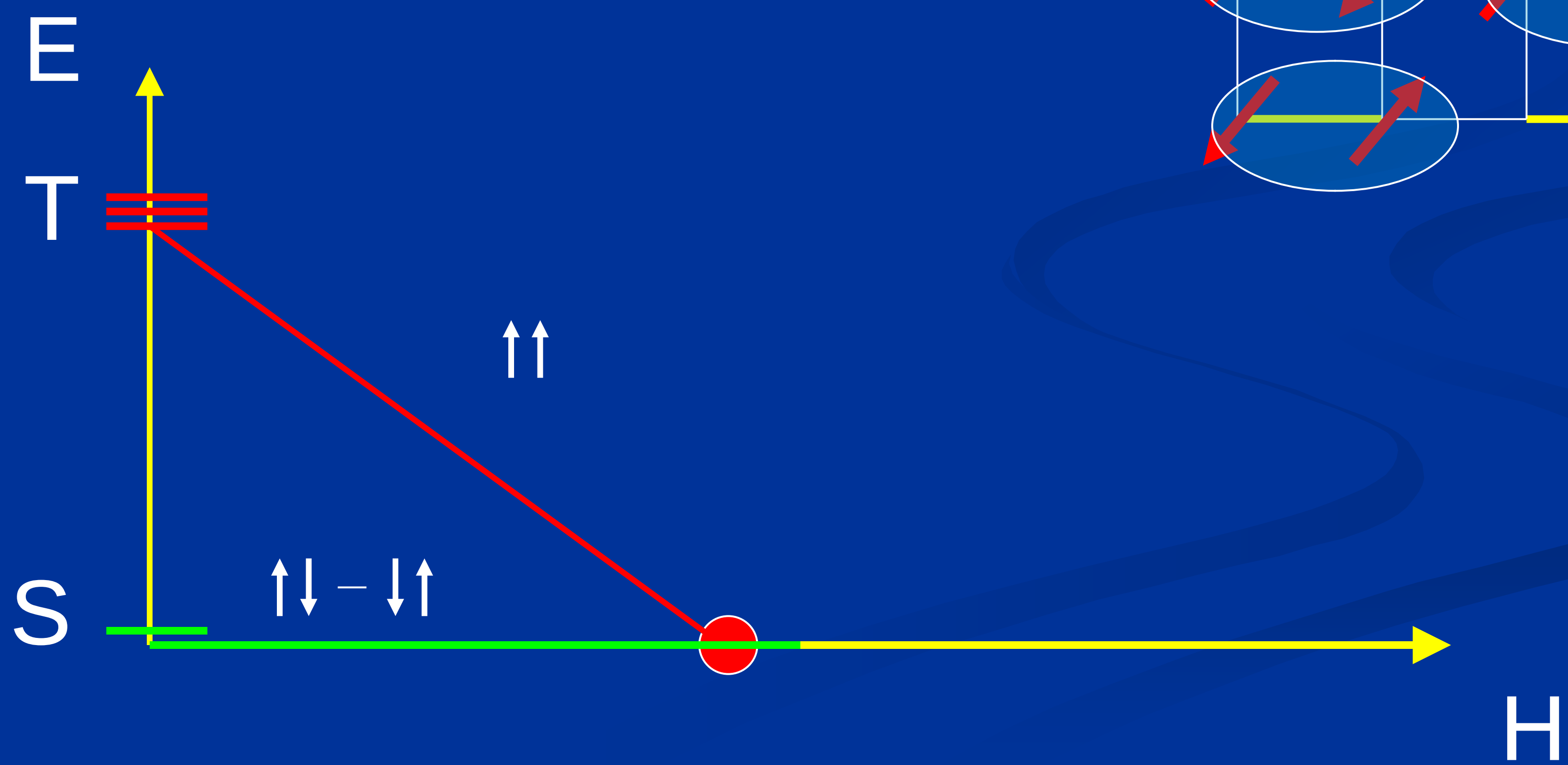

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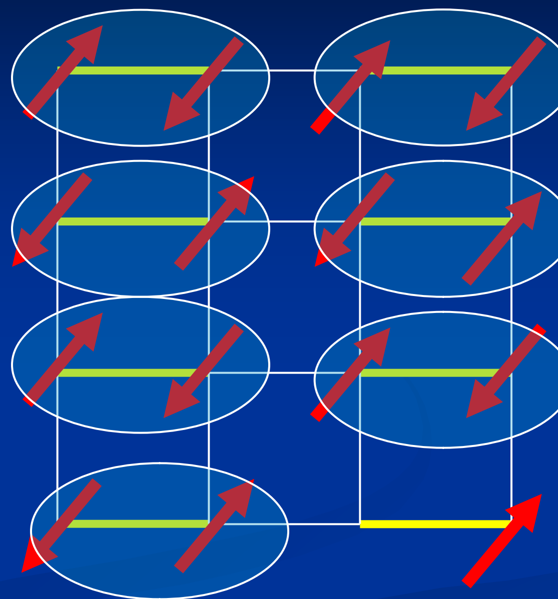

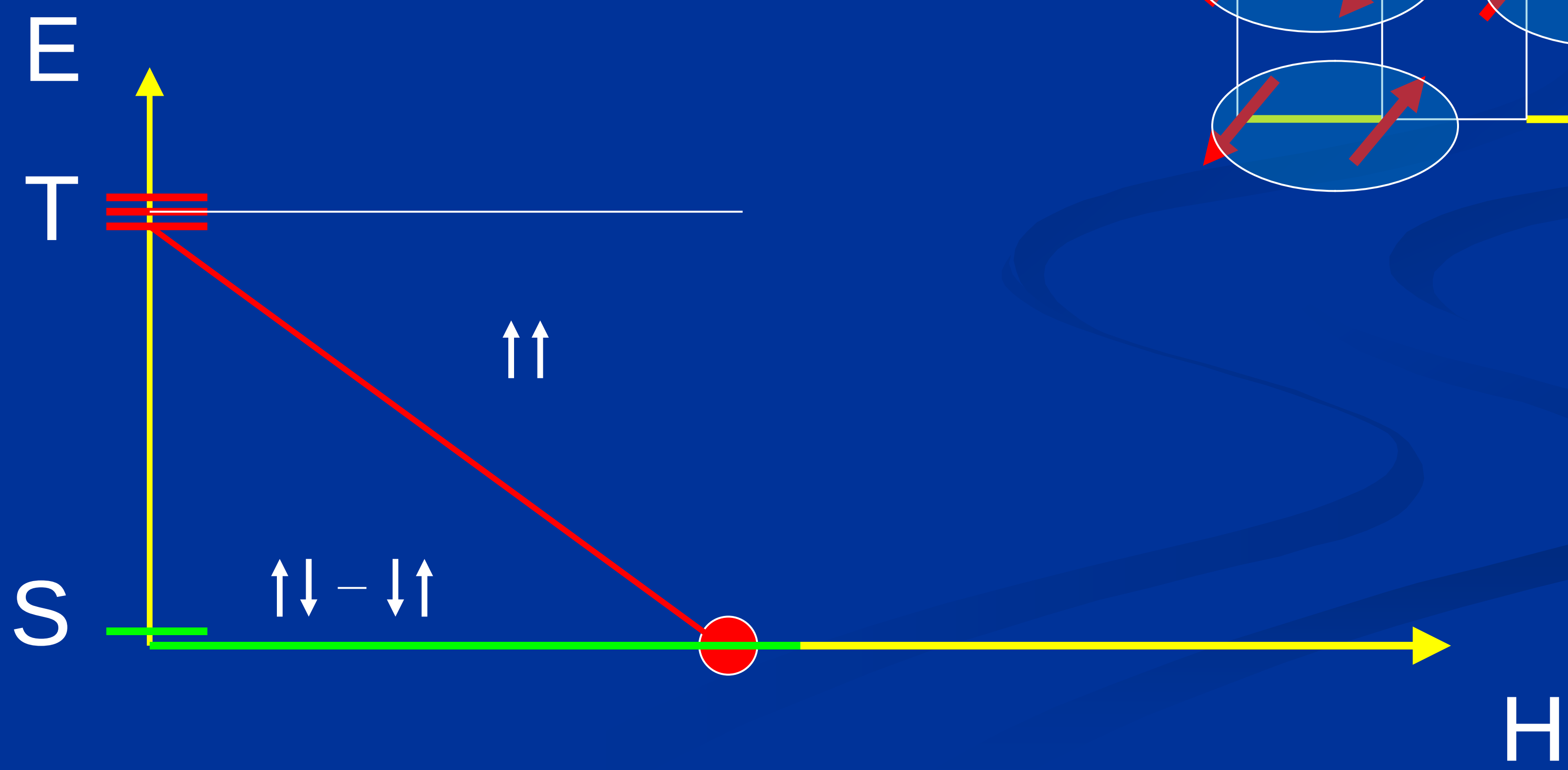

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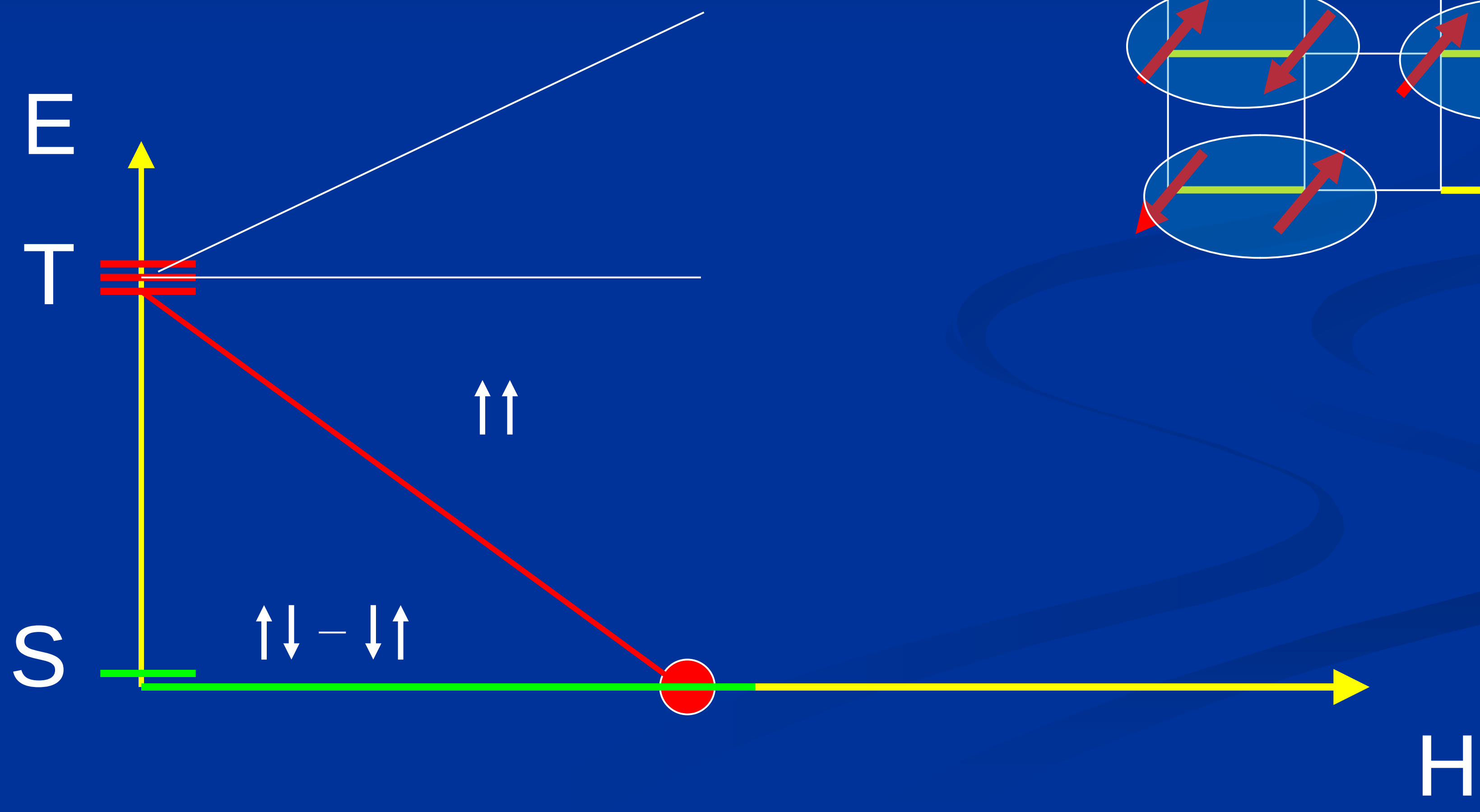

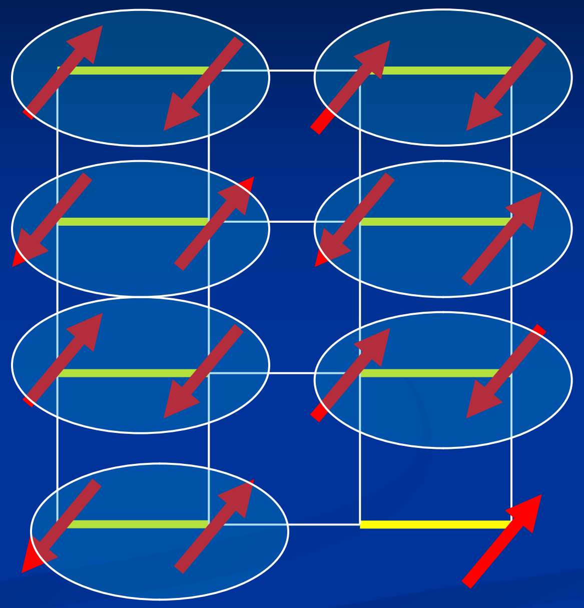

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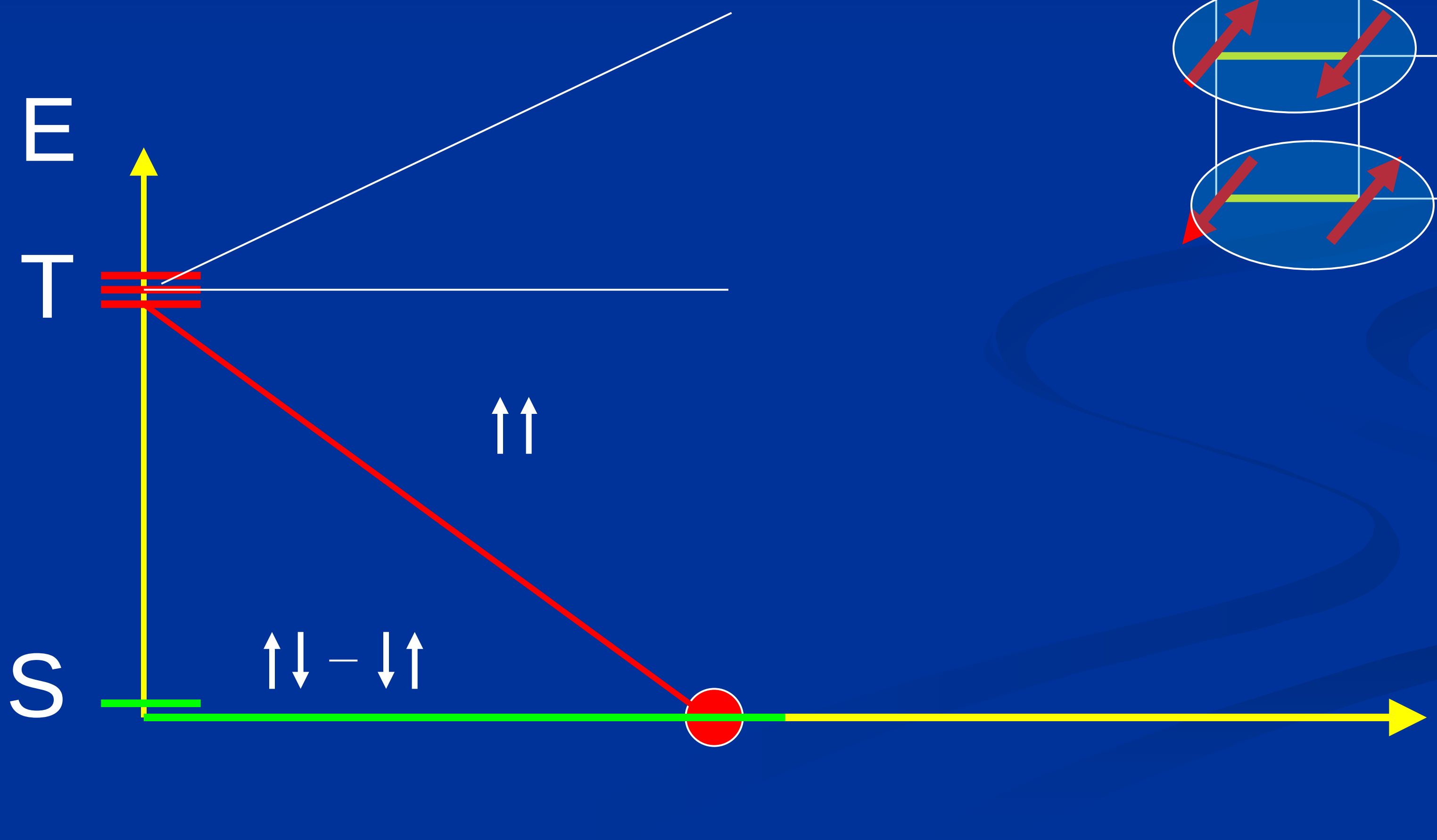

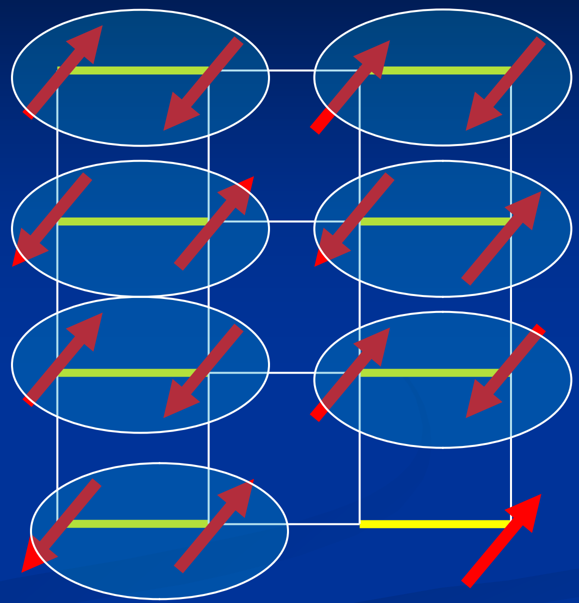

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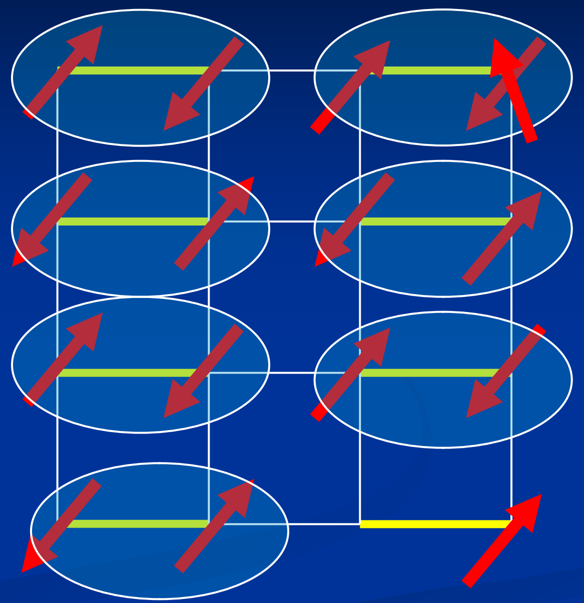

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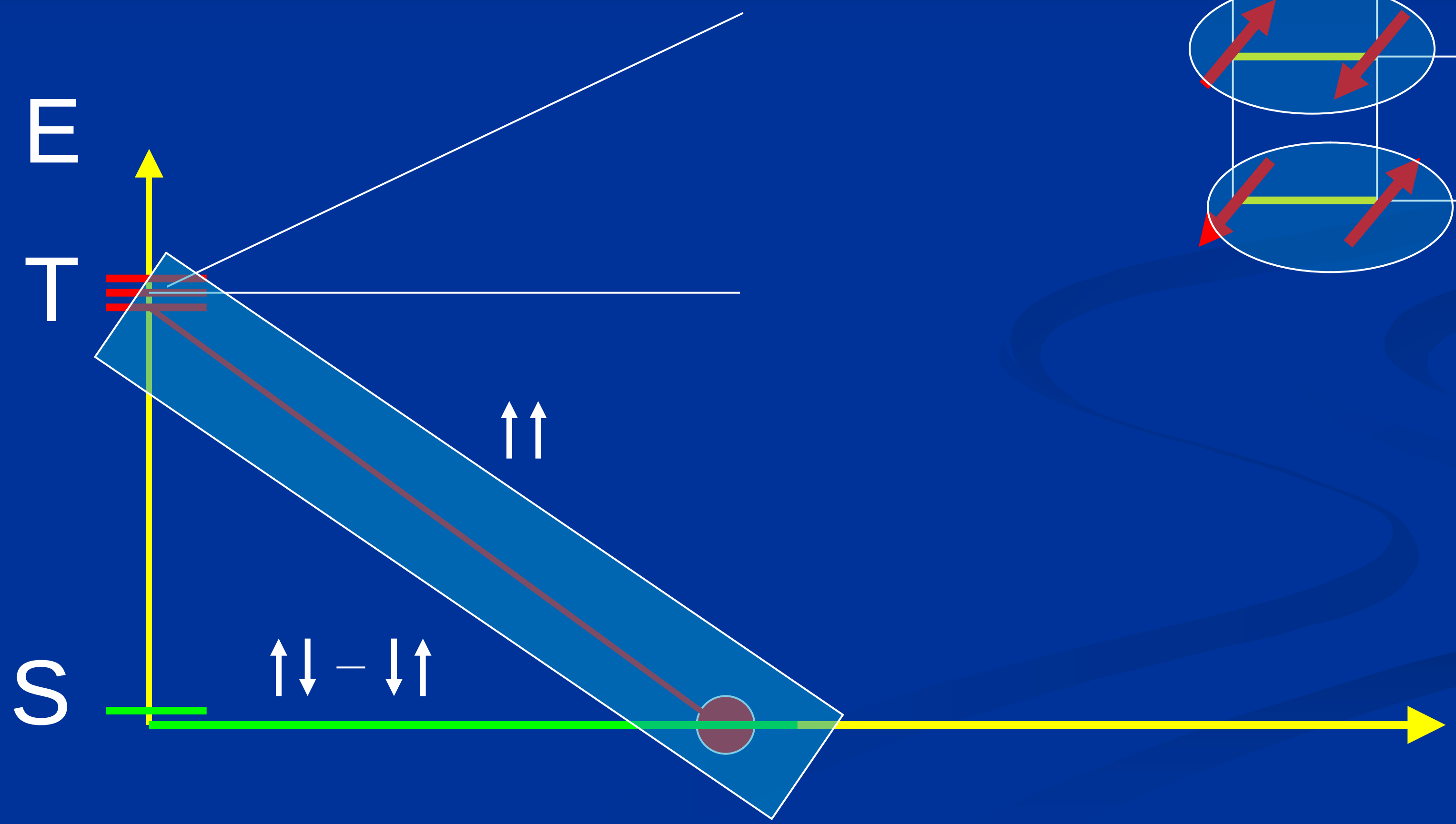

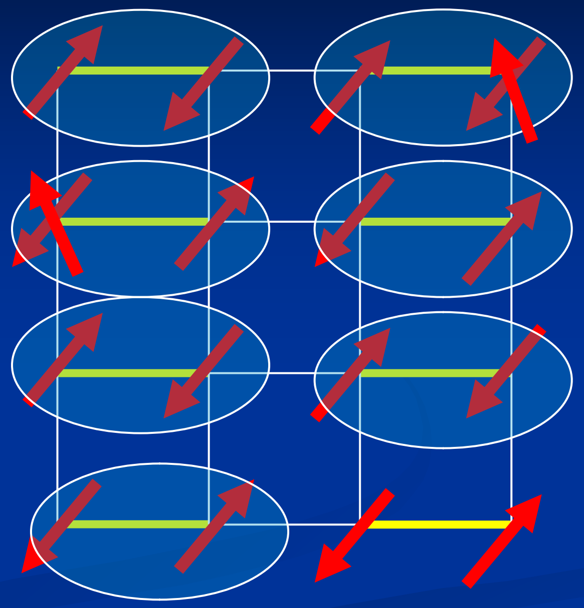

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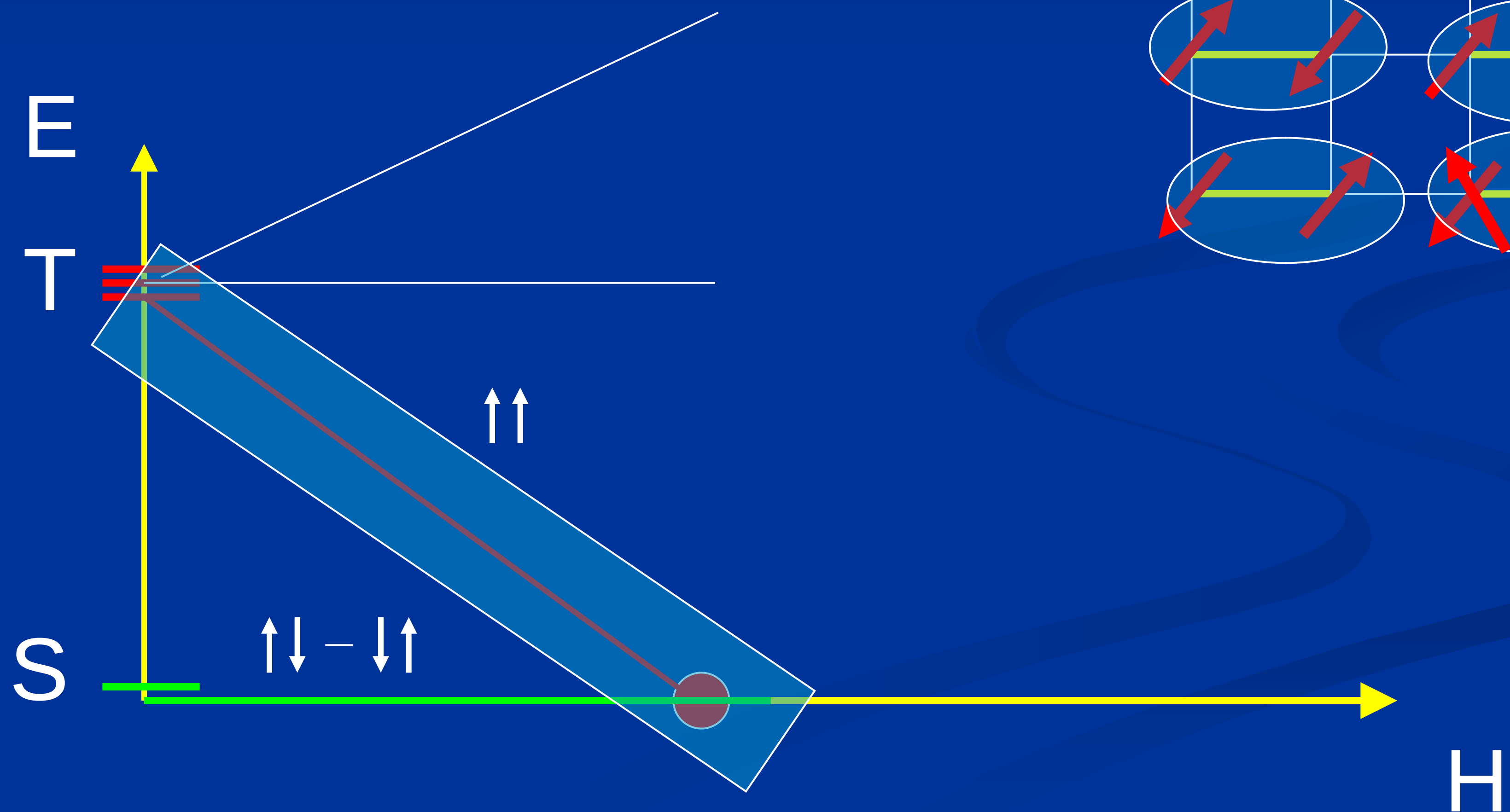

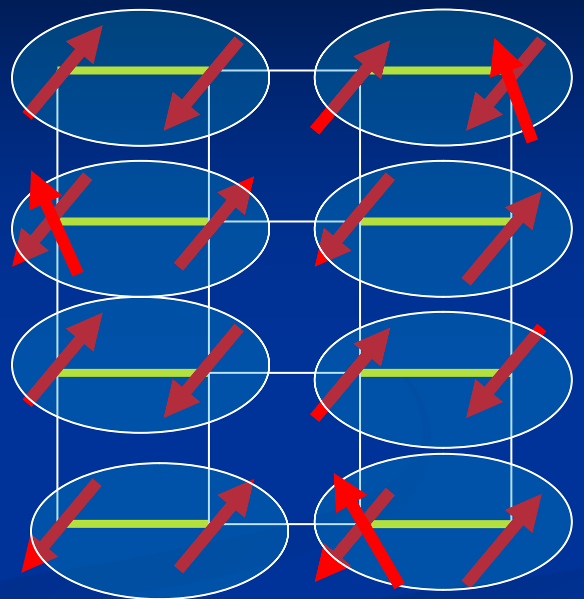

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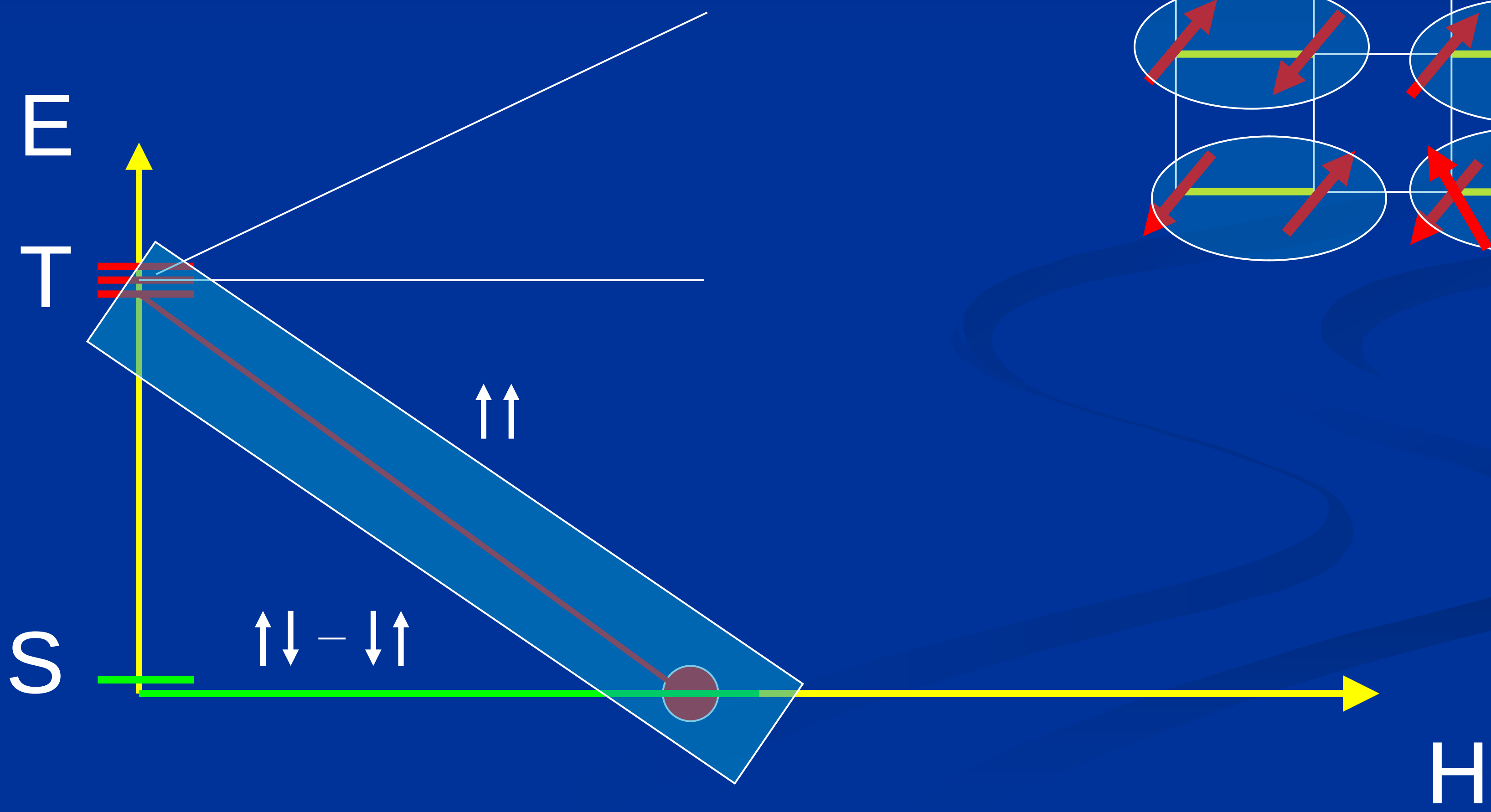

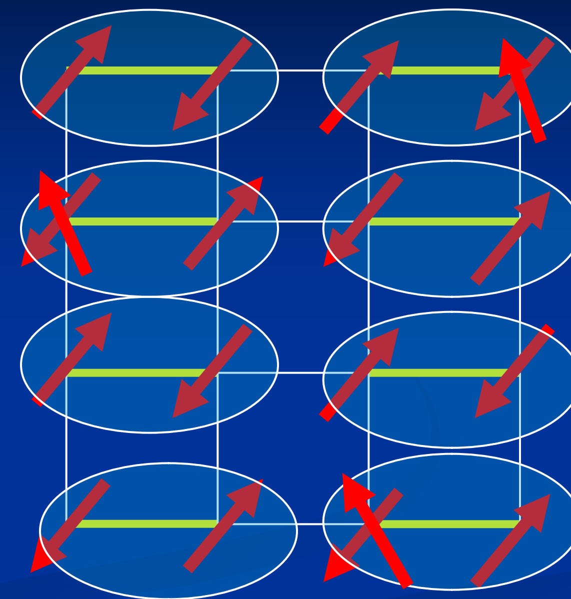

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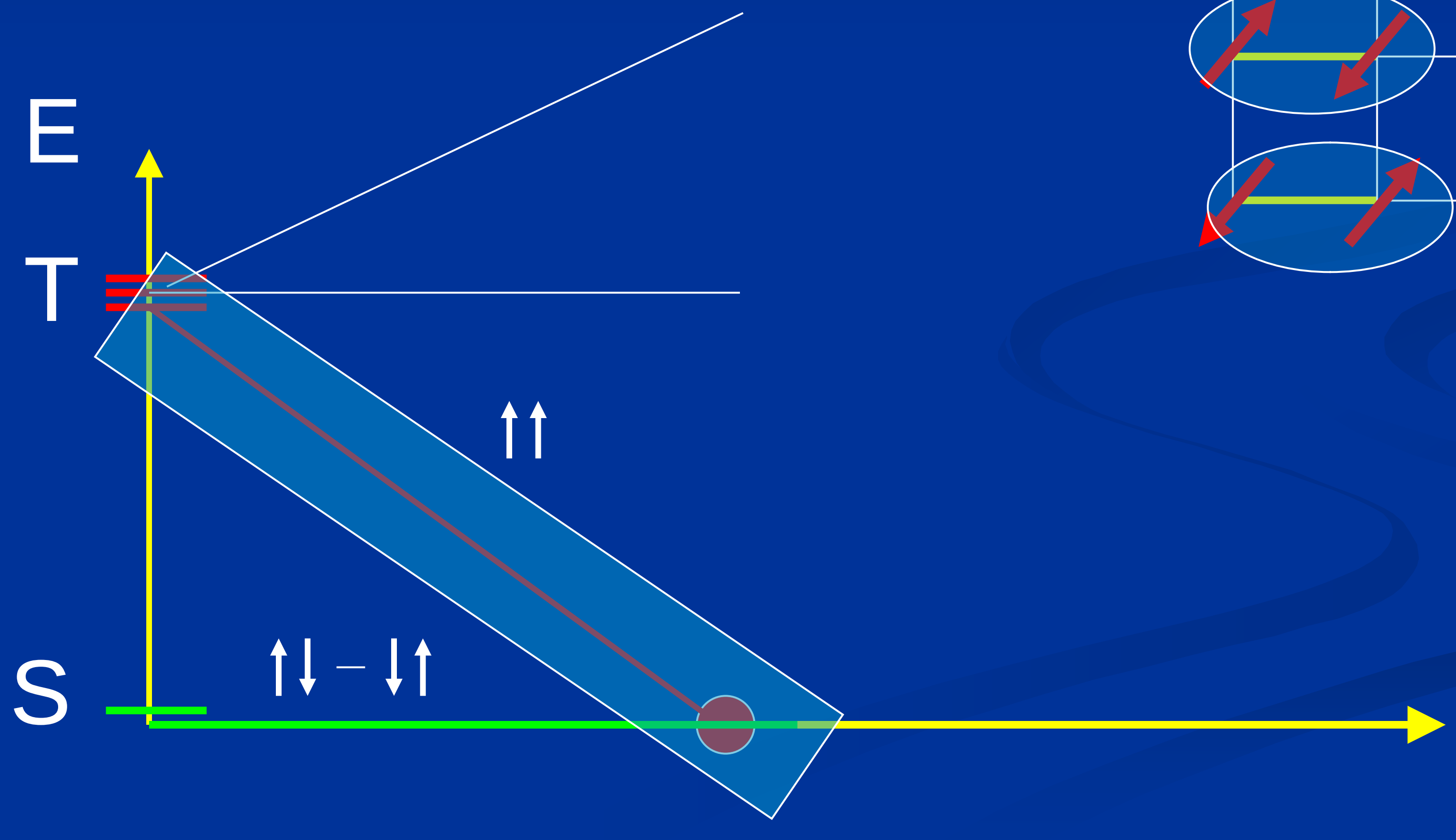

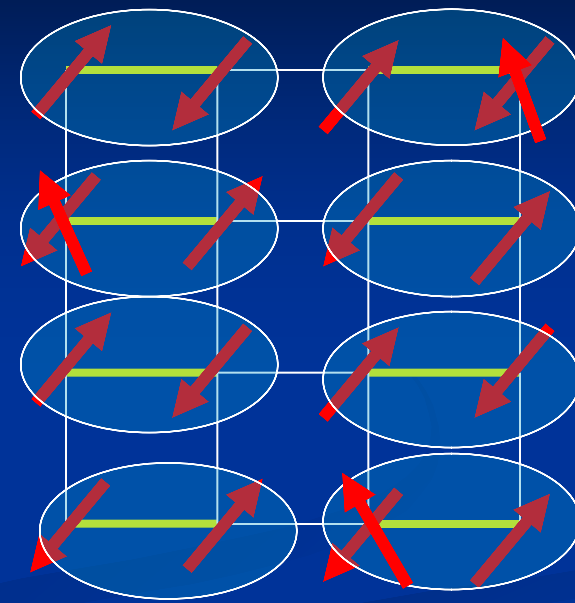

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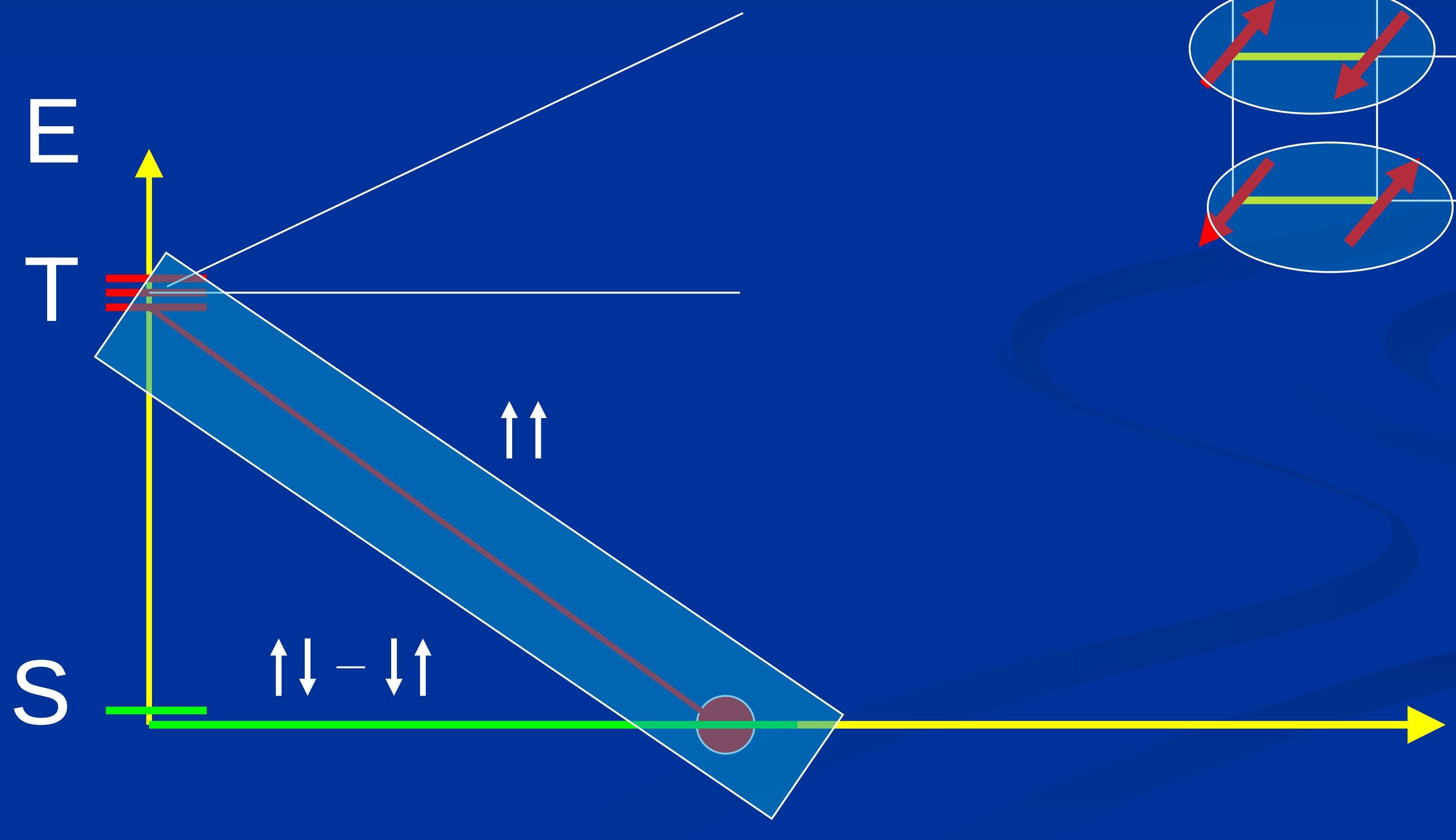

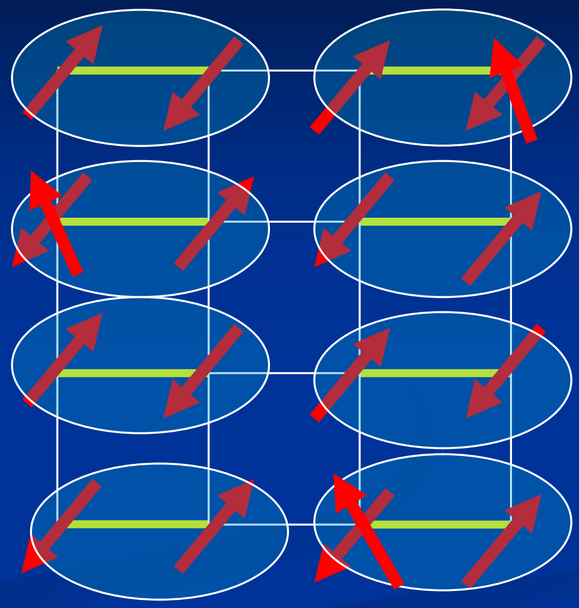

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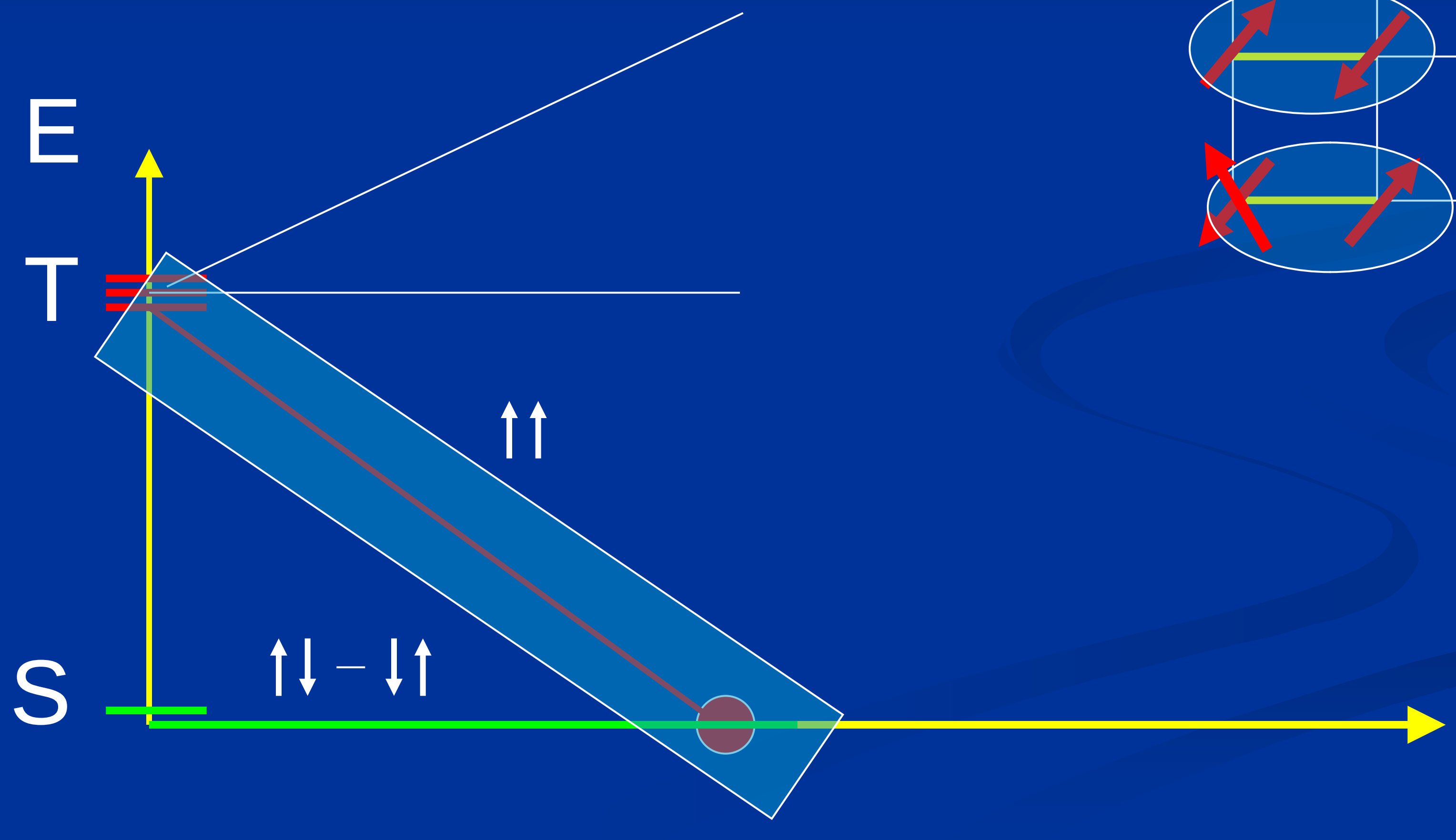

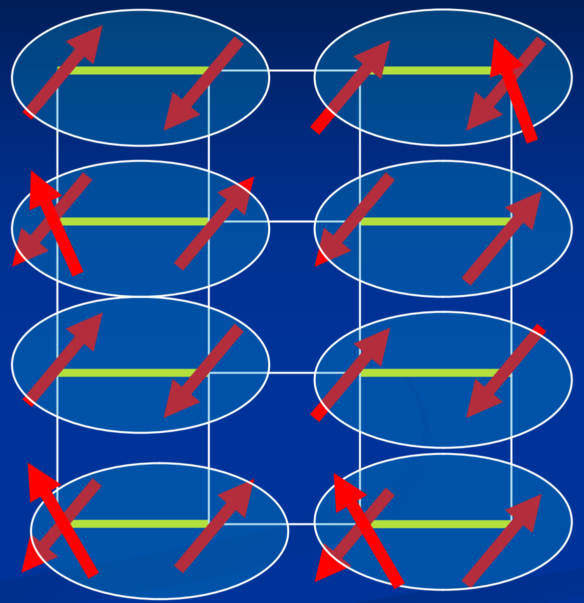

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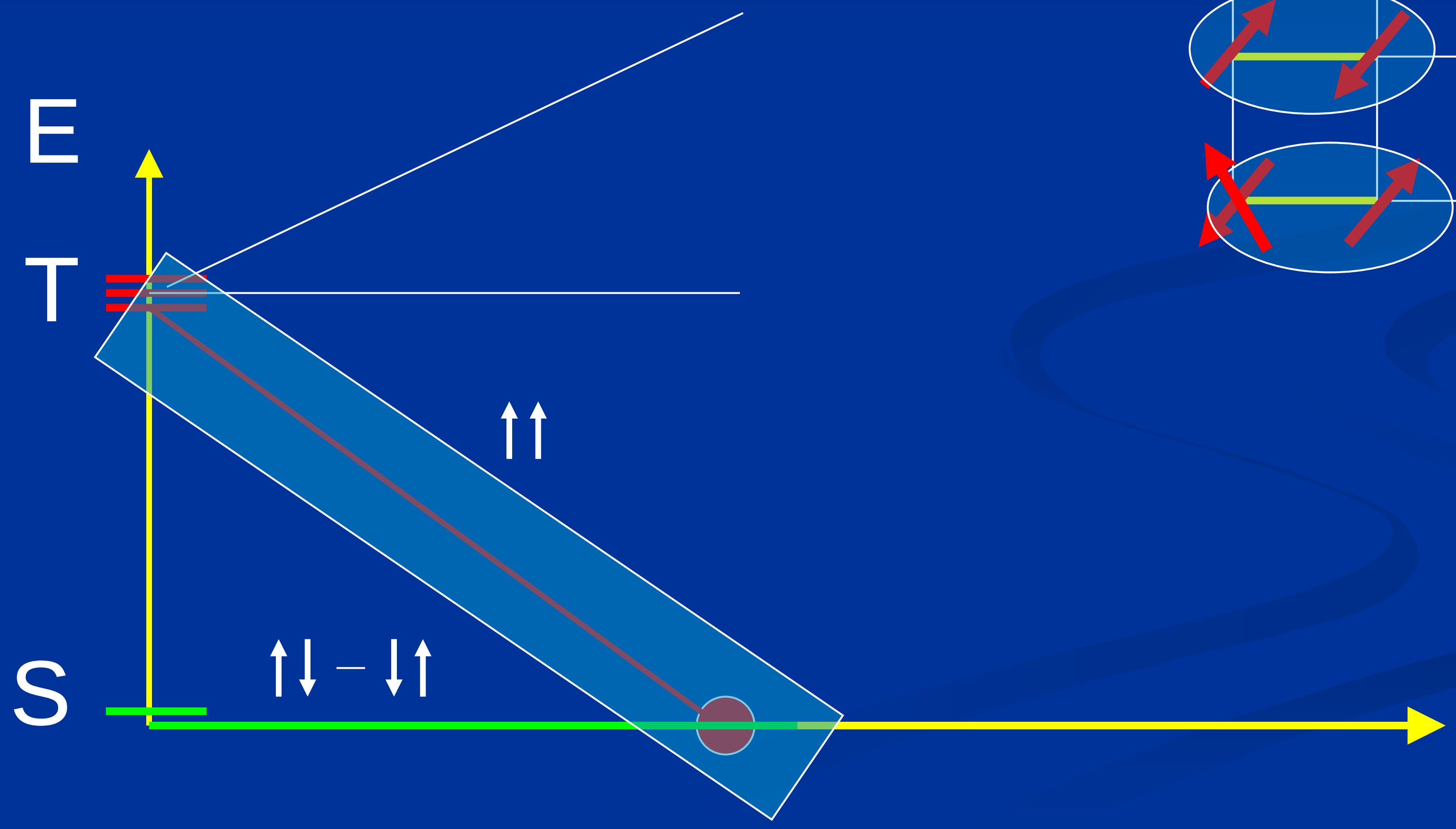

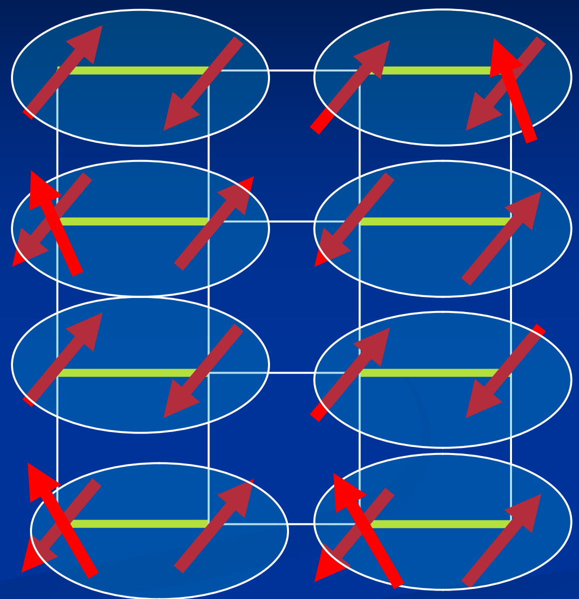

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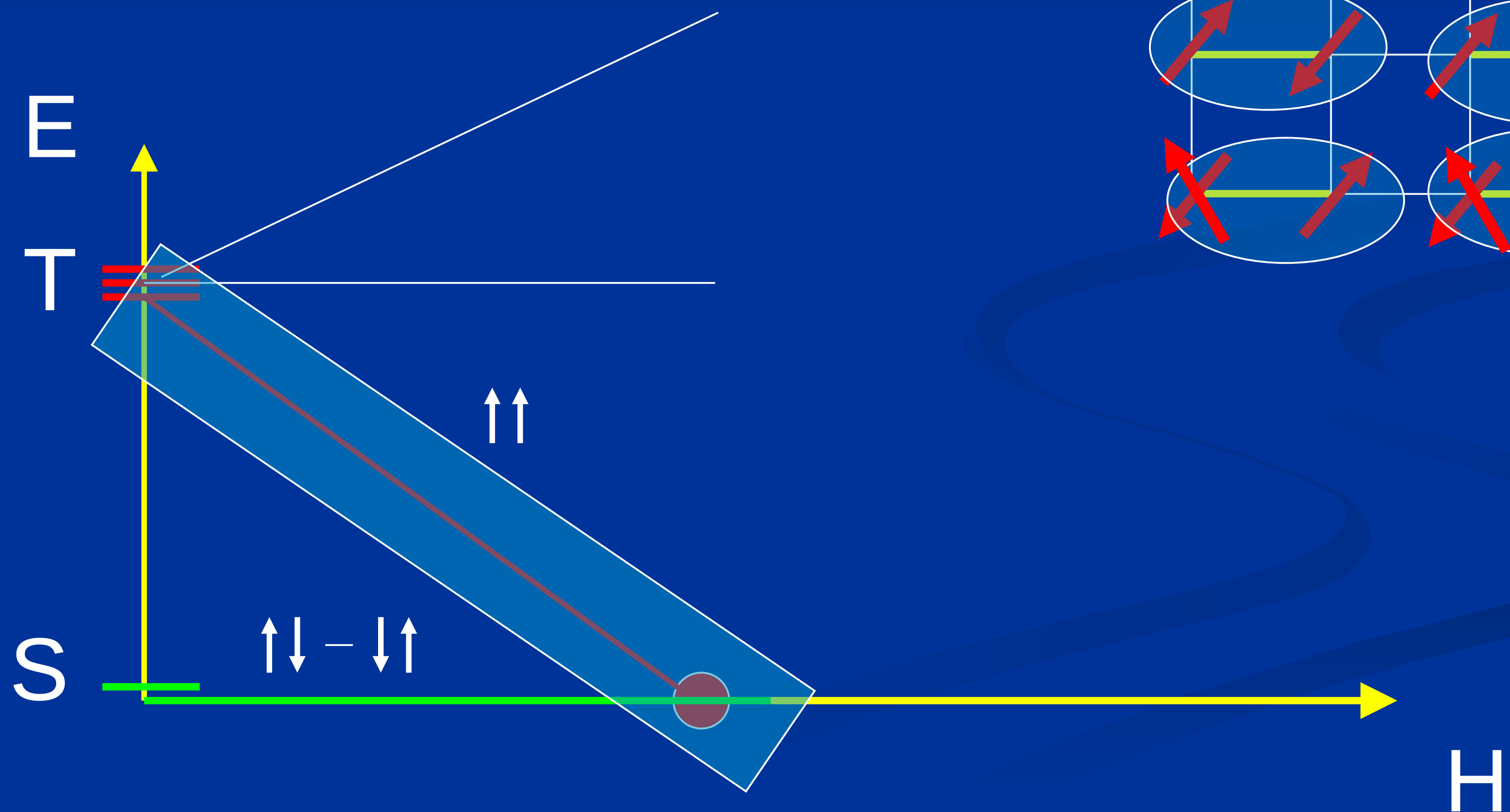
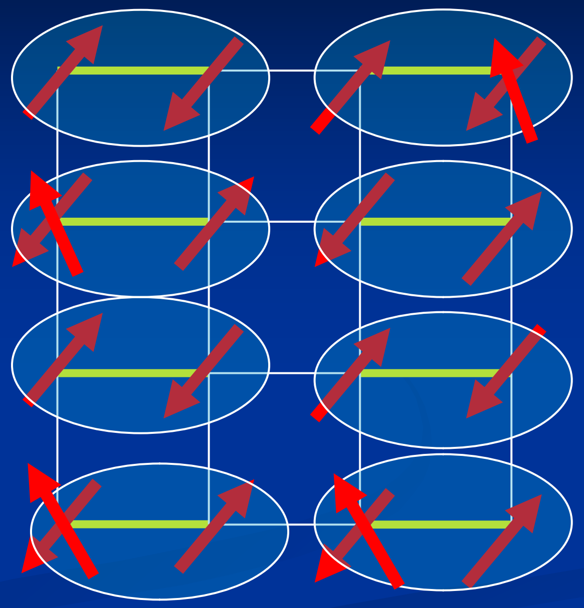
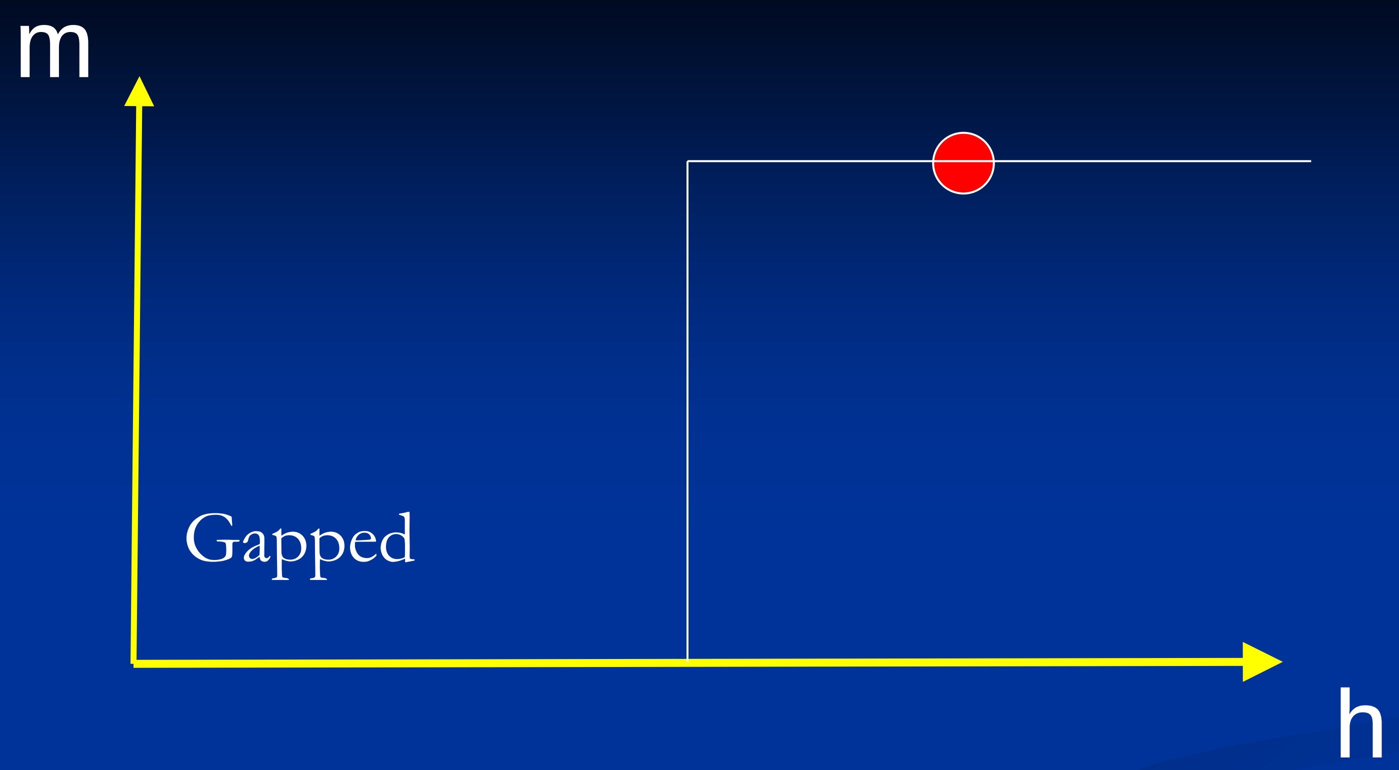





m
Gapped
h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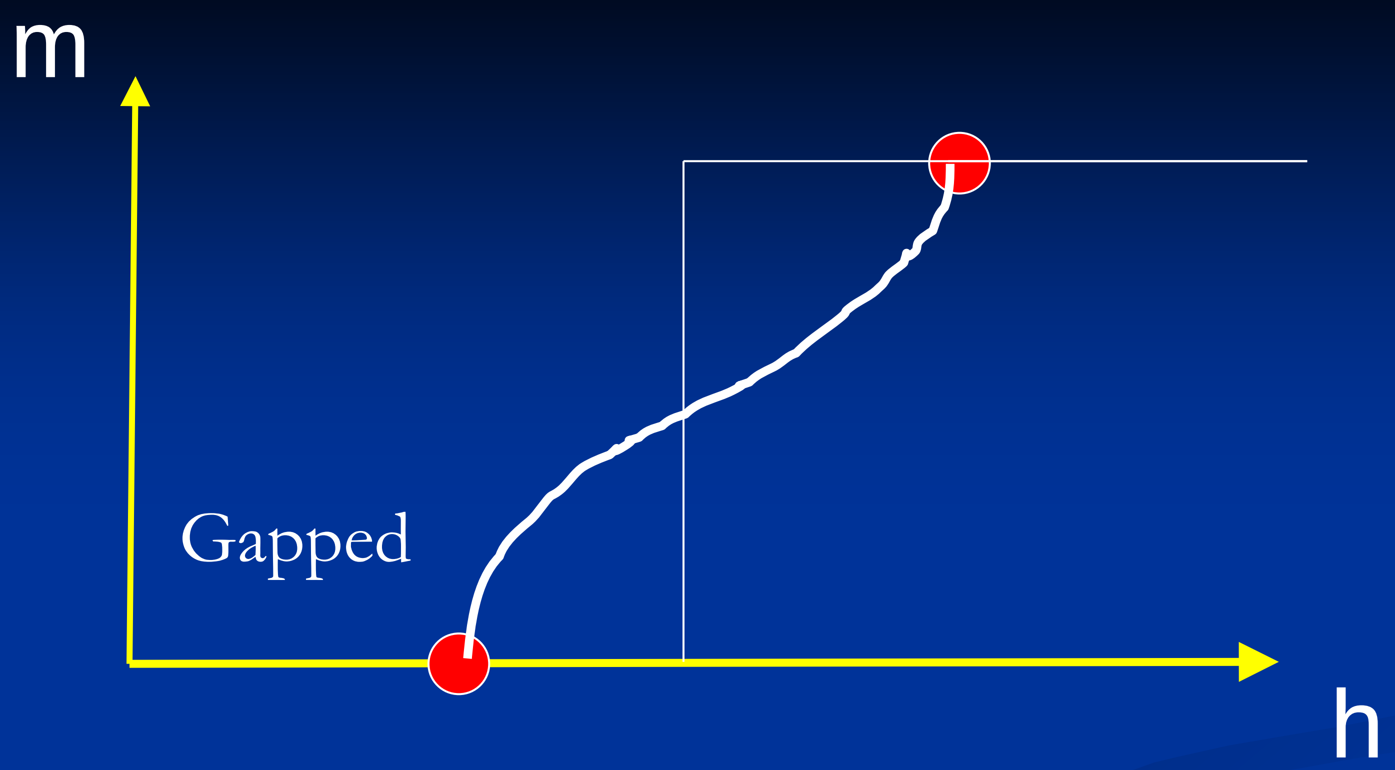

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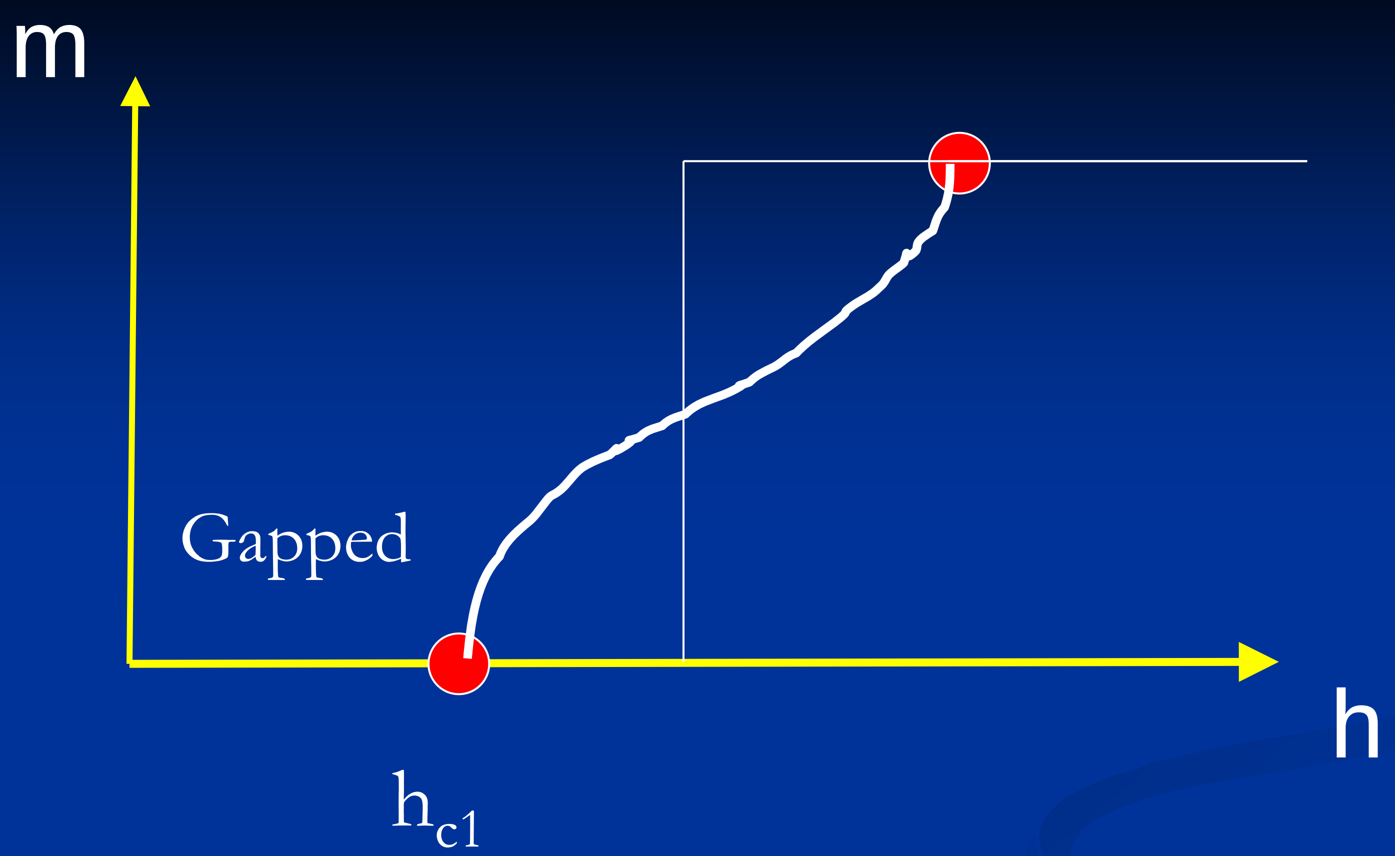
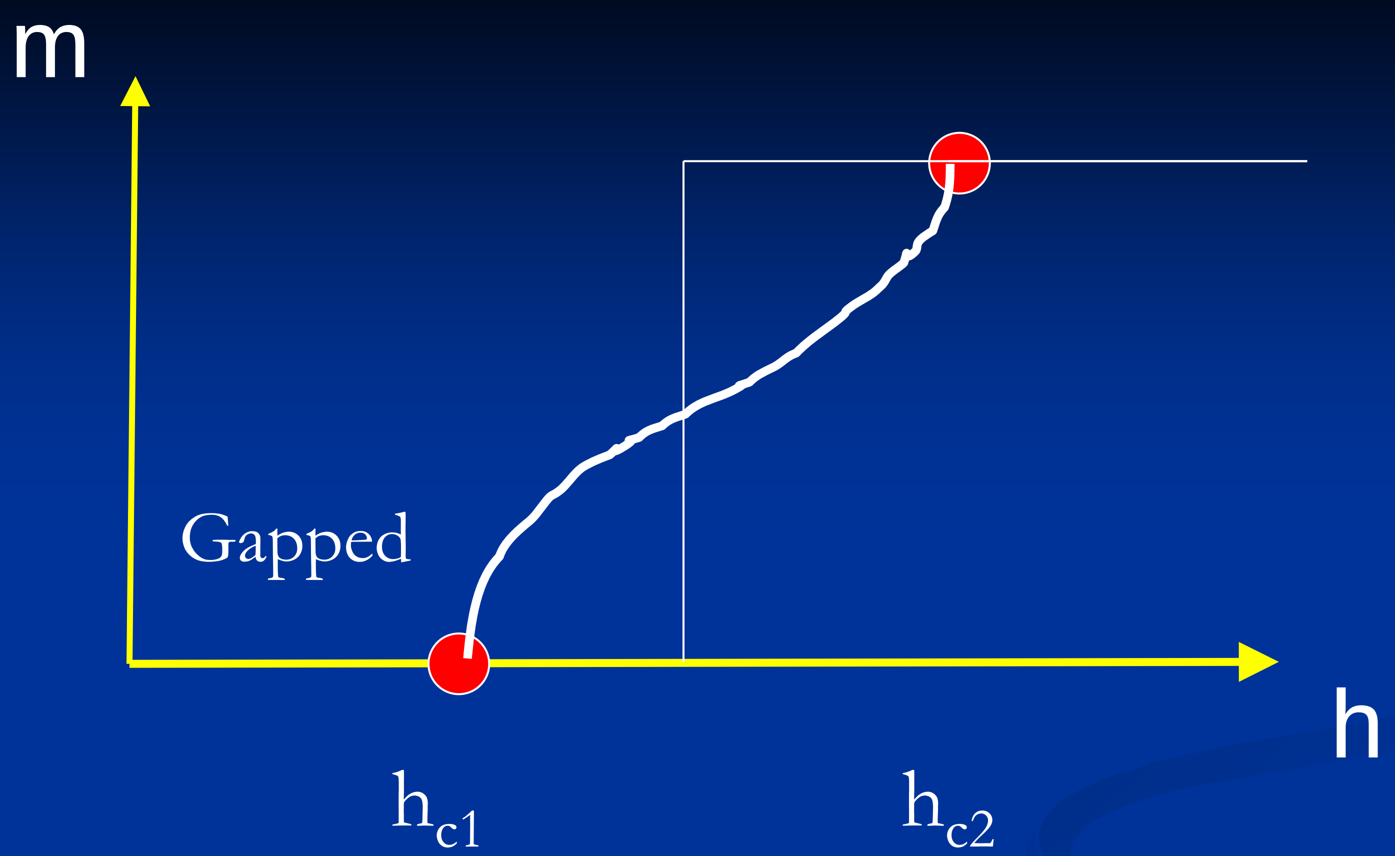
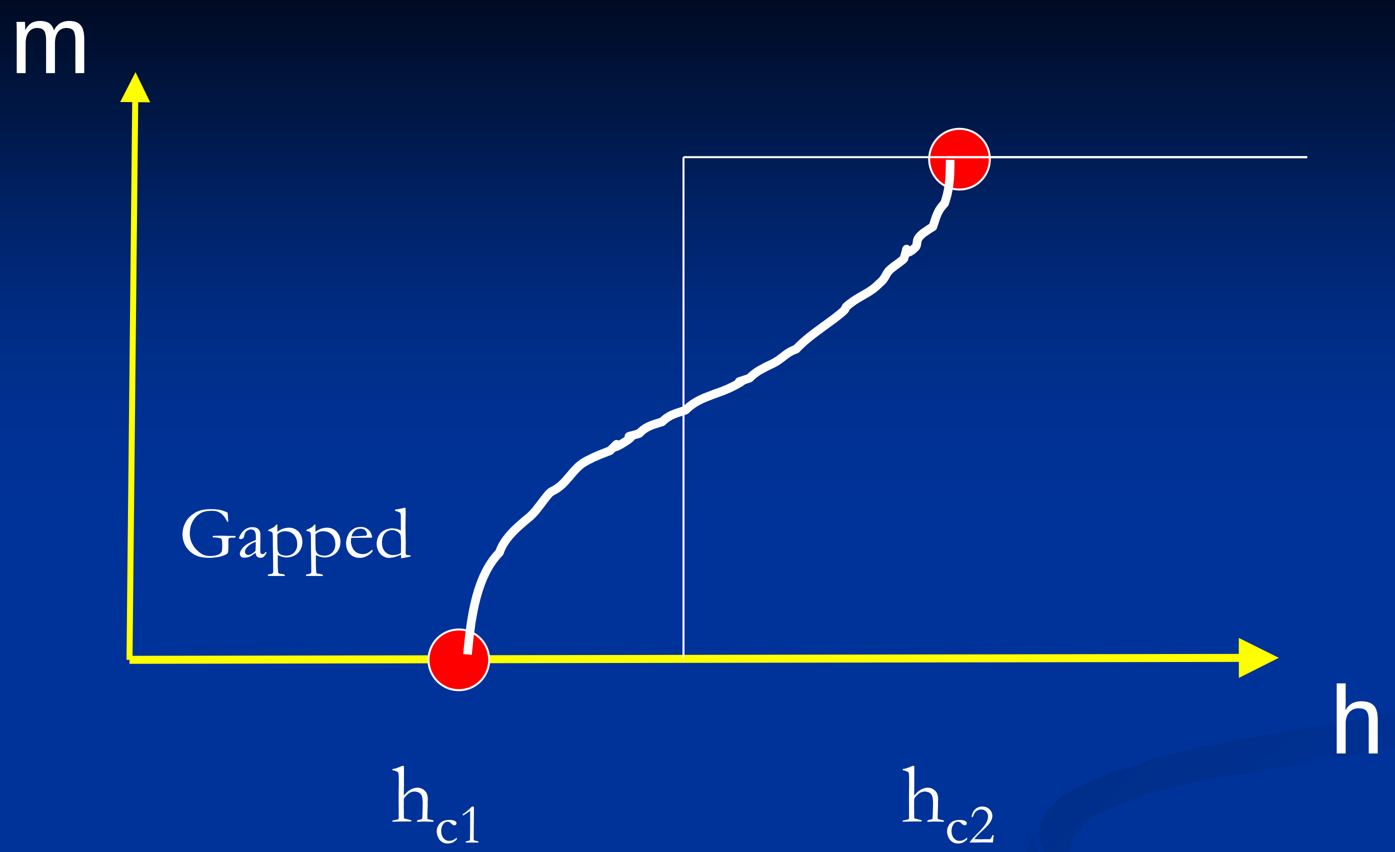

m
Gapped
h
h_{c1}



m
Gapped
h
h_{c1}
h_{c2}



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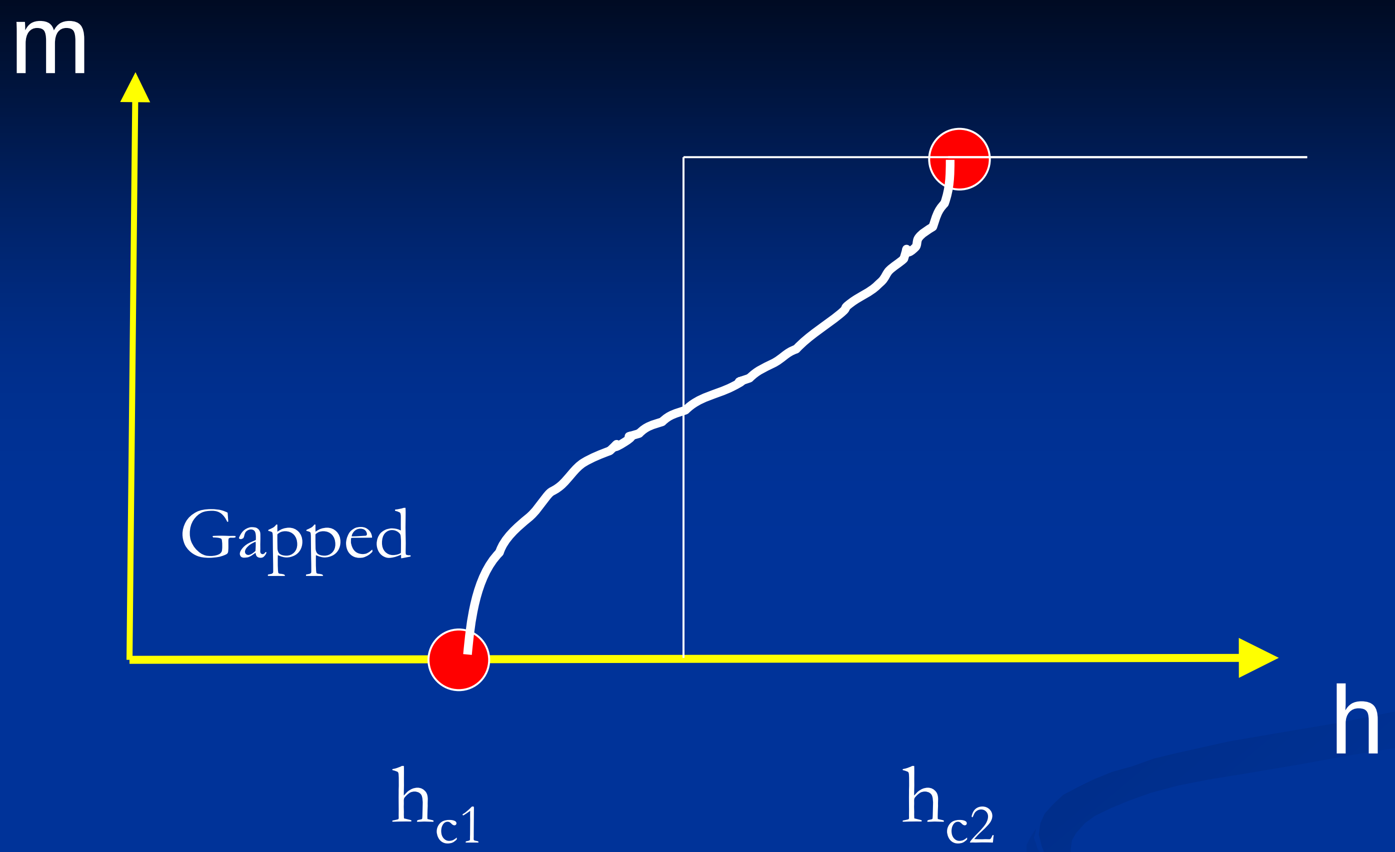
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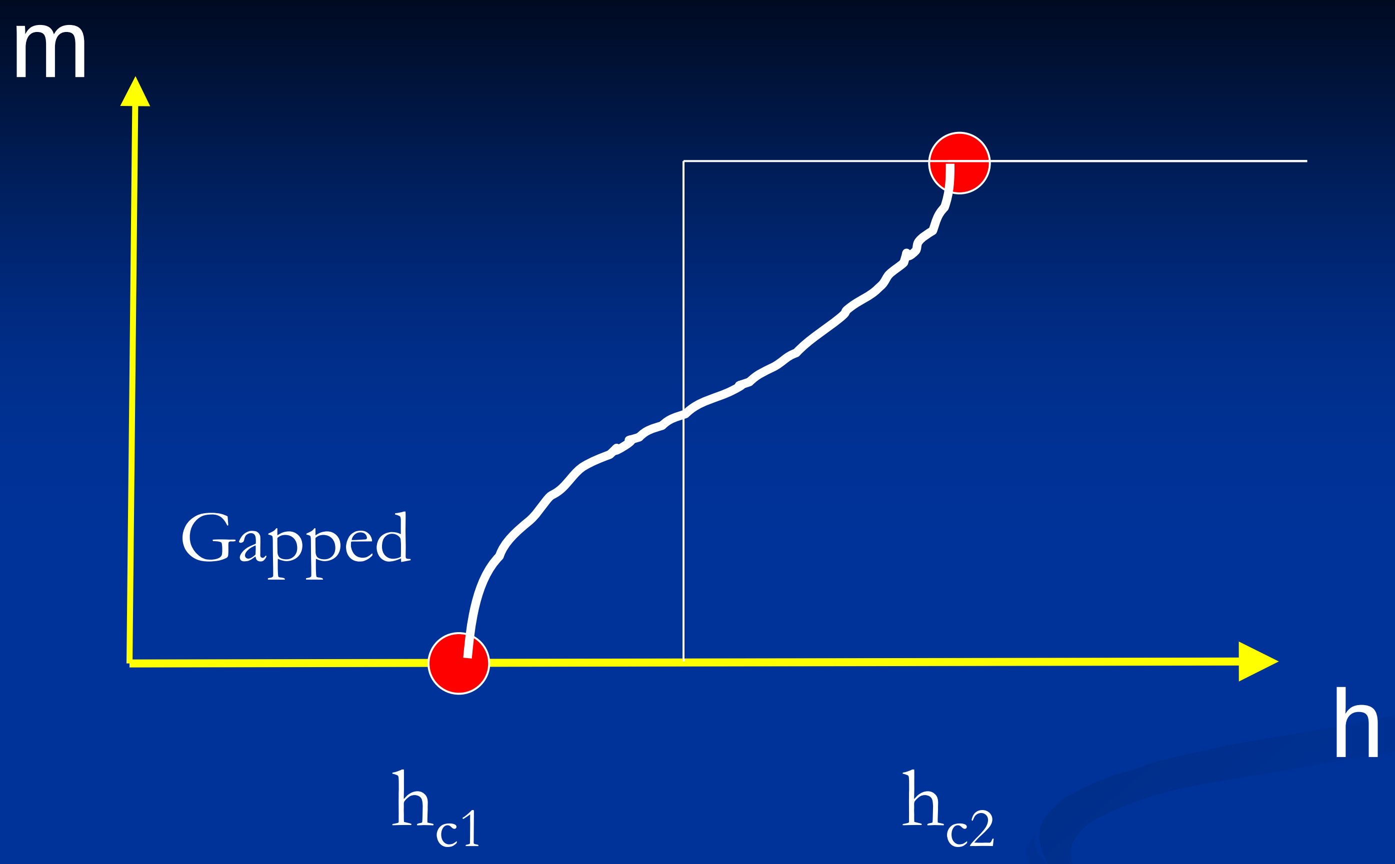
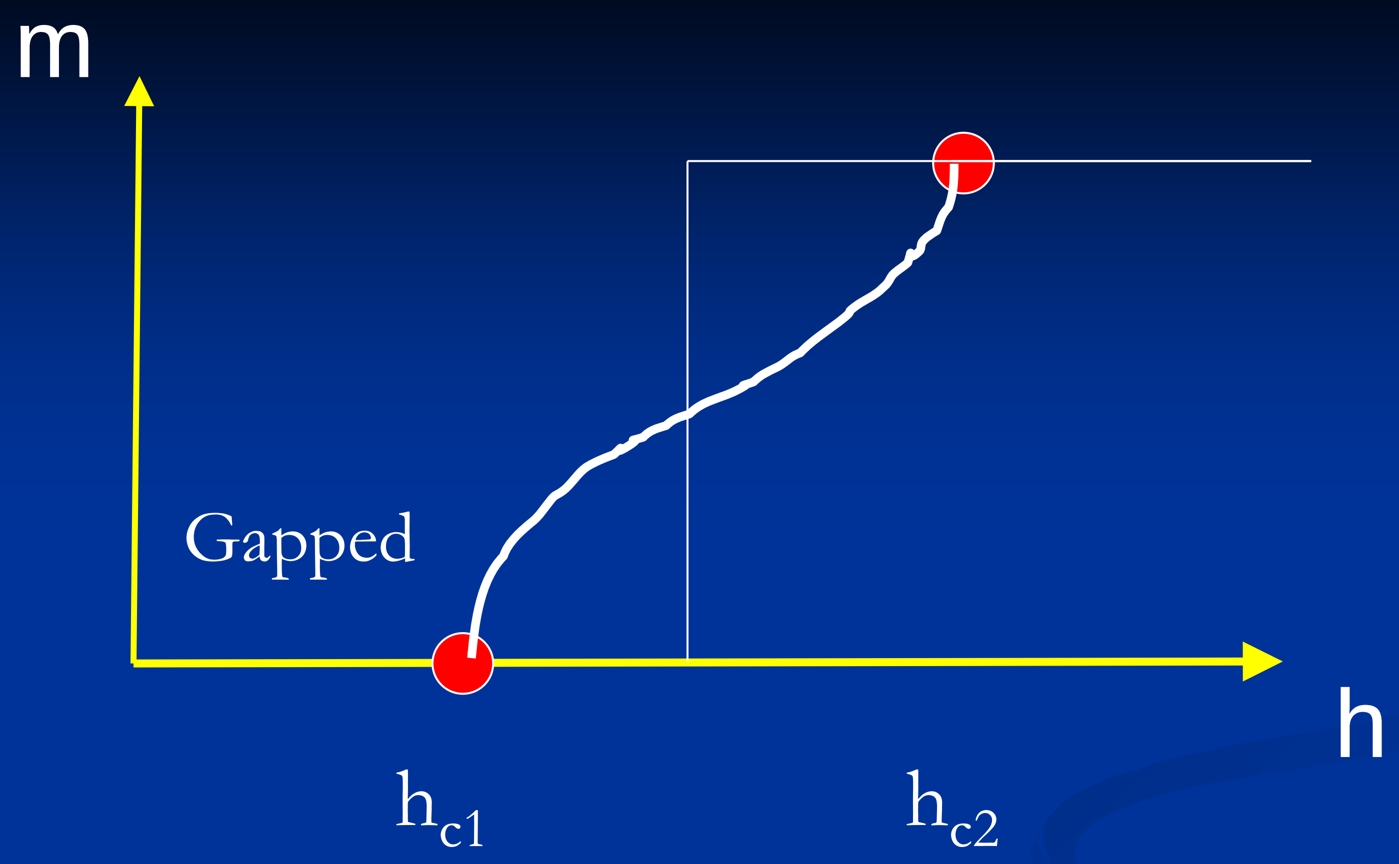


- Quantum phase transition



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- Magnetic field: ``chemical potential” for the triplon band (interacting intinerant ``particles”)

Two examples

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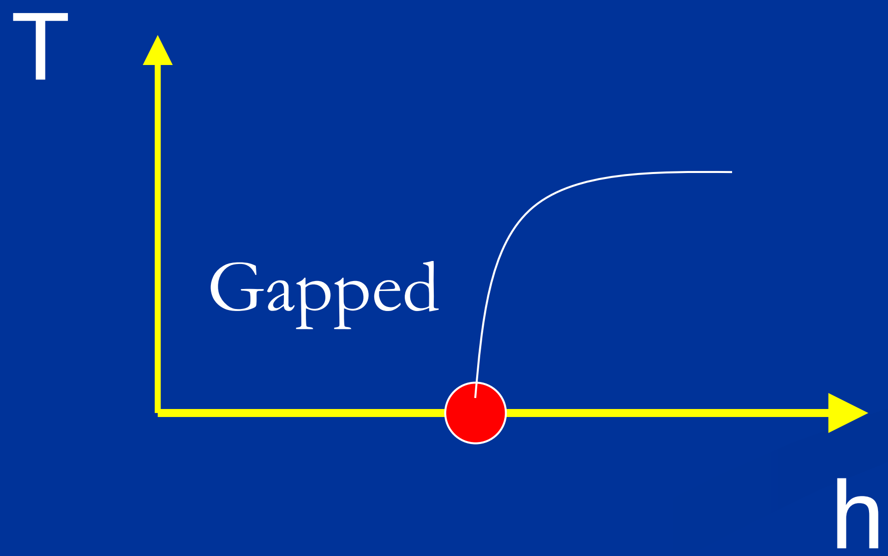
- Tomonaga-Luttinger liquids (d=1)

Bose Einstein condensation



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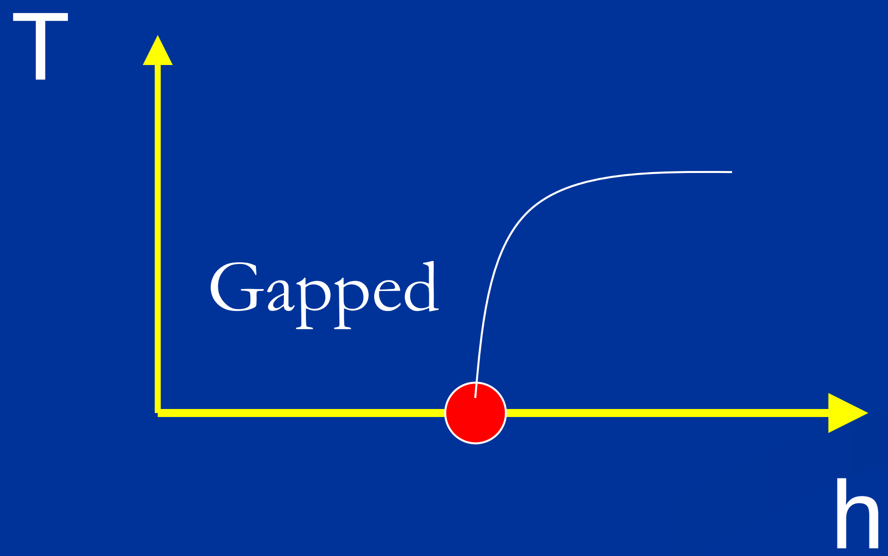
(TG and A. M. Tsvelik PRB 59 11398 (1999))



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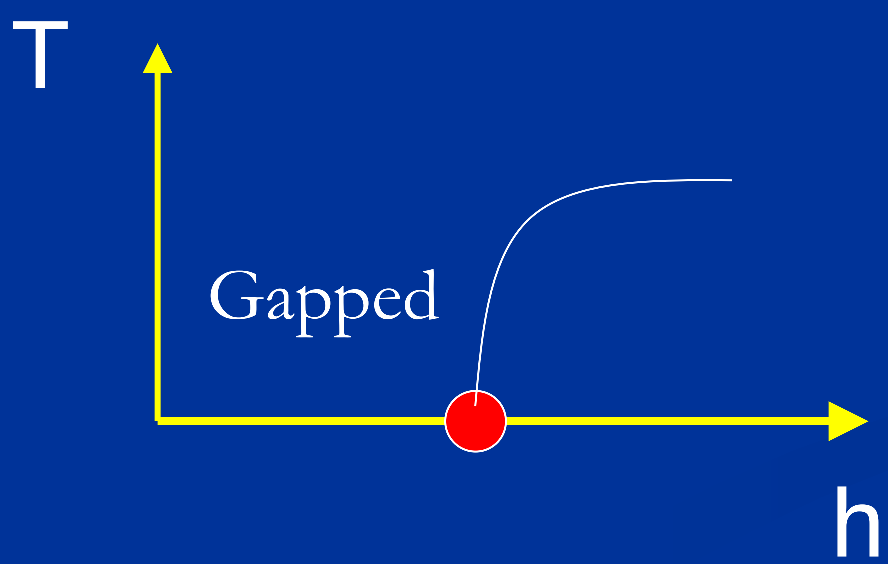

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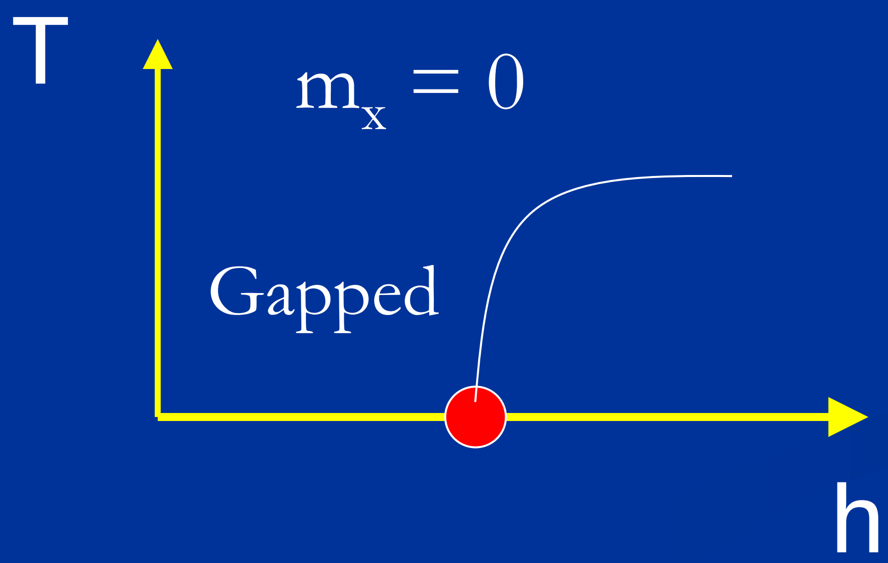

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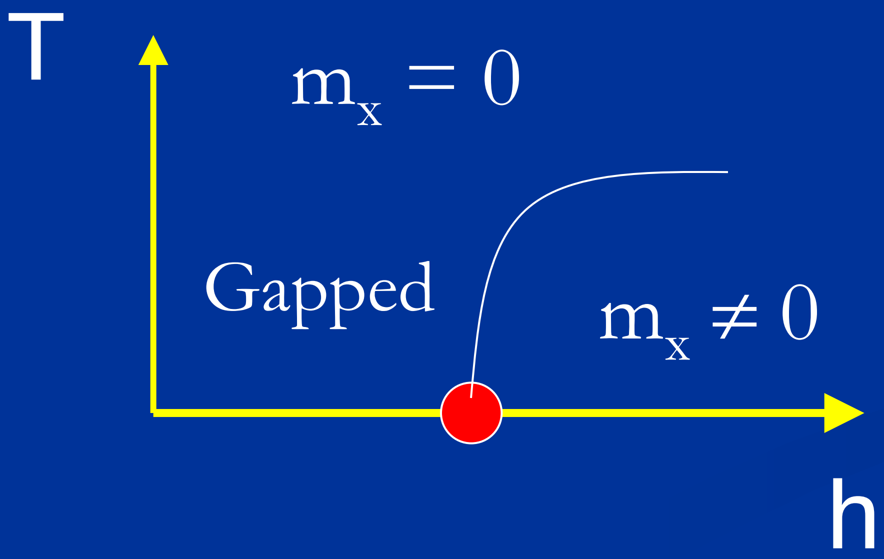

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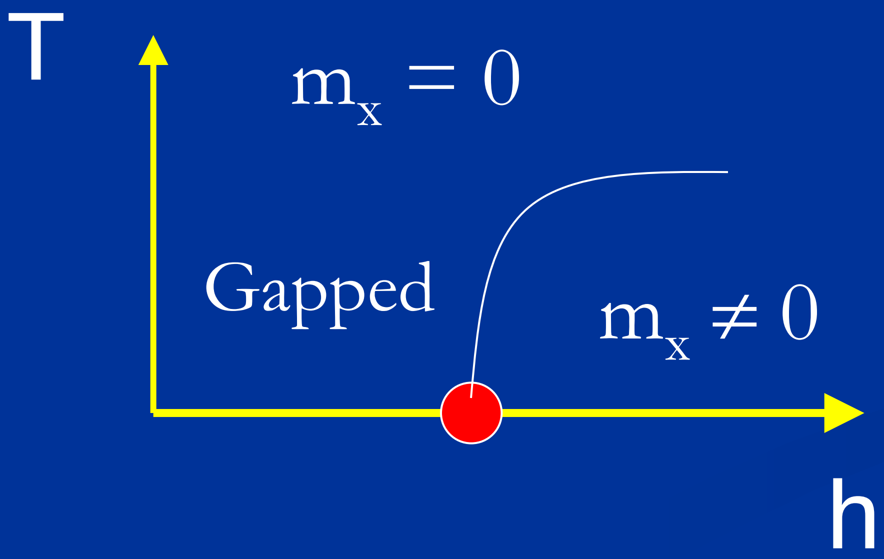

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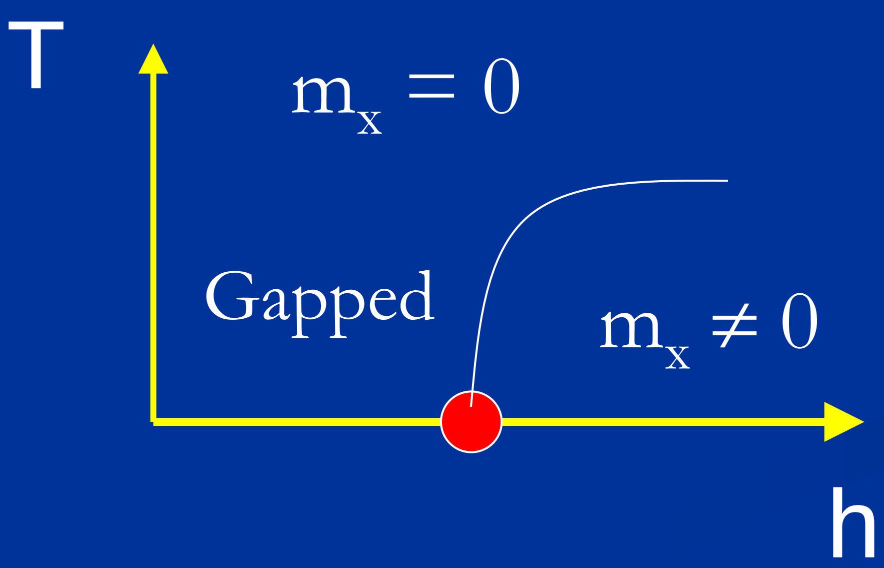

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- NMR relaxation time

$$\frac{1}{T_1} \propto \frac{T}{\sqrt{\Lambda}},$$

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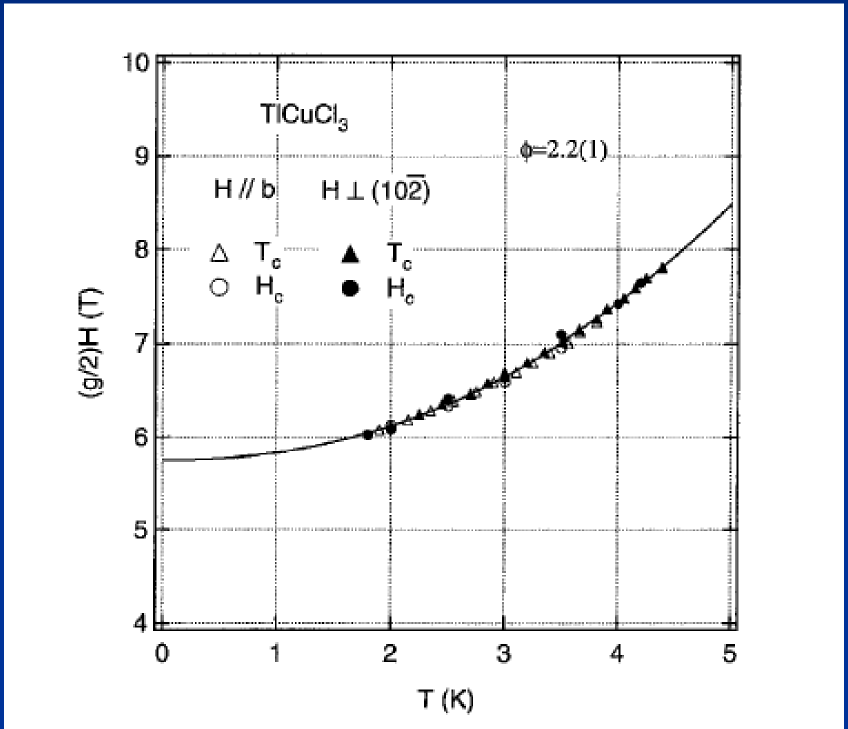

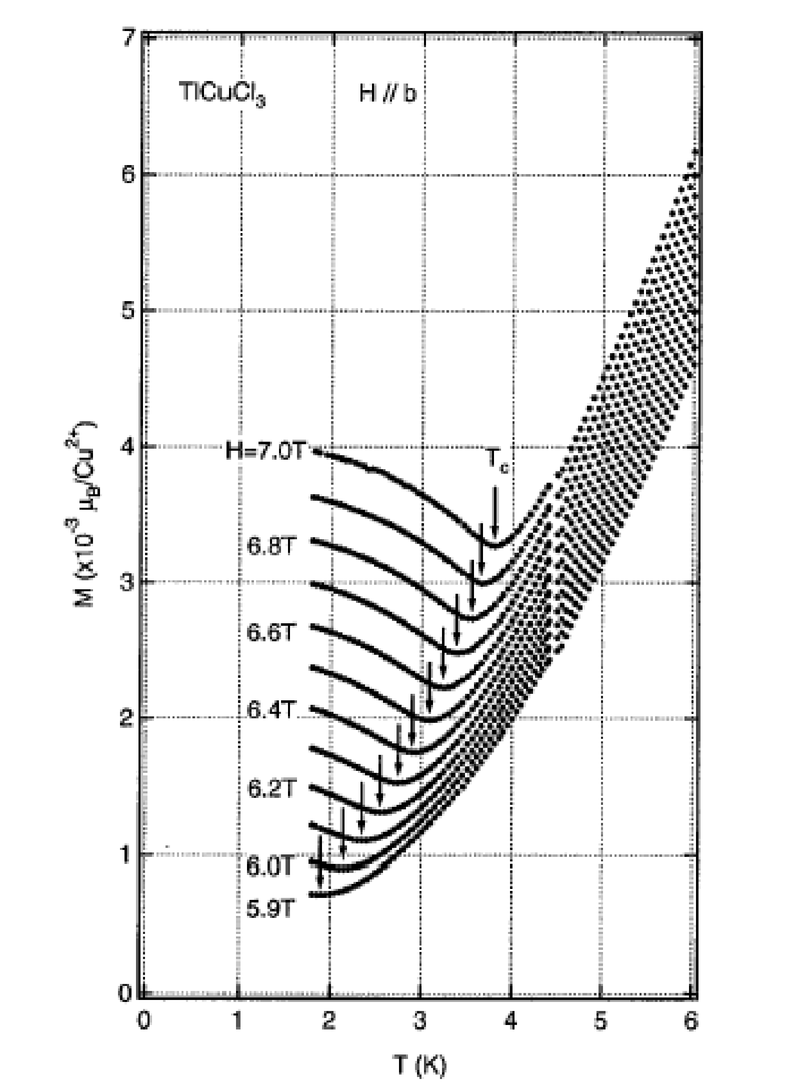

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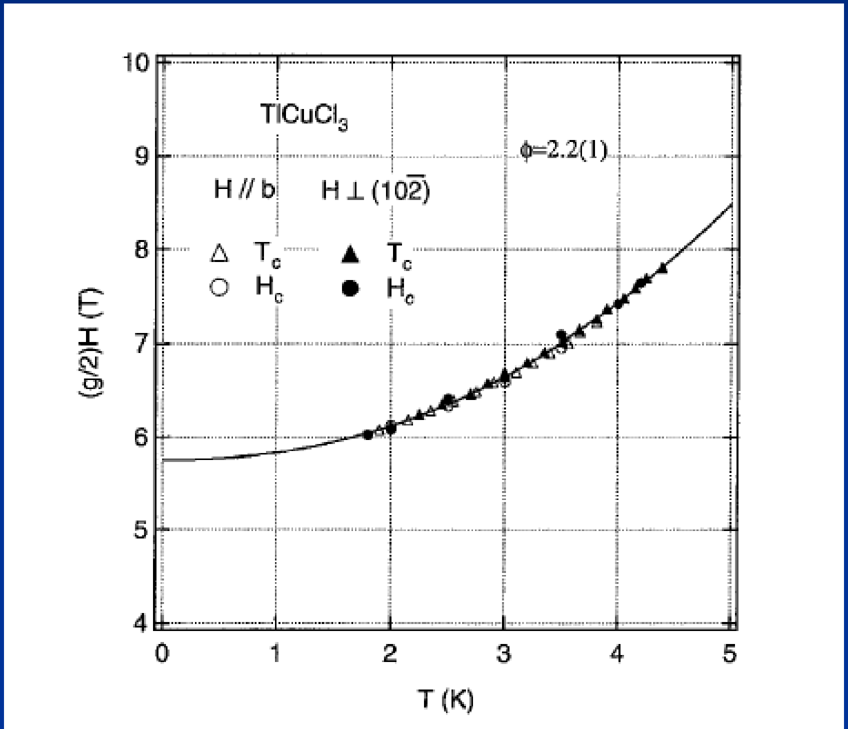

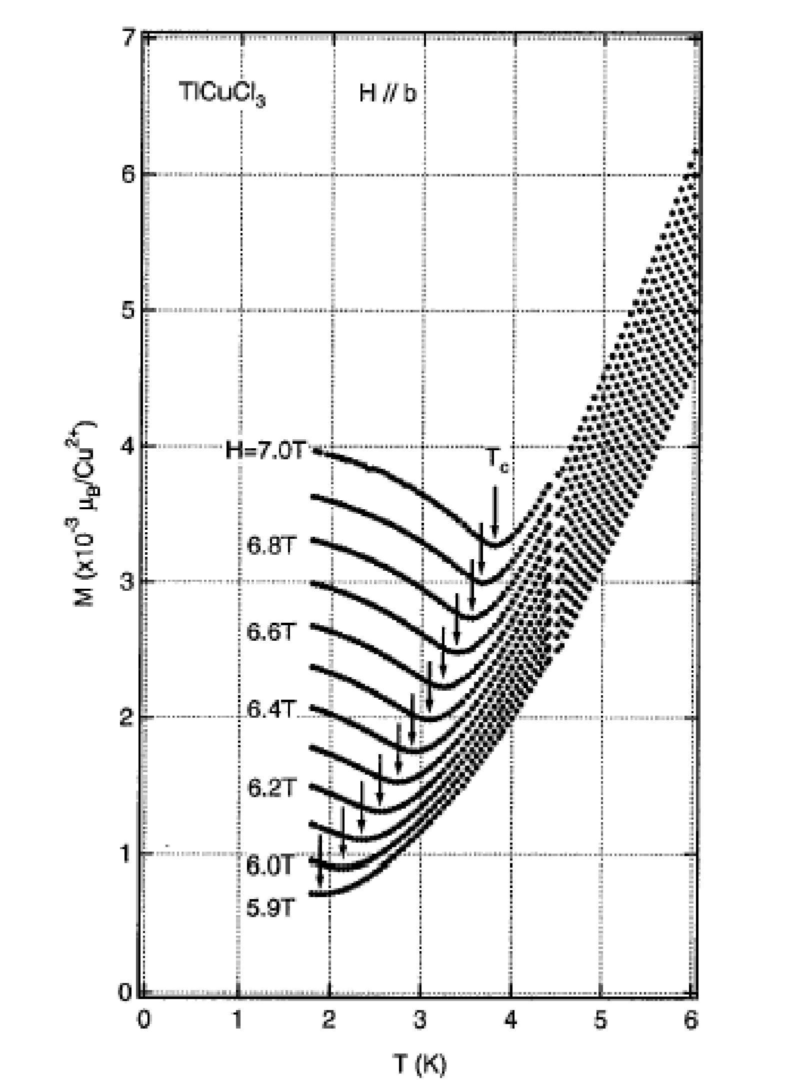

T. Nikuni et al., PRL **84** 5868 (2000)

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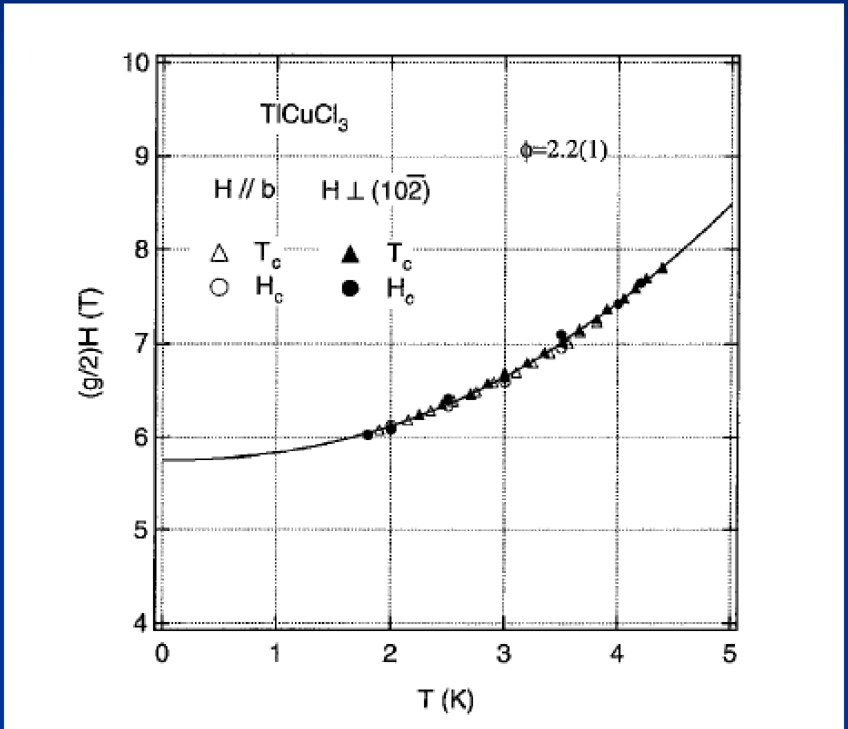

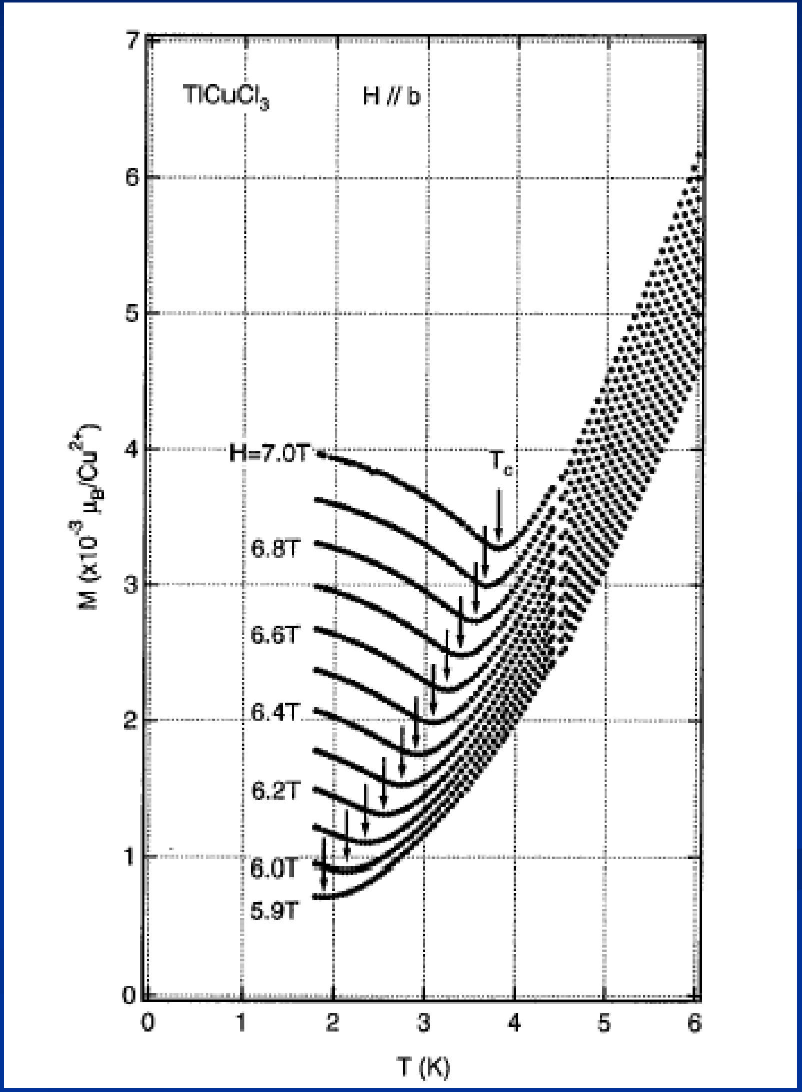

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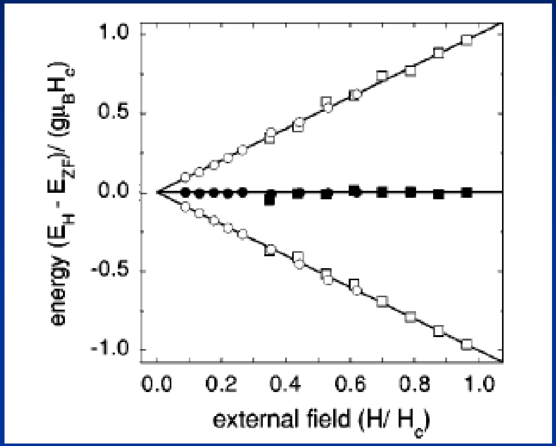

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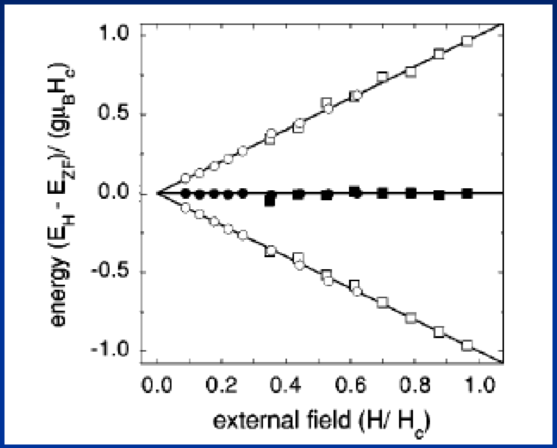

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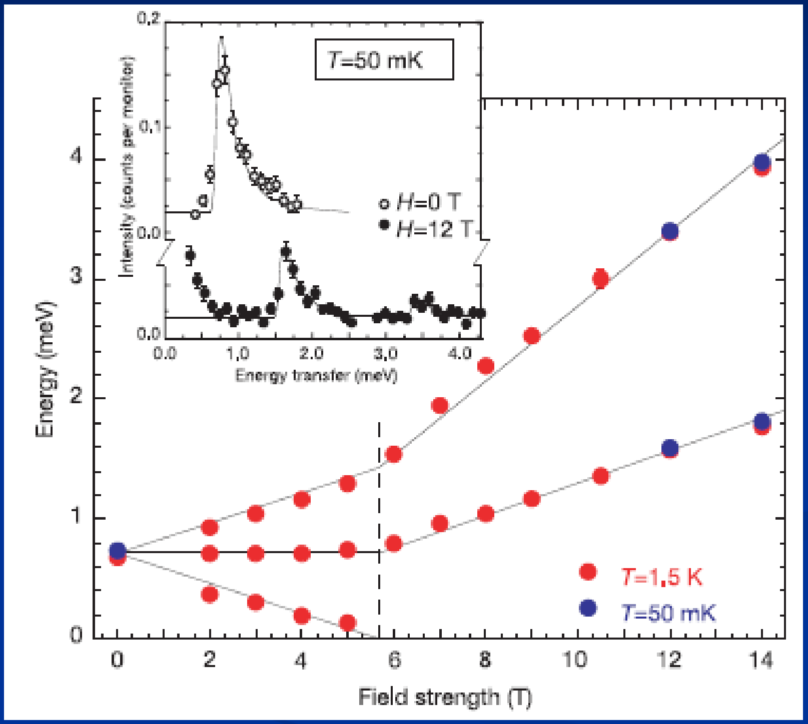

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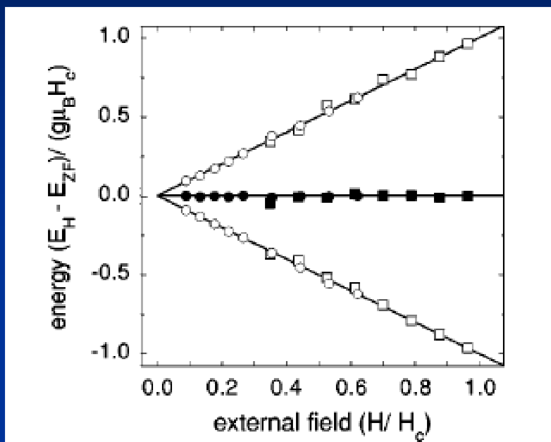

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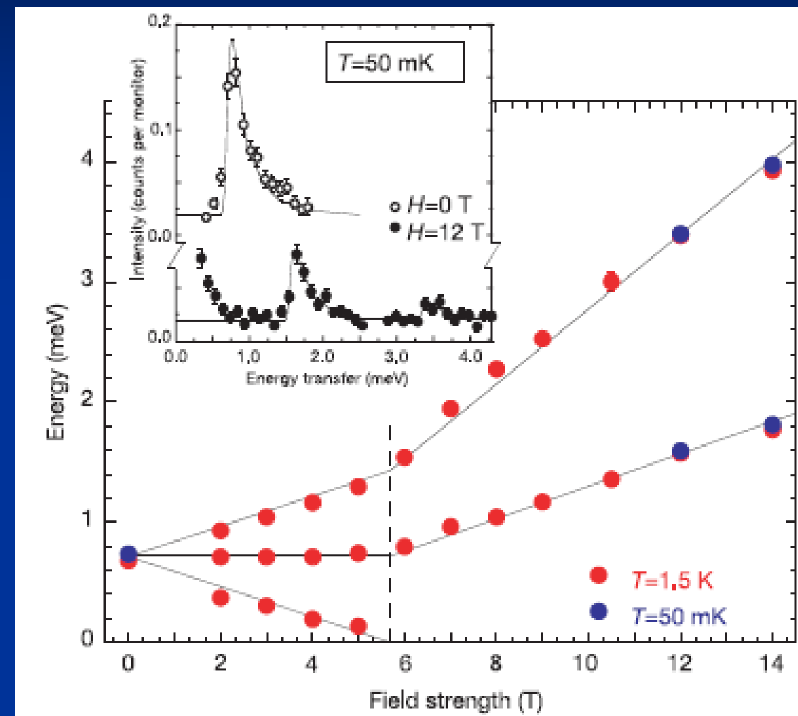

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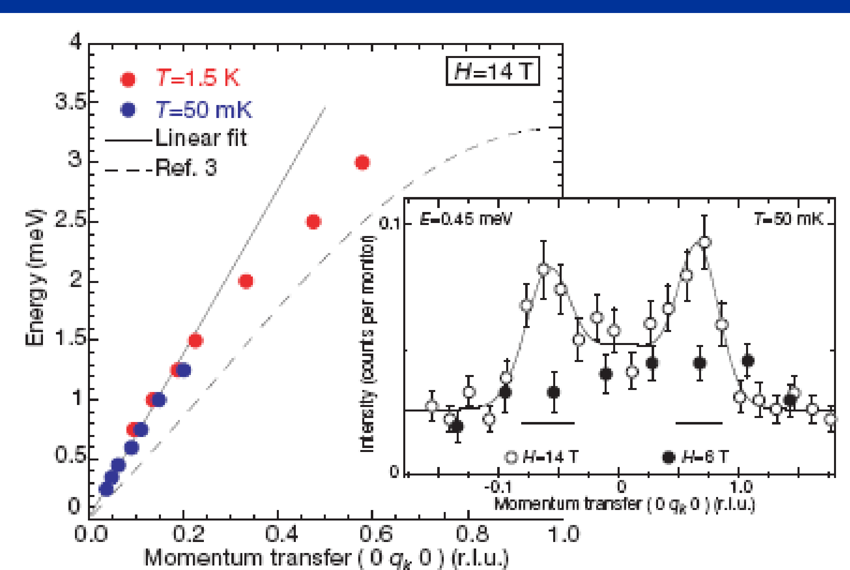

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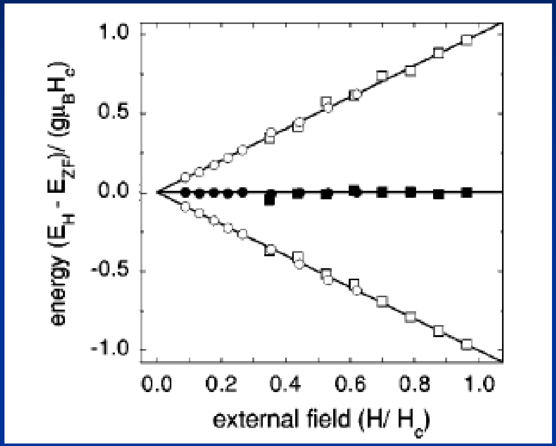

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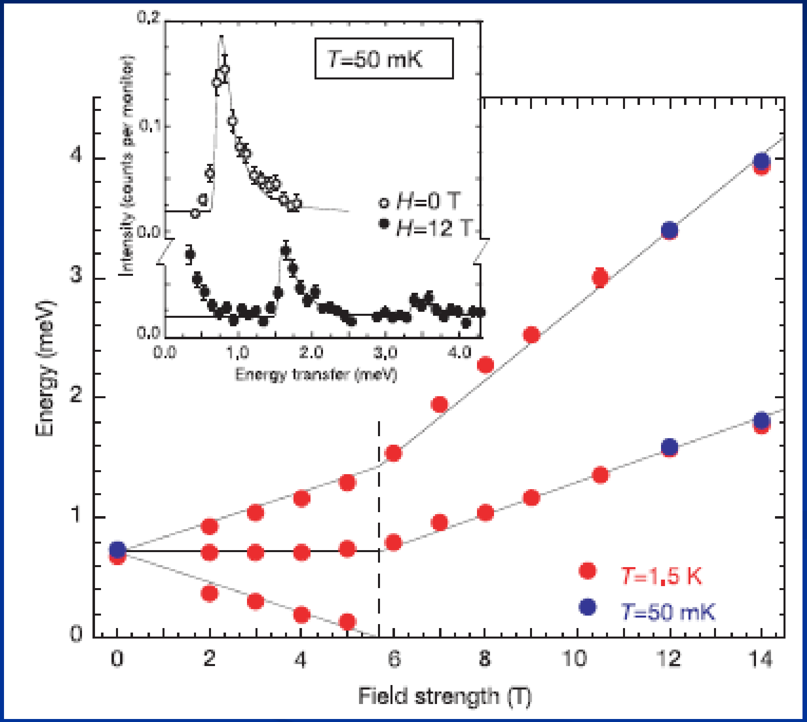

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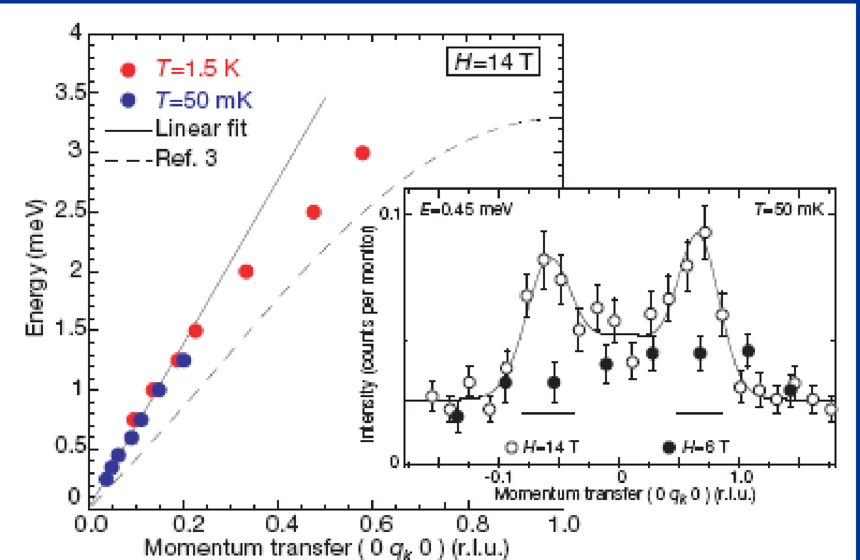

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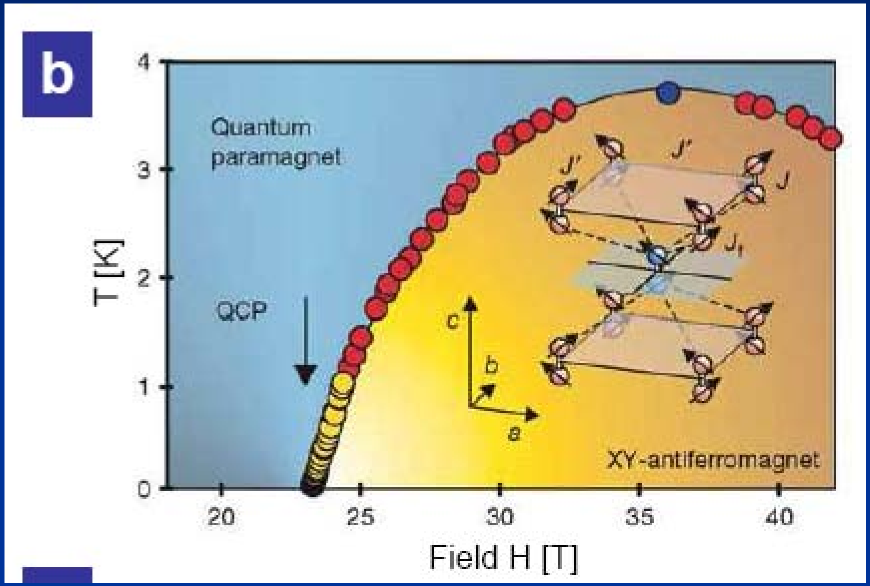

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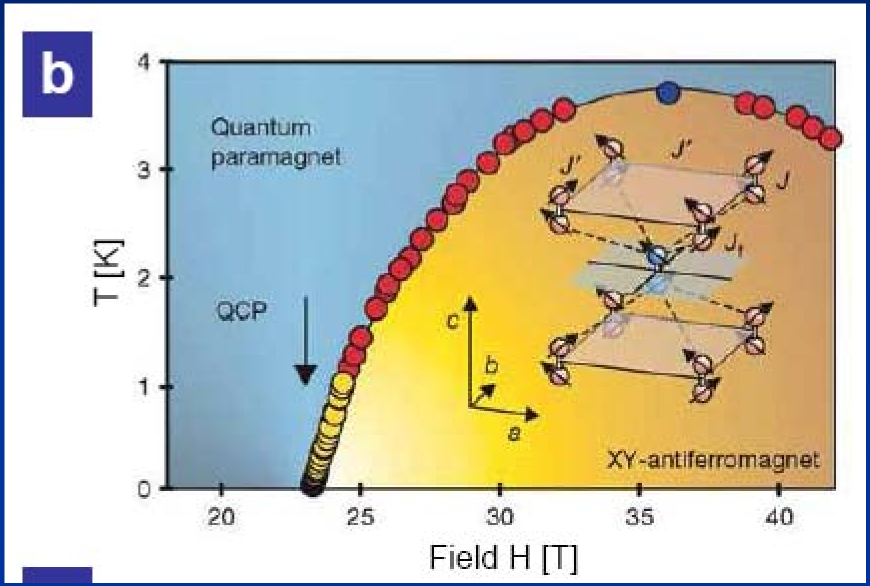

$BaCuSi_2O_6$

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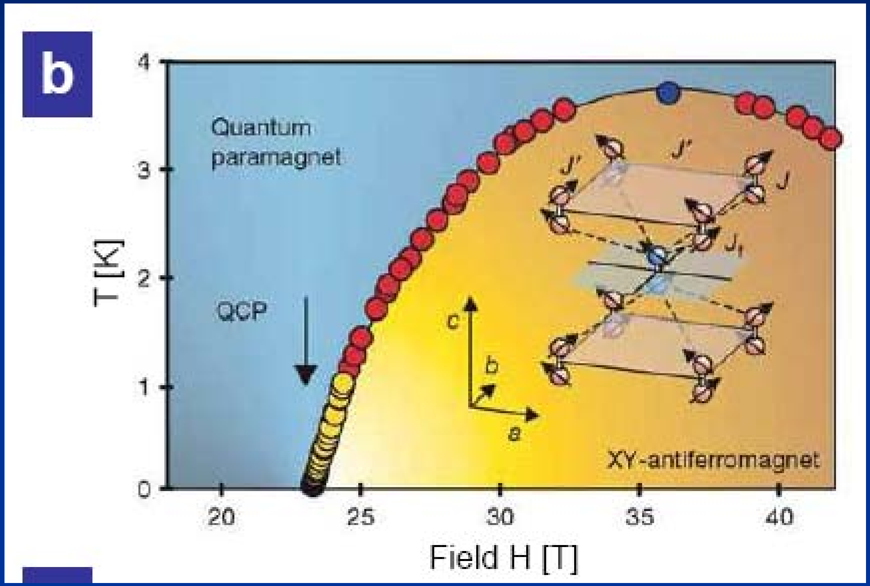

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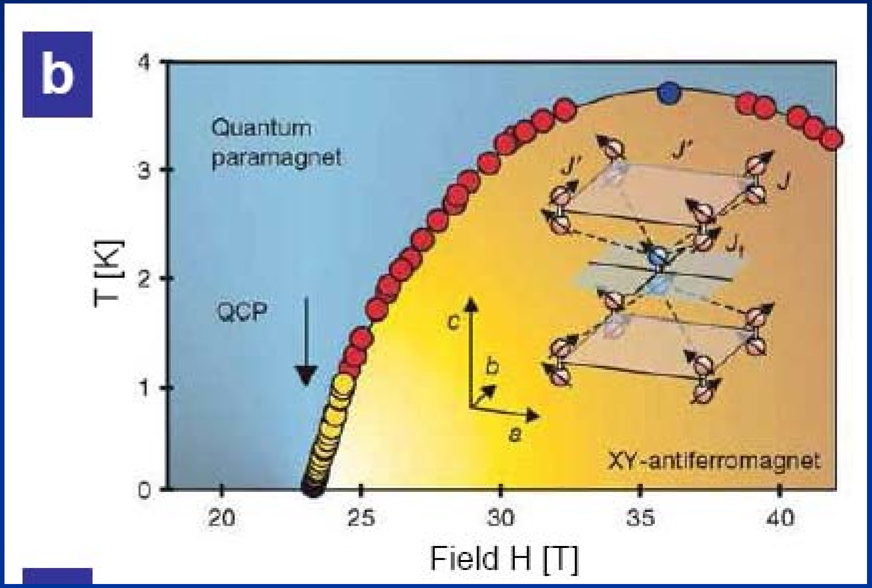

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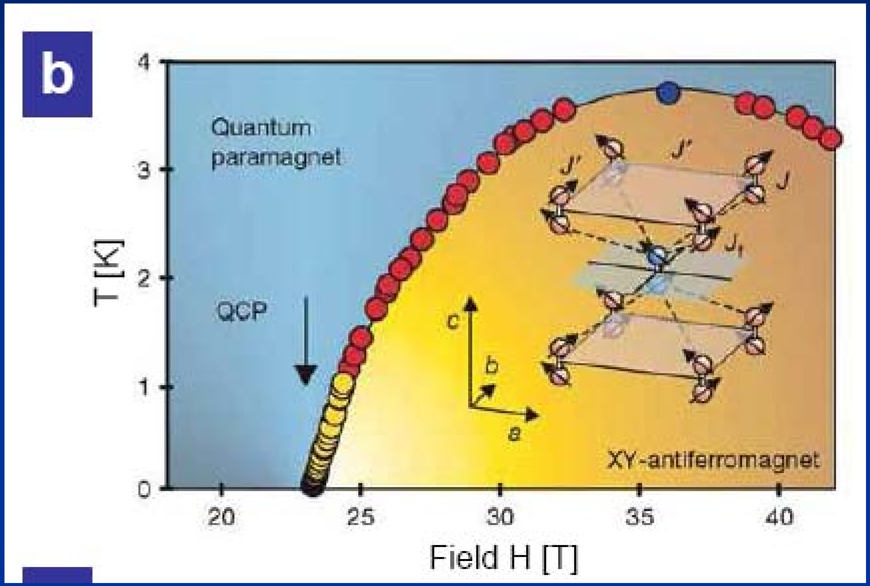

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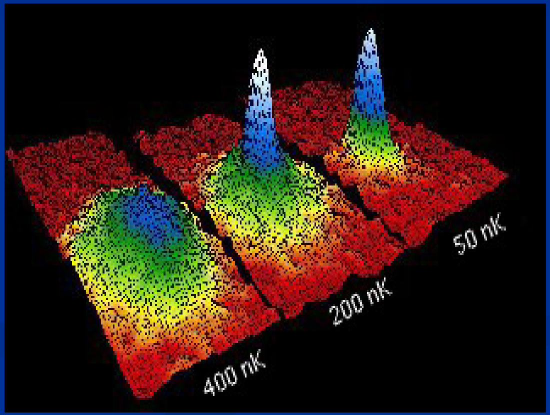
  - probes



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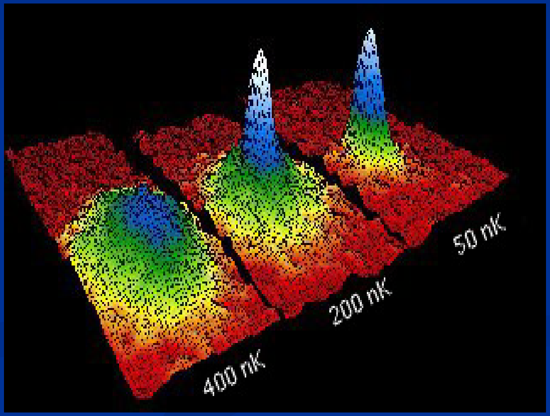

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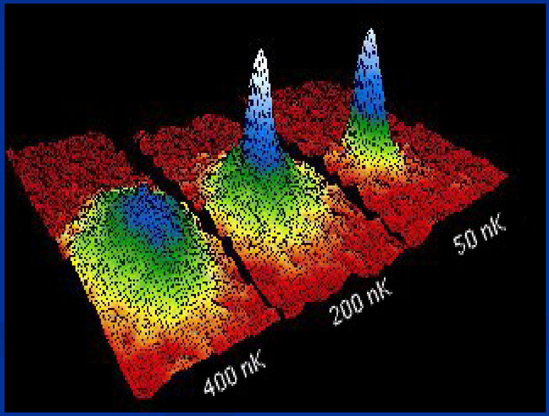

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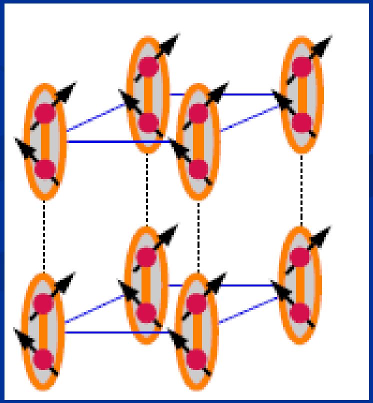

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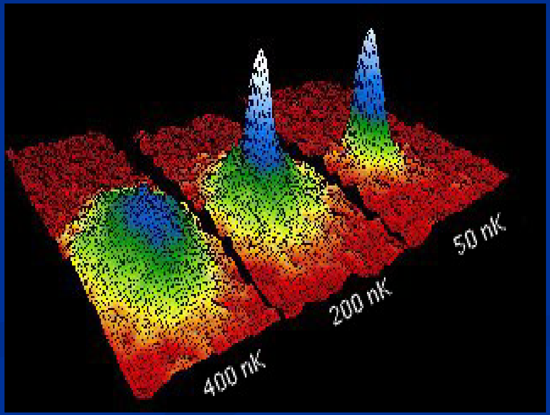

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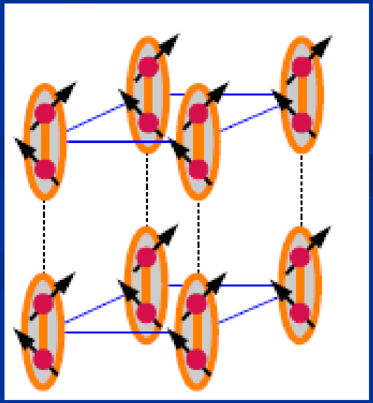

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TG + H. J. Schulz PRB 37 325 (1988);
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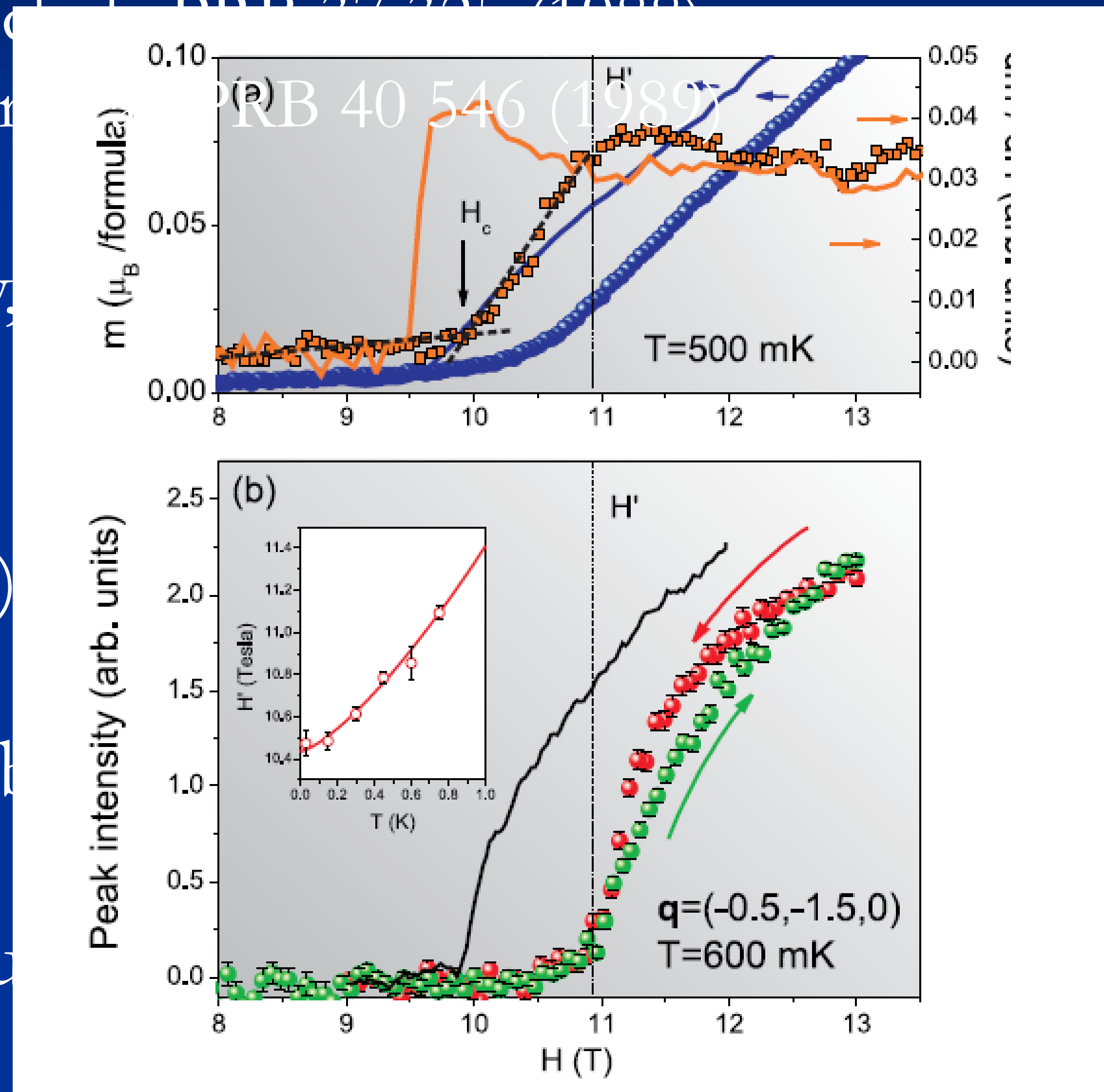
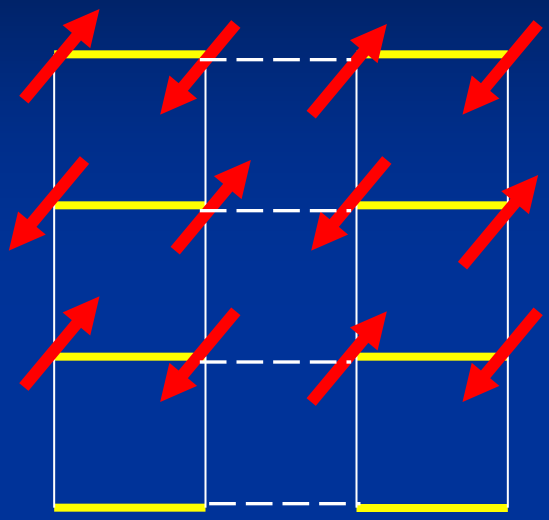


Tomonaga Luttinger Liquid

What about d= 1 ?



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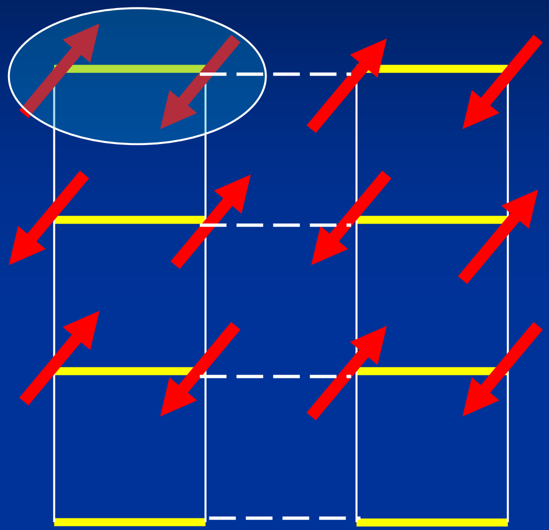

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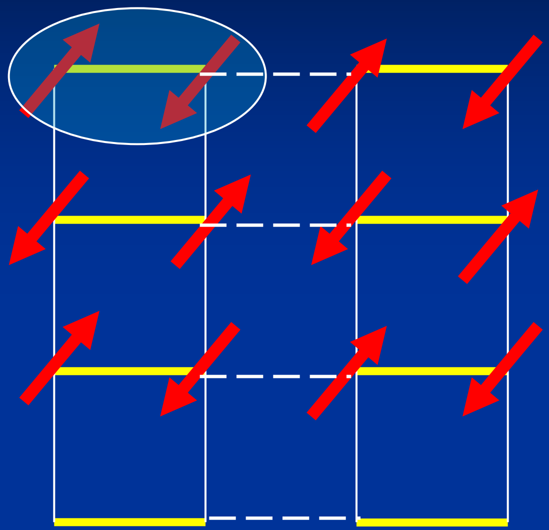

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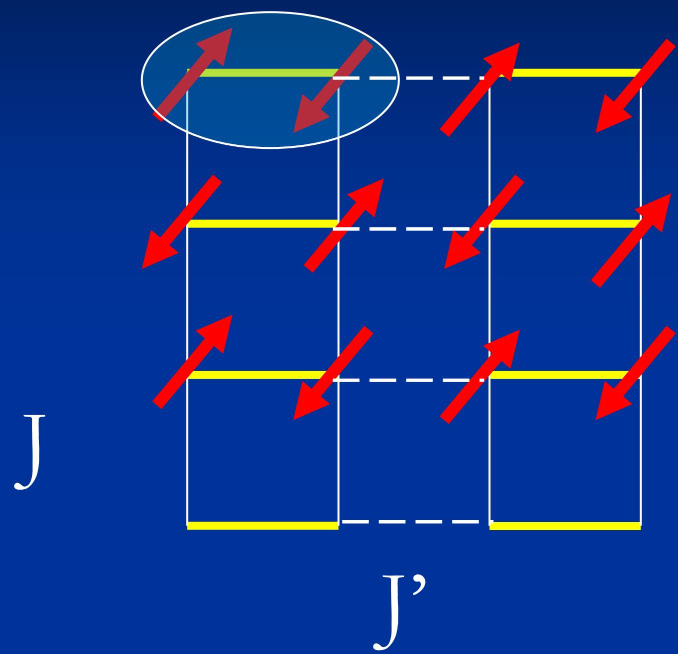

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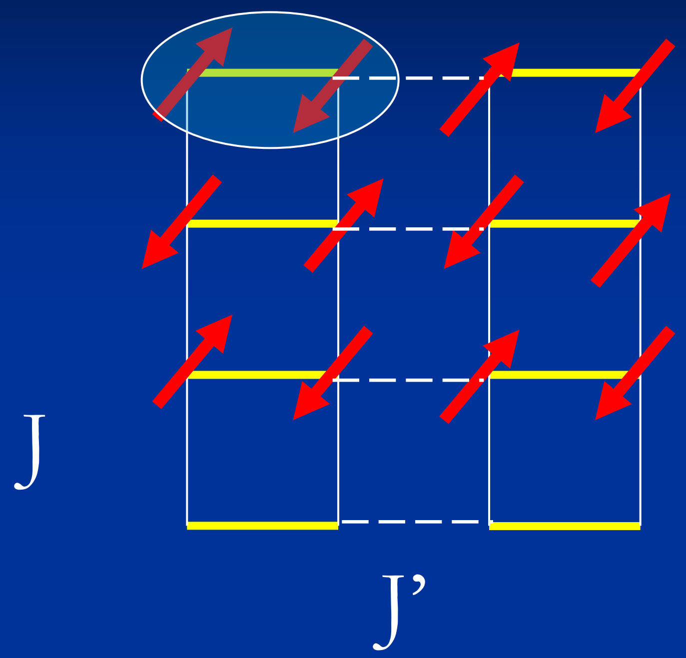

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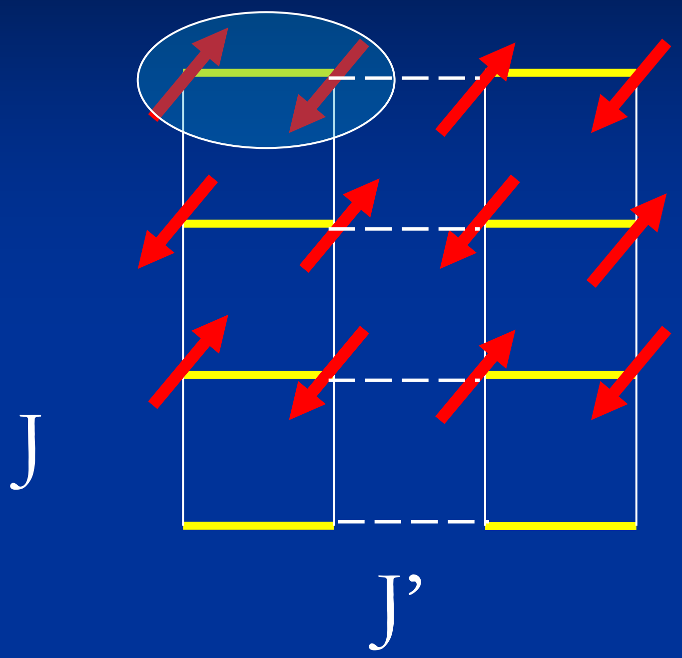

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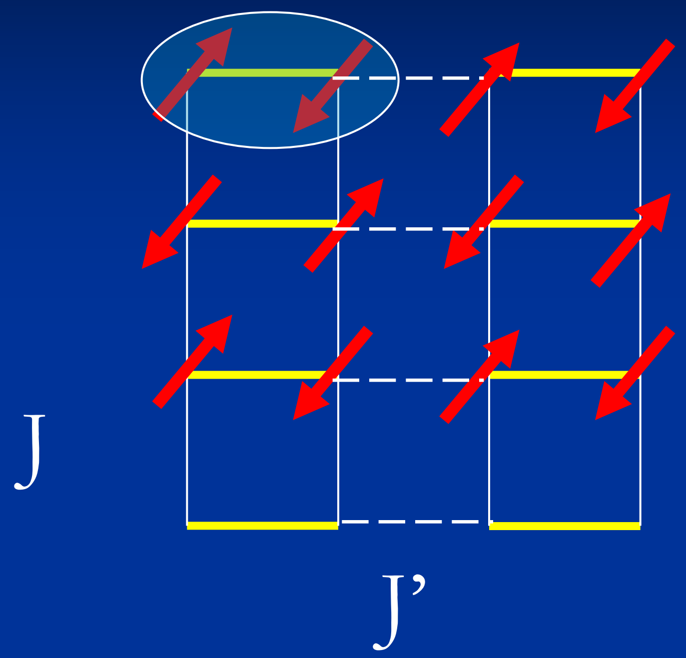

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- Allow to study interacting fermions/bosons: Tomonaga Luttinger liquid

Tomonaga Luttinger liquid theory

$$\left\langle S_z(x) S_z(0) \right\rangle = \frac{1}{x^2} + \cos(\pi x / a)\left(\frac{1}{x}\right)^{2K}$$

Tomonaga Luttinger liquid theory

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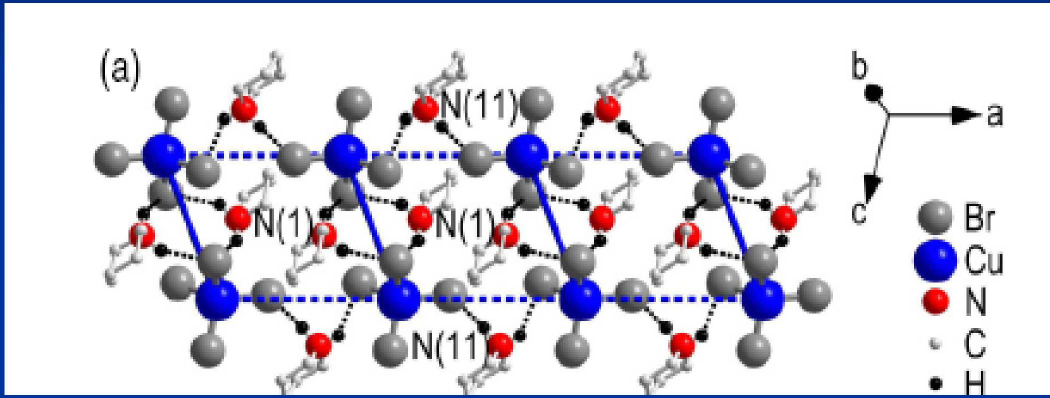

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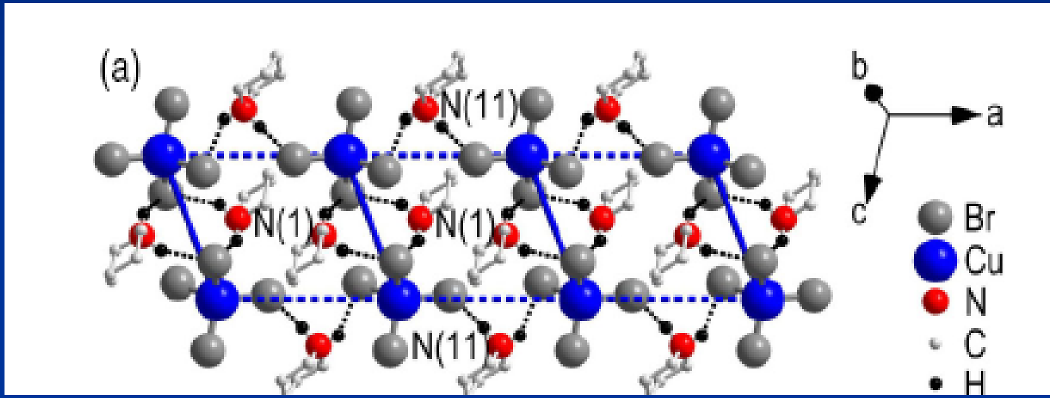

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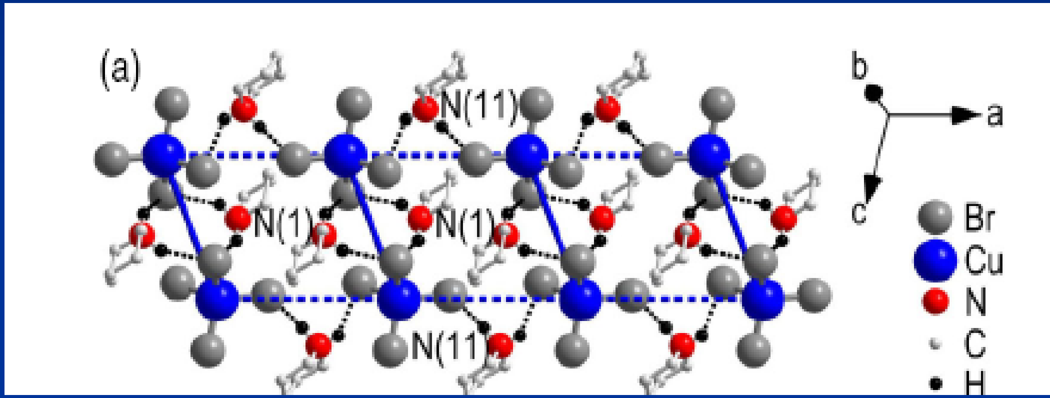

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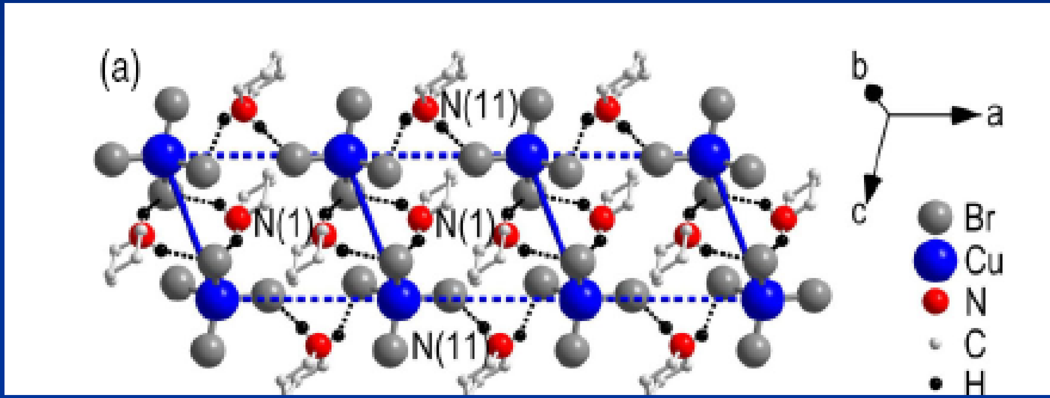

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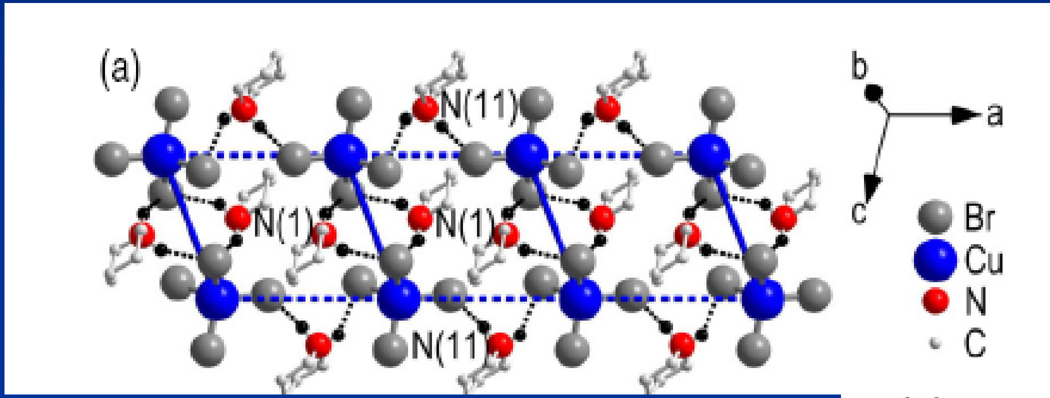

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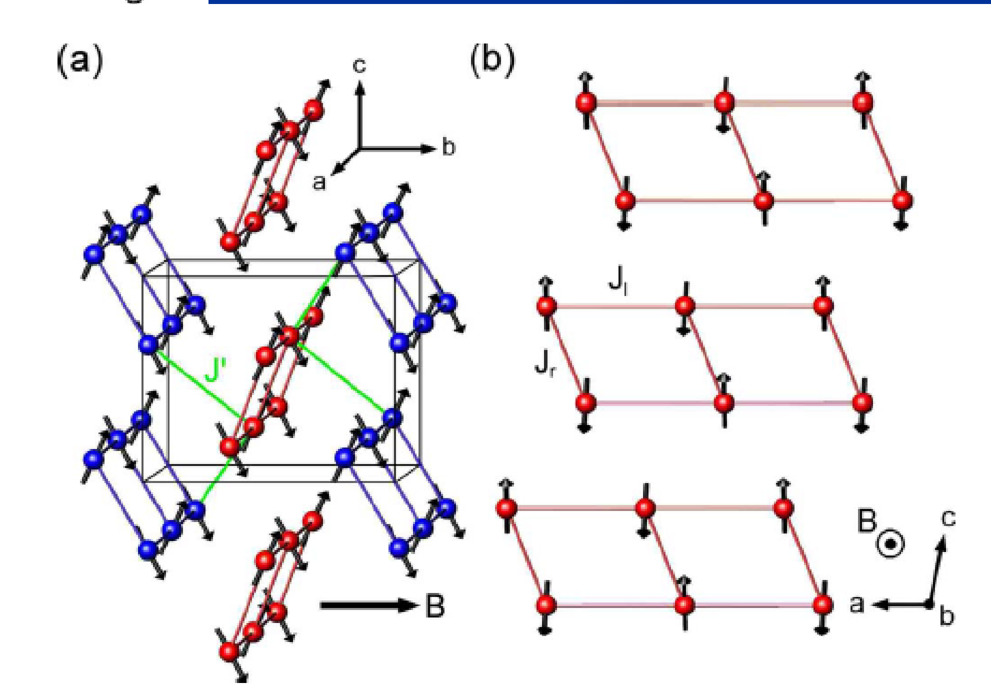

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S. Sachdev , T. Senthil, R. Shankar PRB 50 258 (94); R. Chitra, TG PRB 55 5816 (97); TG, A.M. Tsvelik PRB 59 11398 (99)

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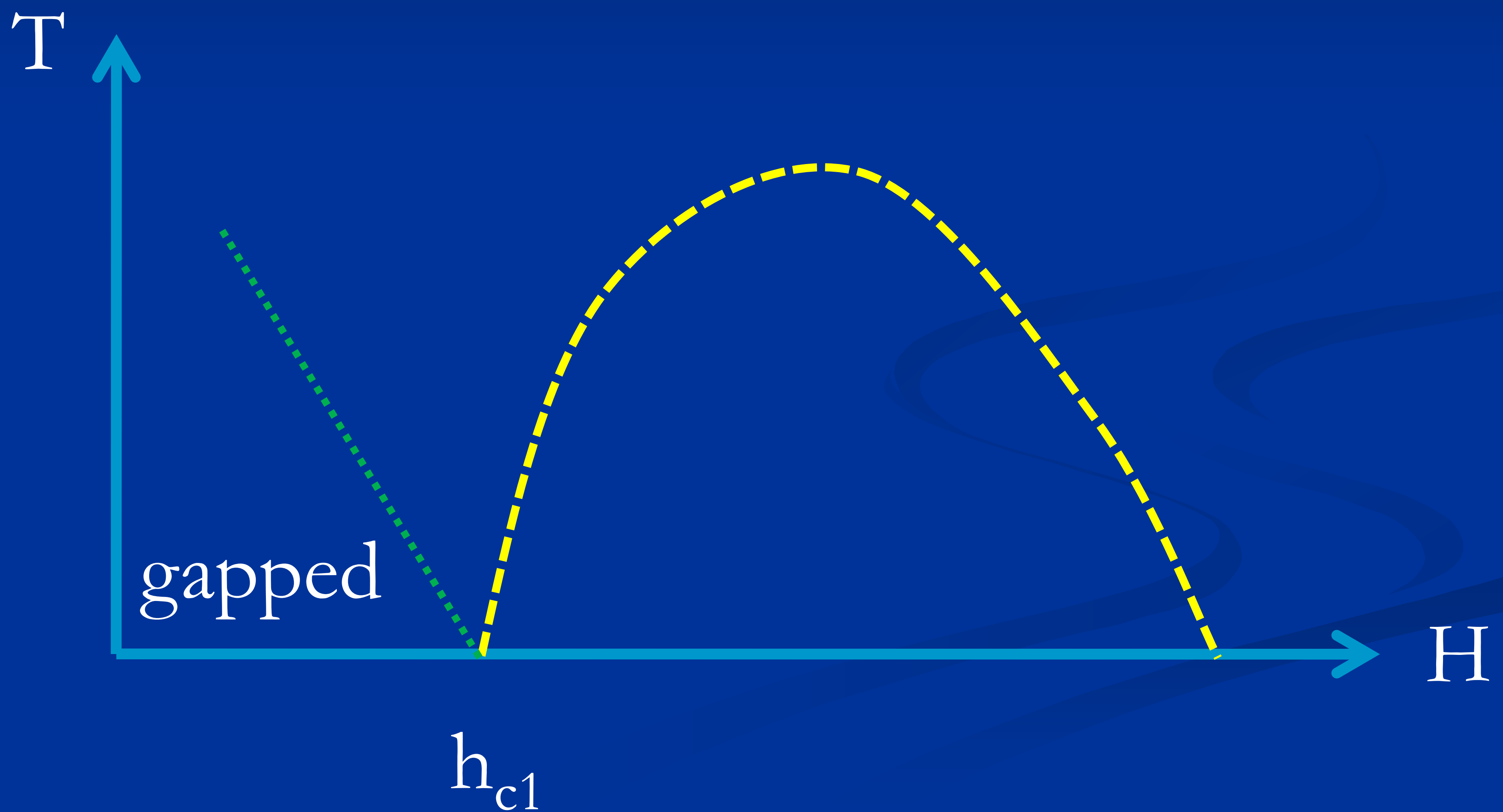

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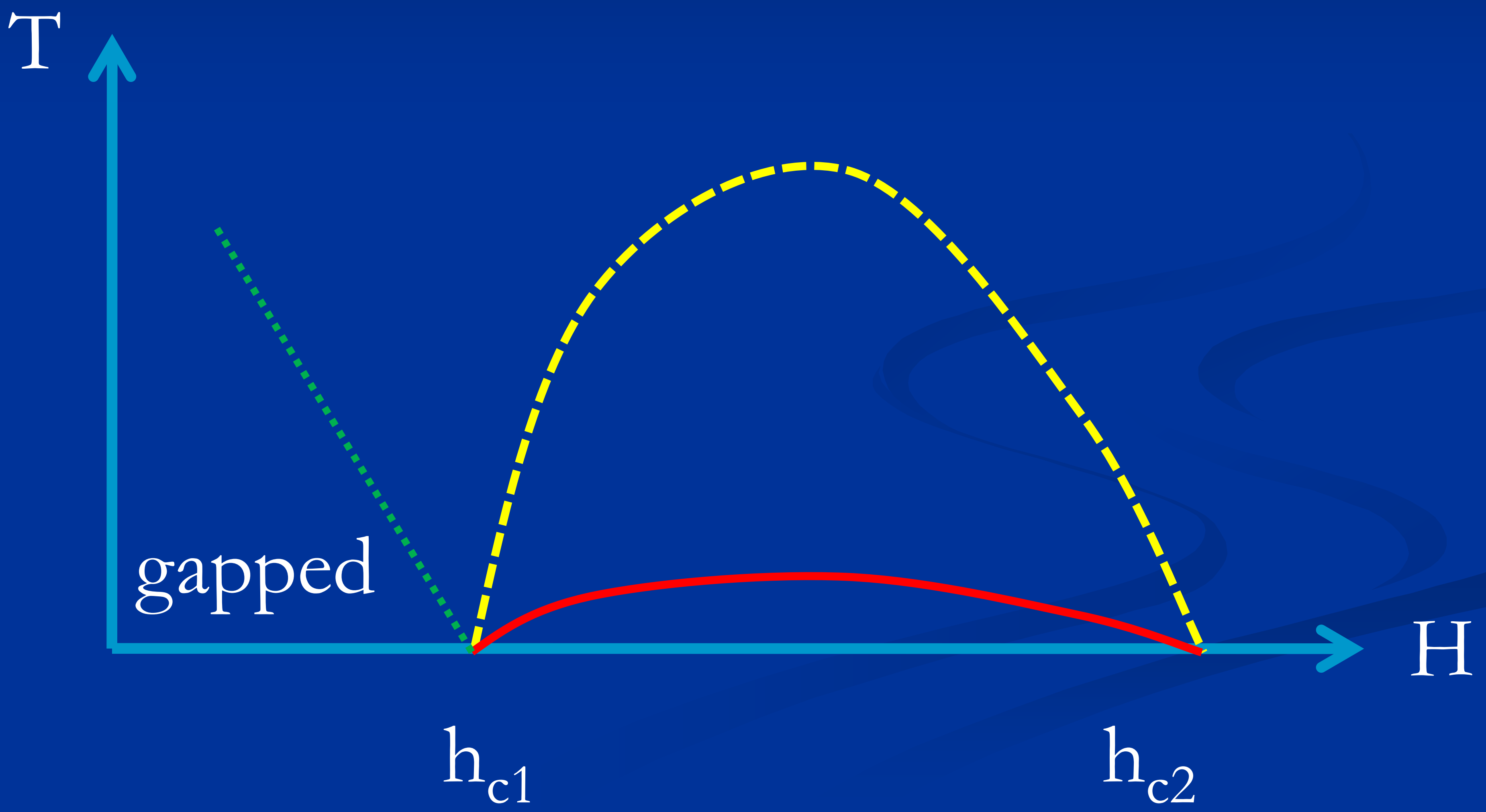

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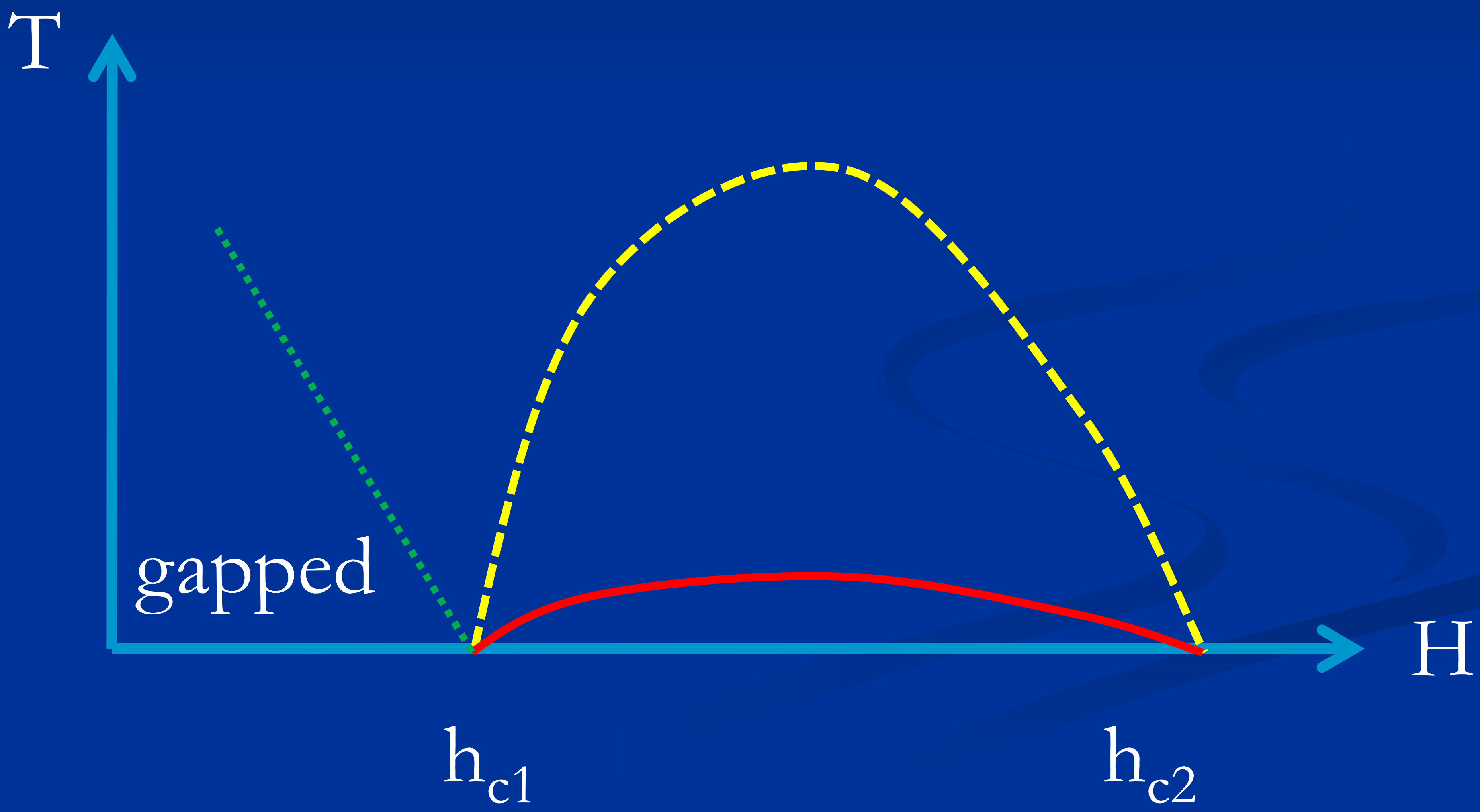

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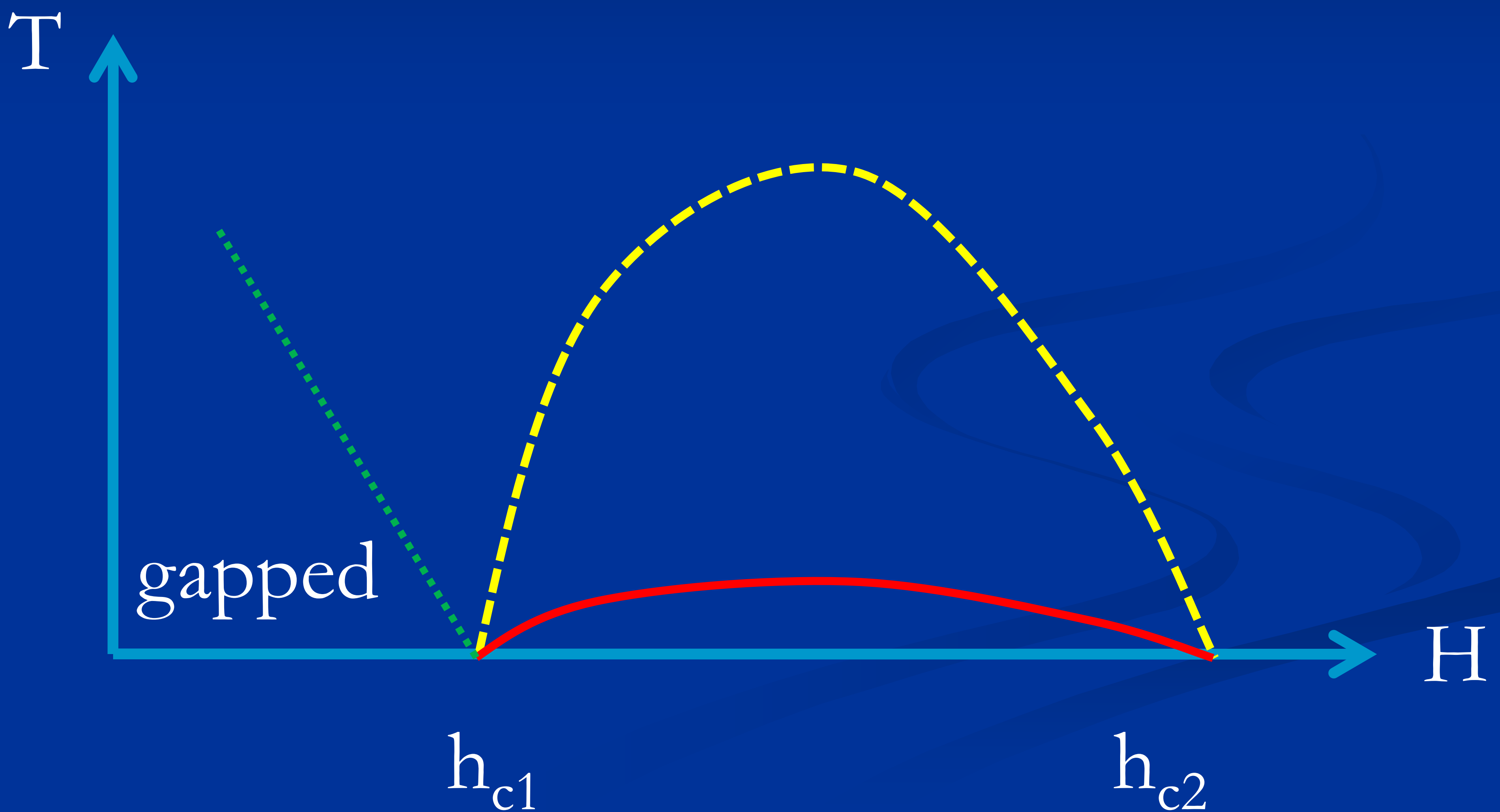

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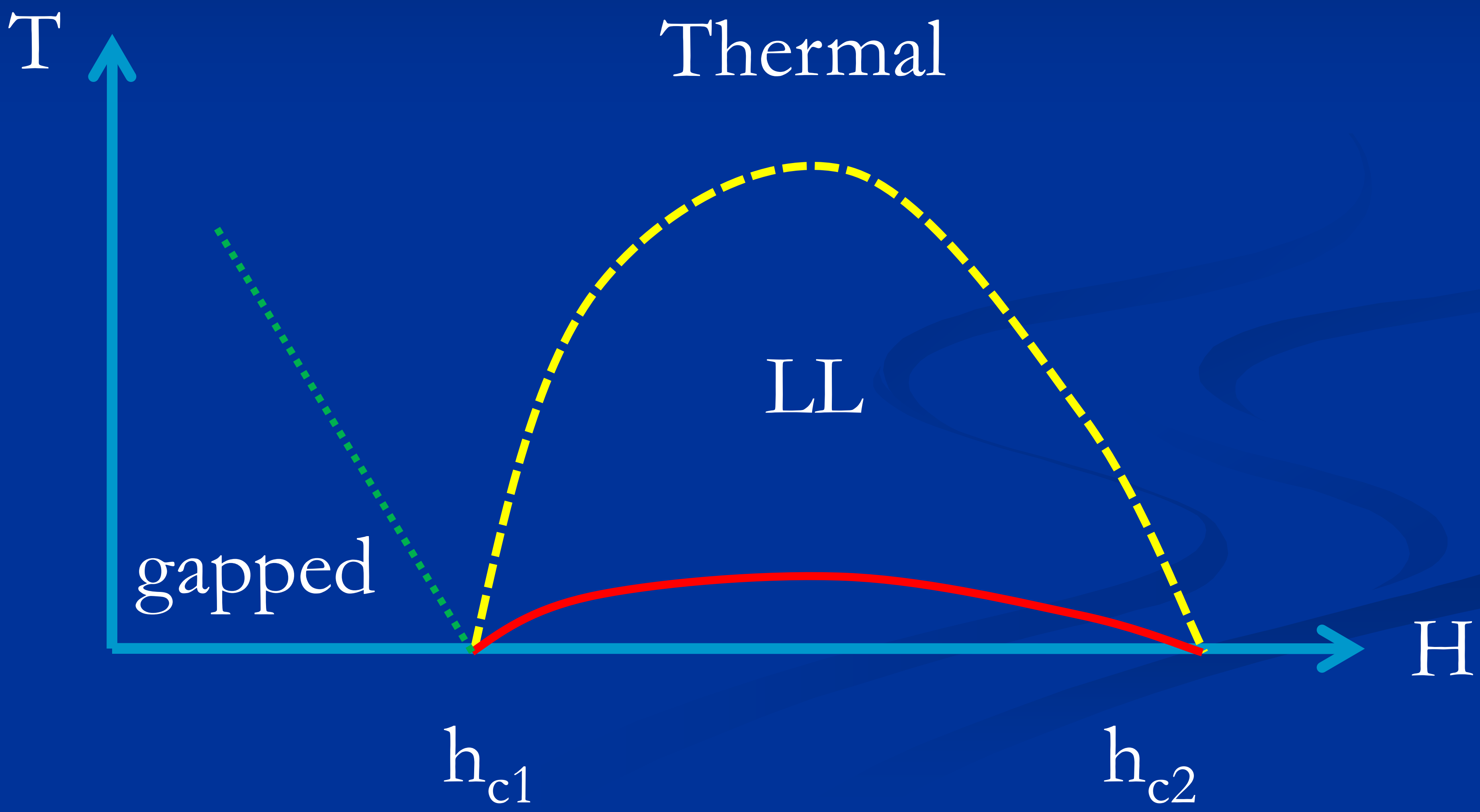

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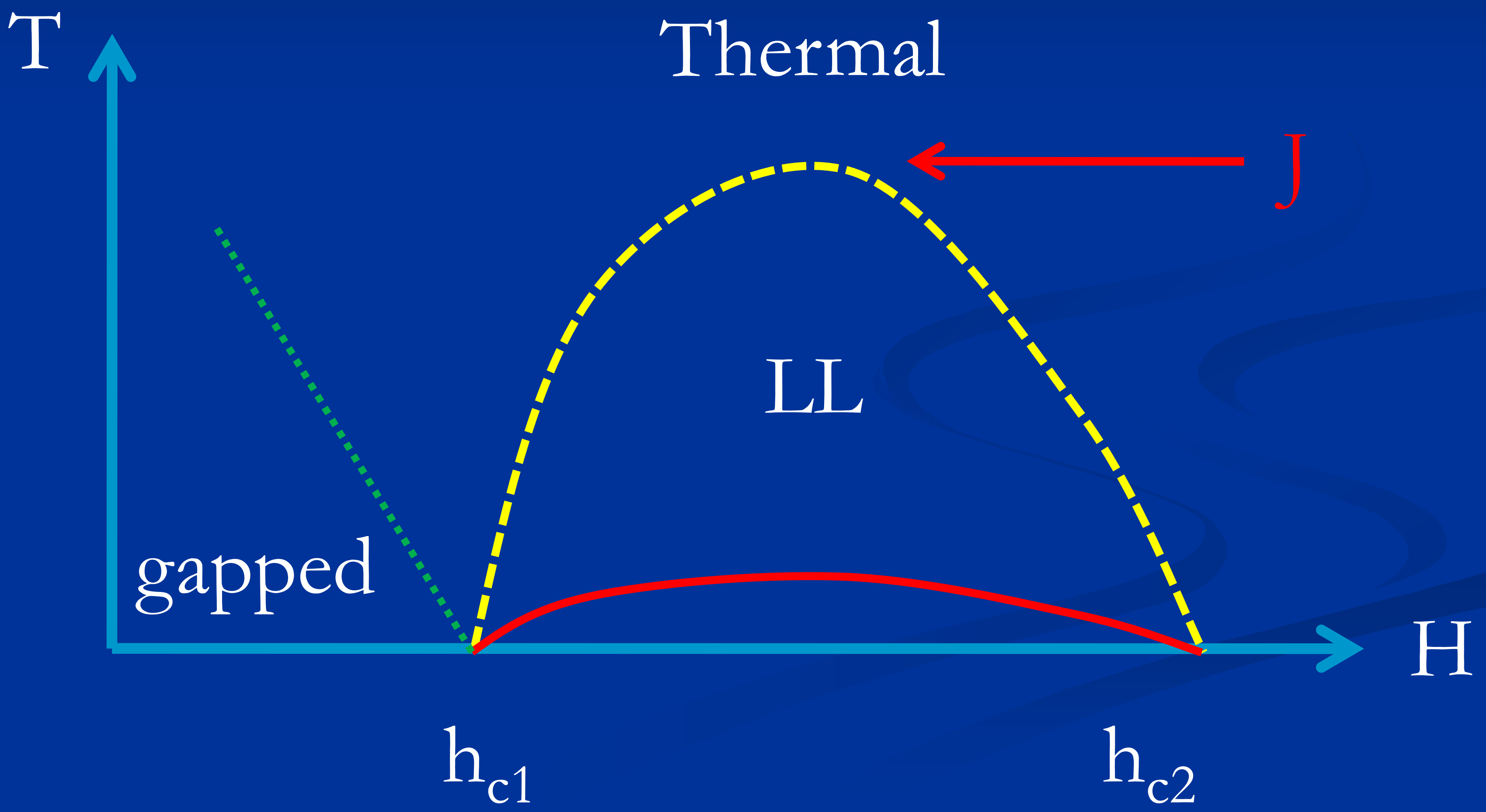

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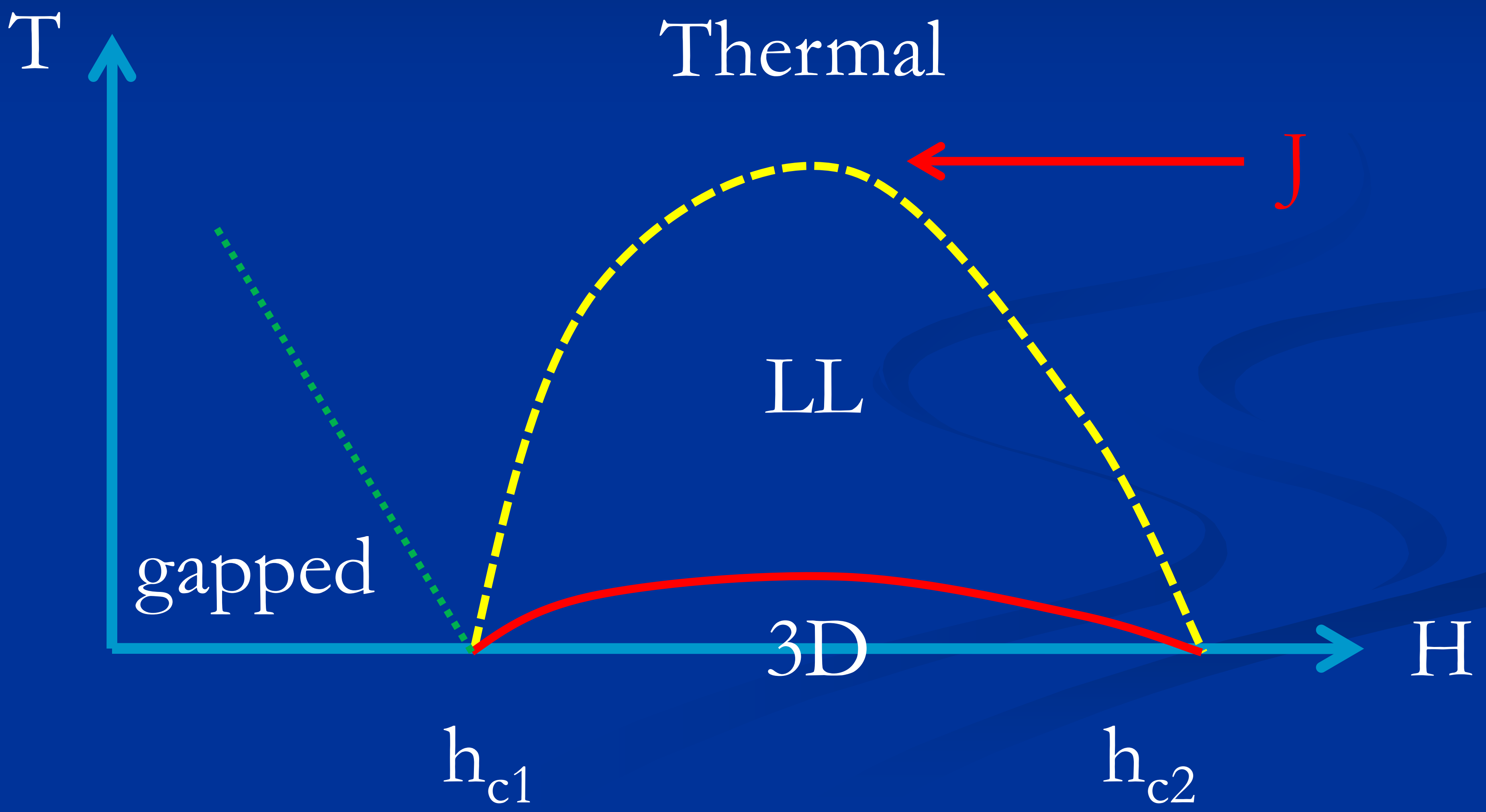

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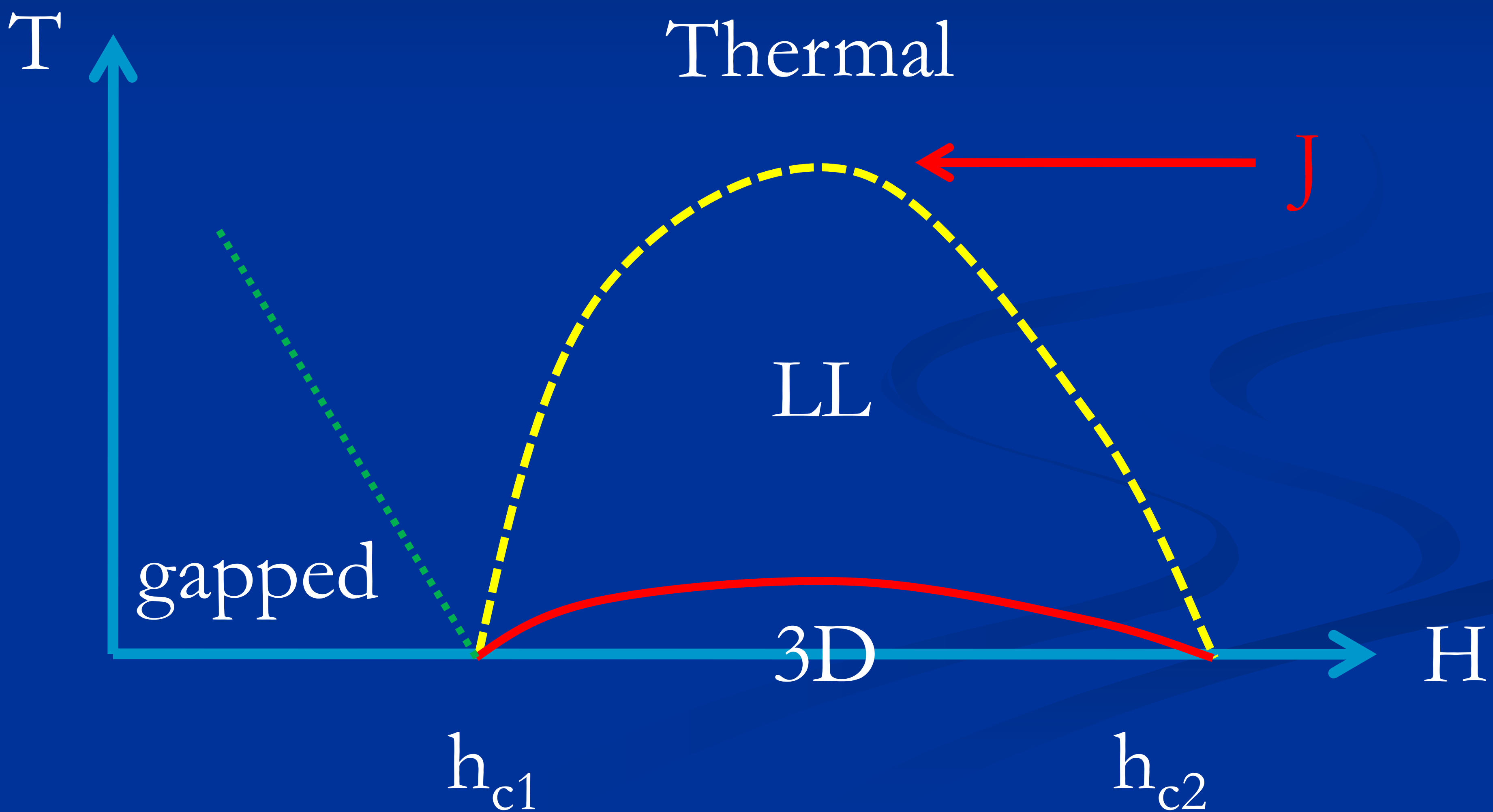

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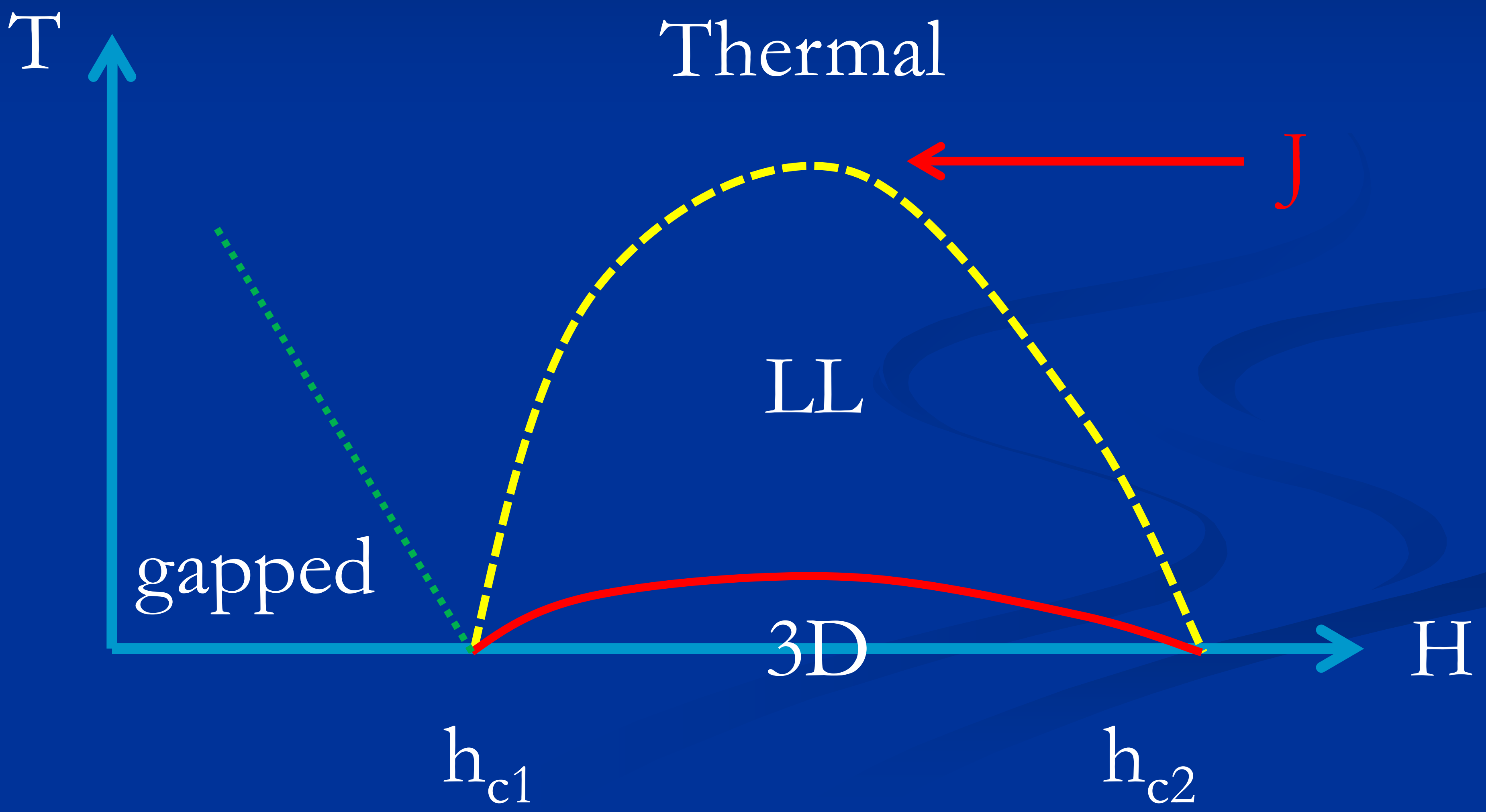

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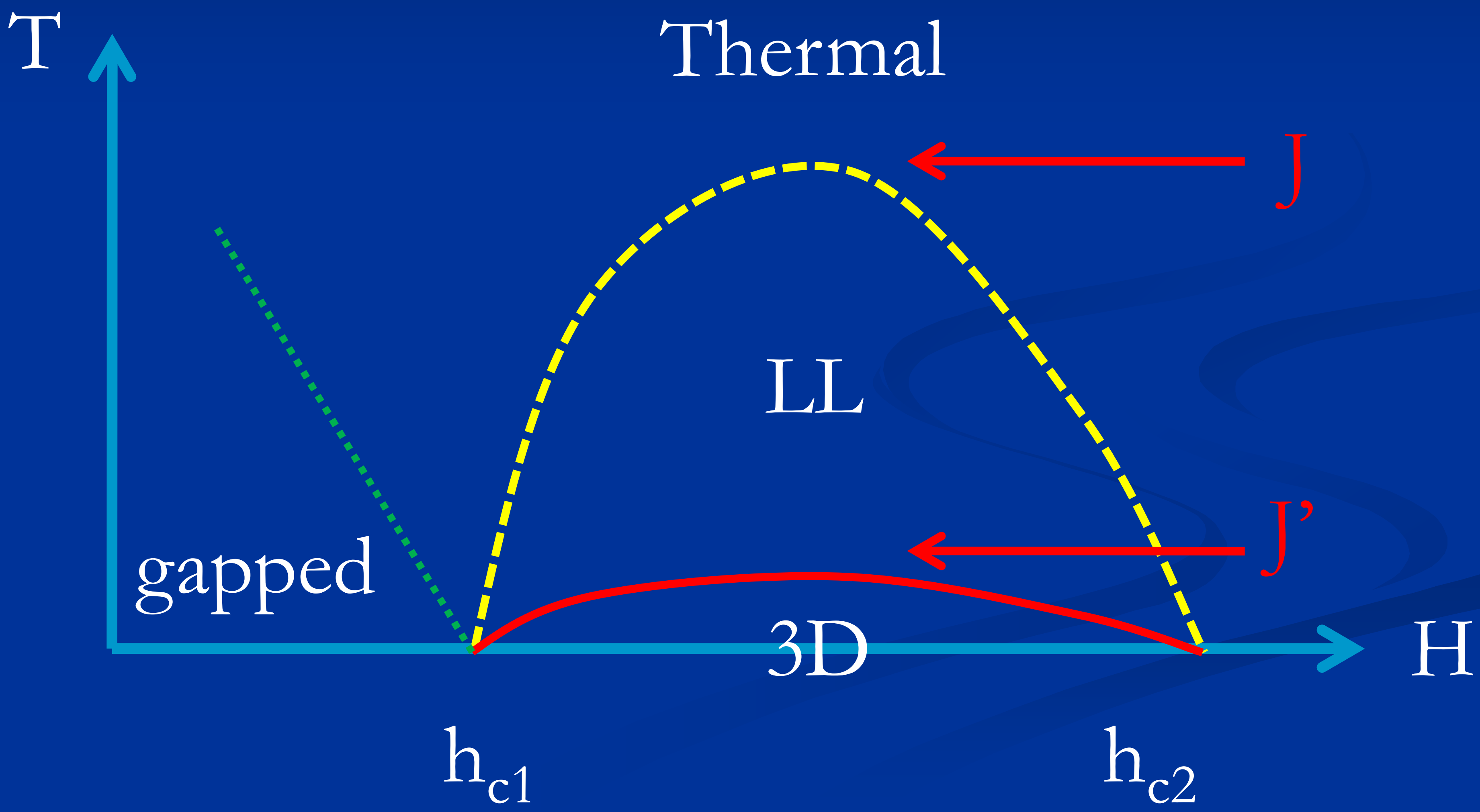

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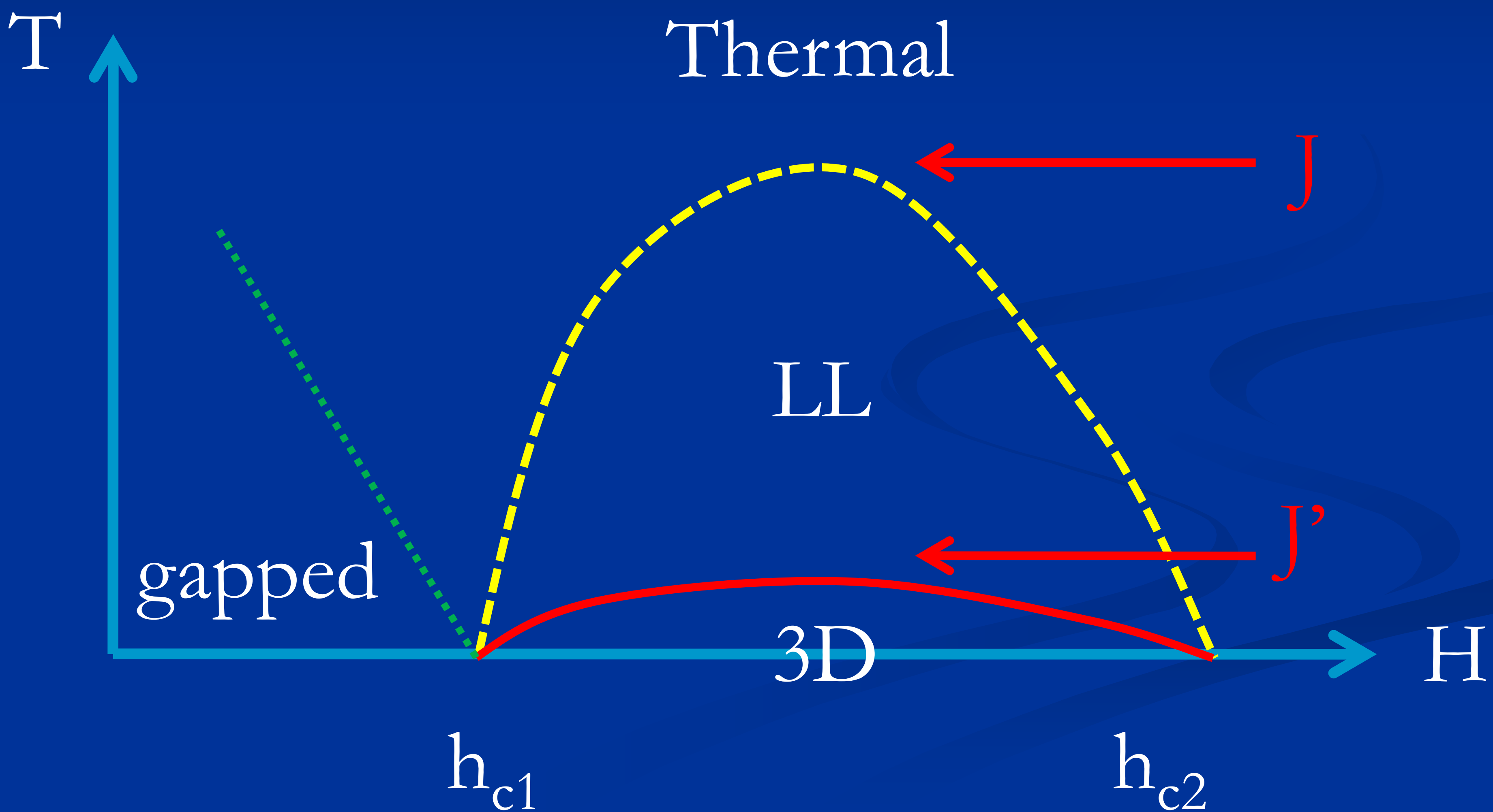

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- Analytical calculations (Luttinger liquid + BA)

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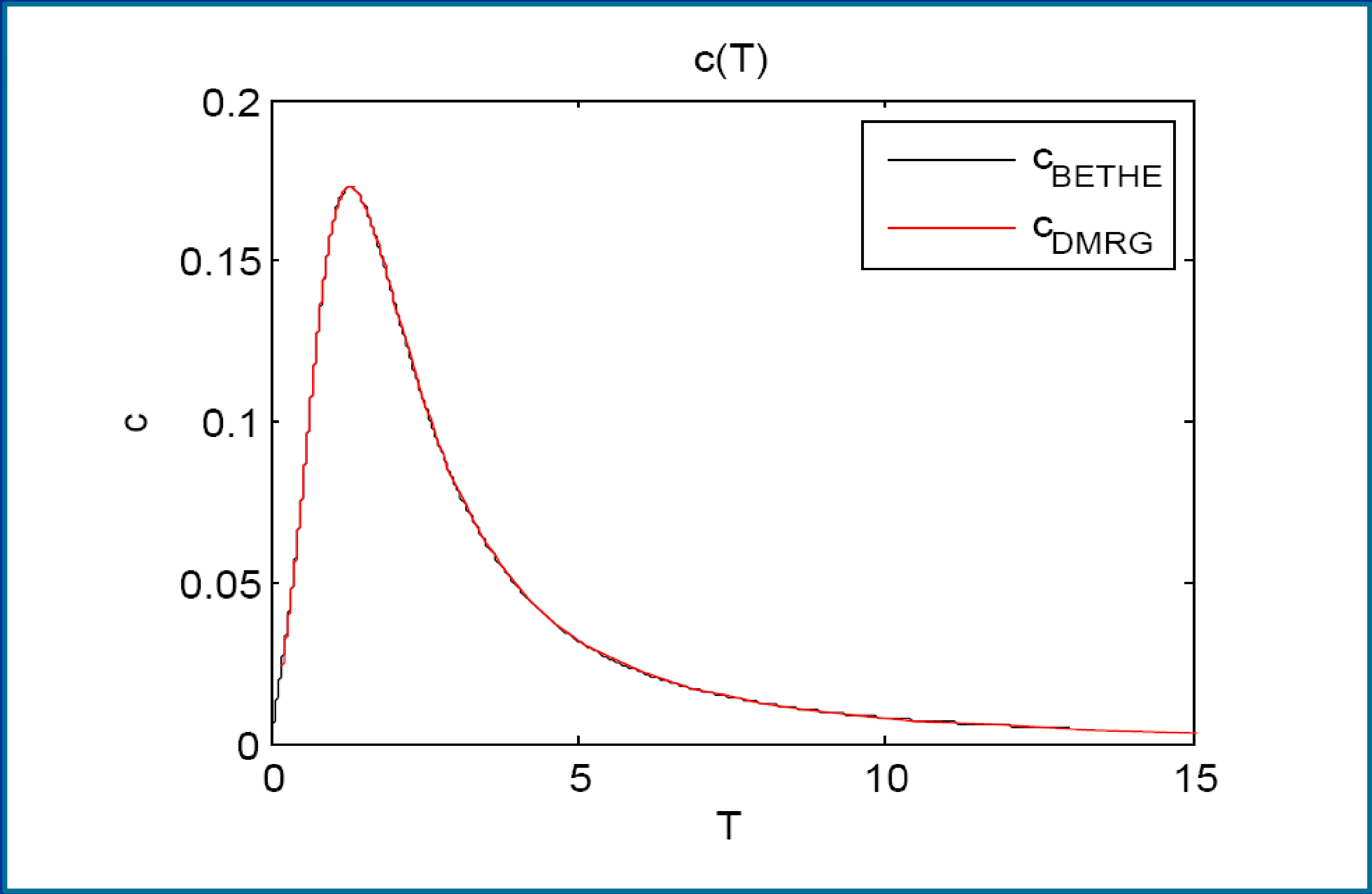
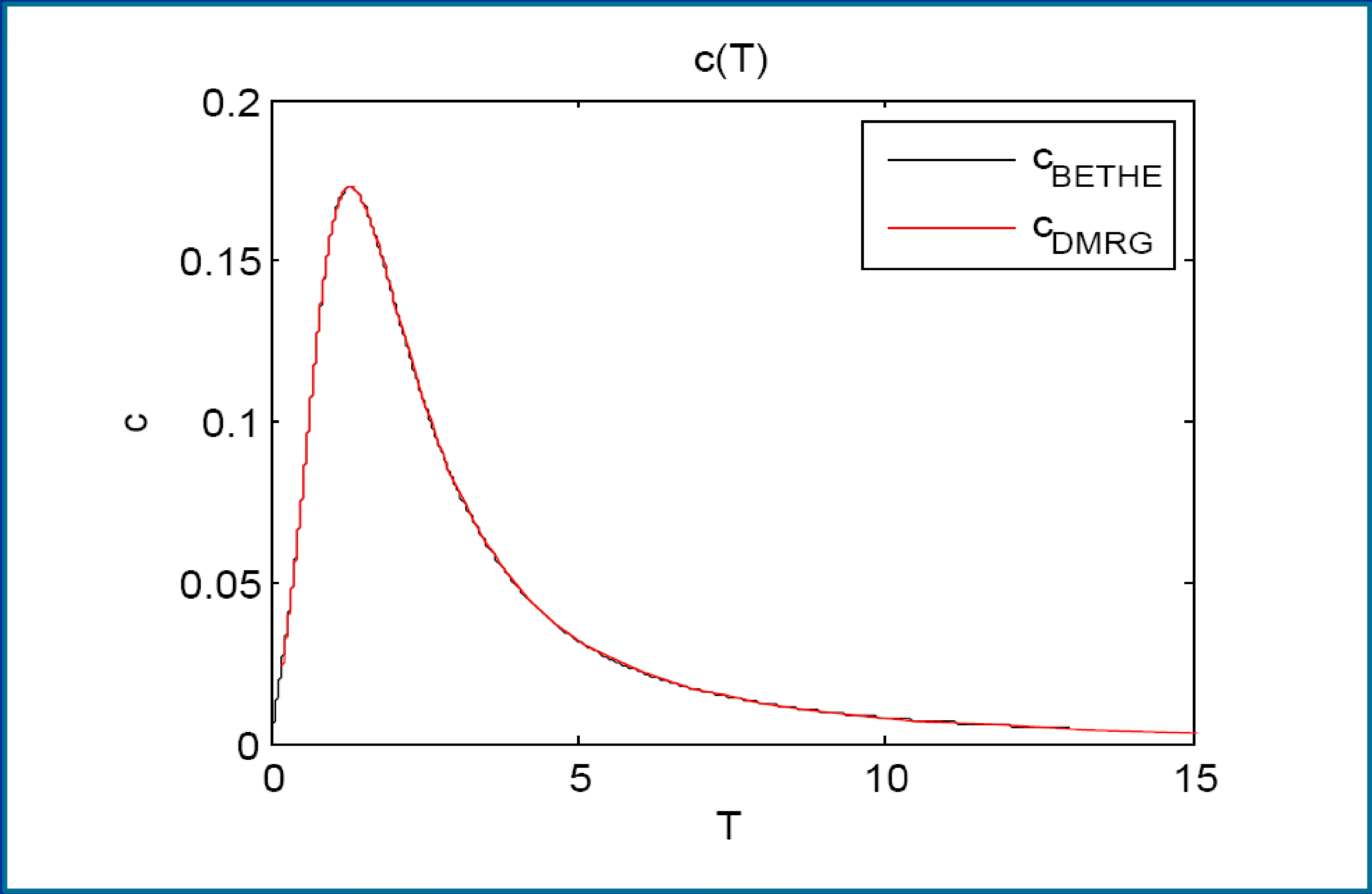
S. White

Finite temperature DMRG



Finite temperature DMRG

Specific heat of spin chain



Magnetization

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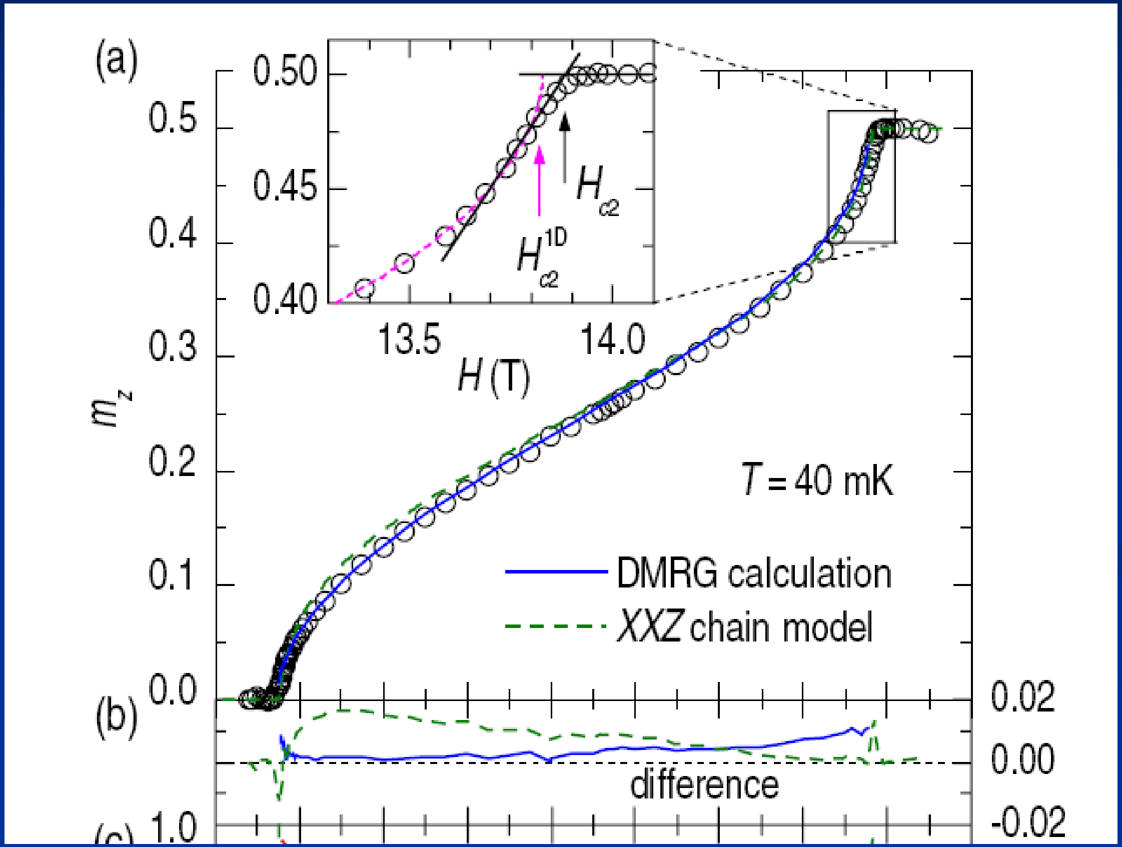

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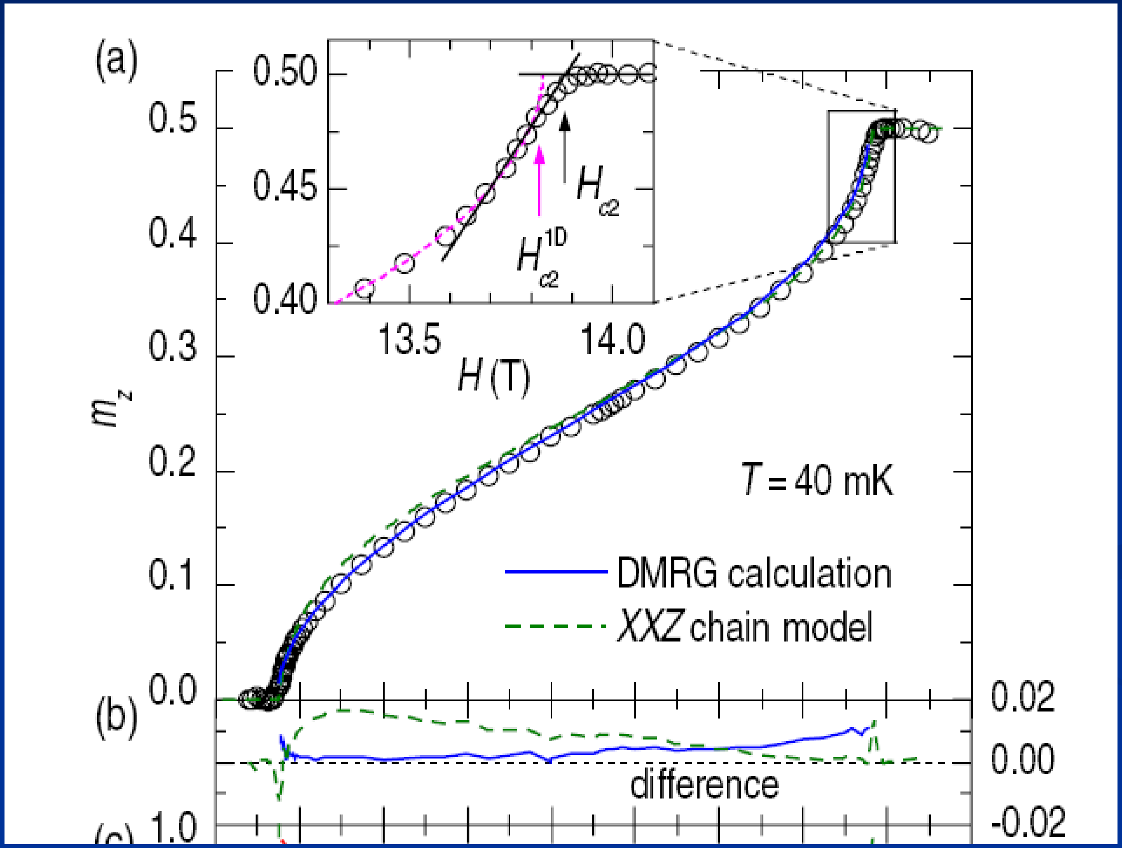

M. Klanjsek et al., PRL 101 137207 (2008)

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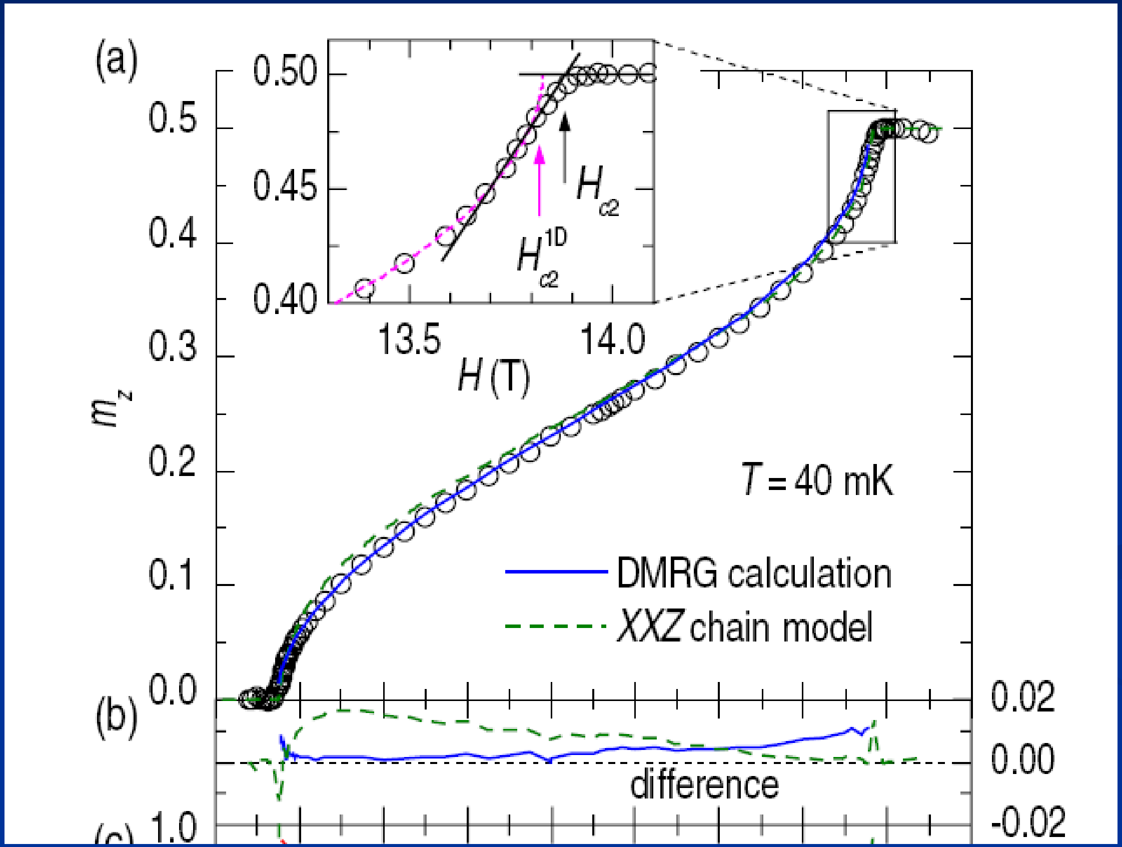

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J_r = 12.9 K

J = 3.6 K

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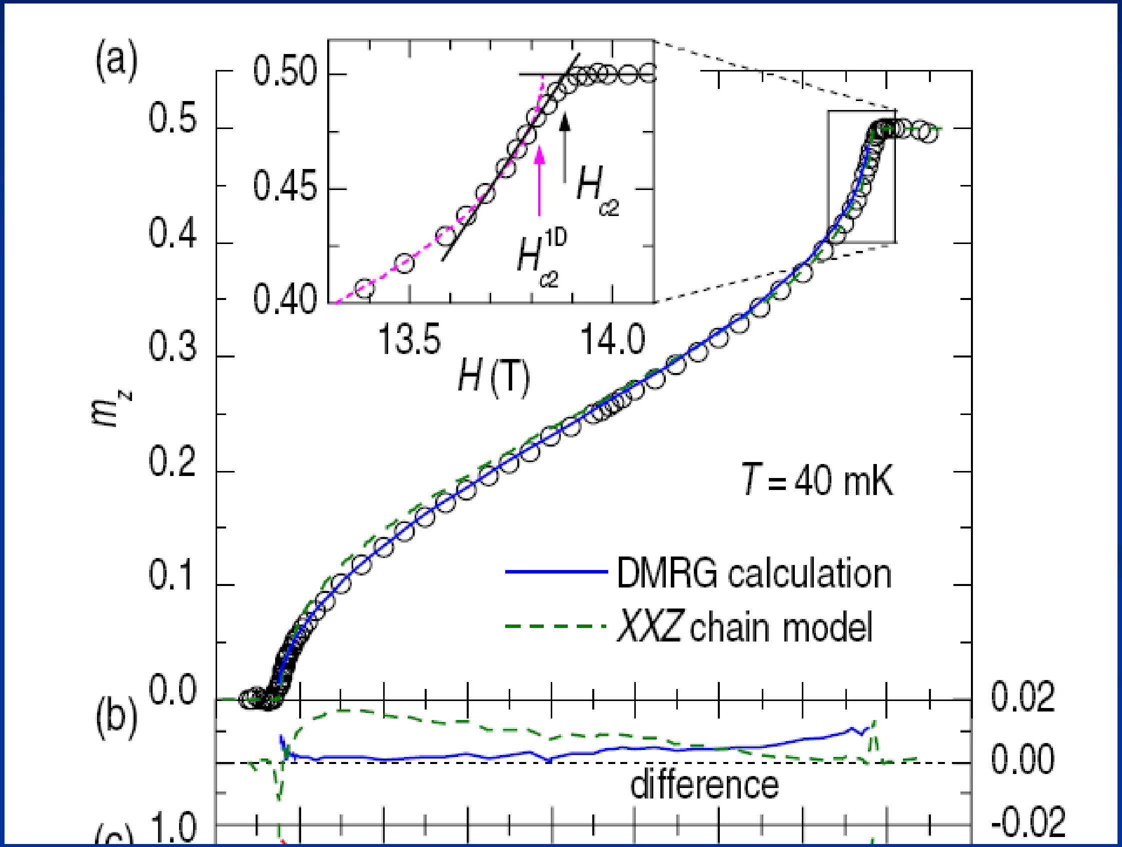

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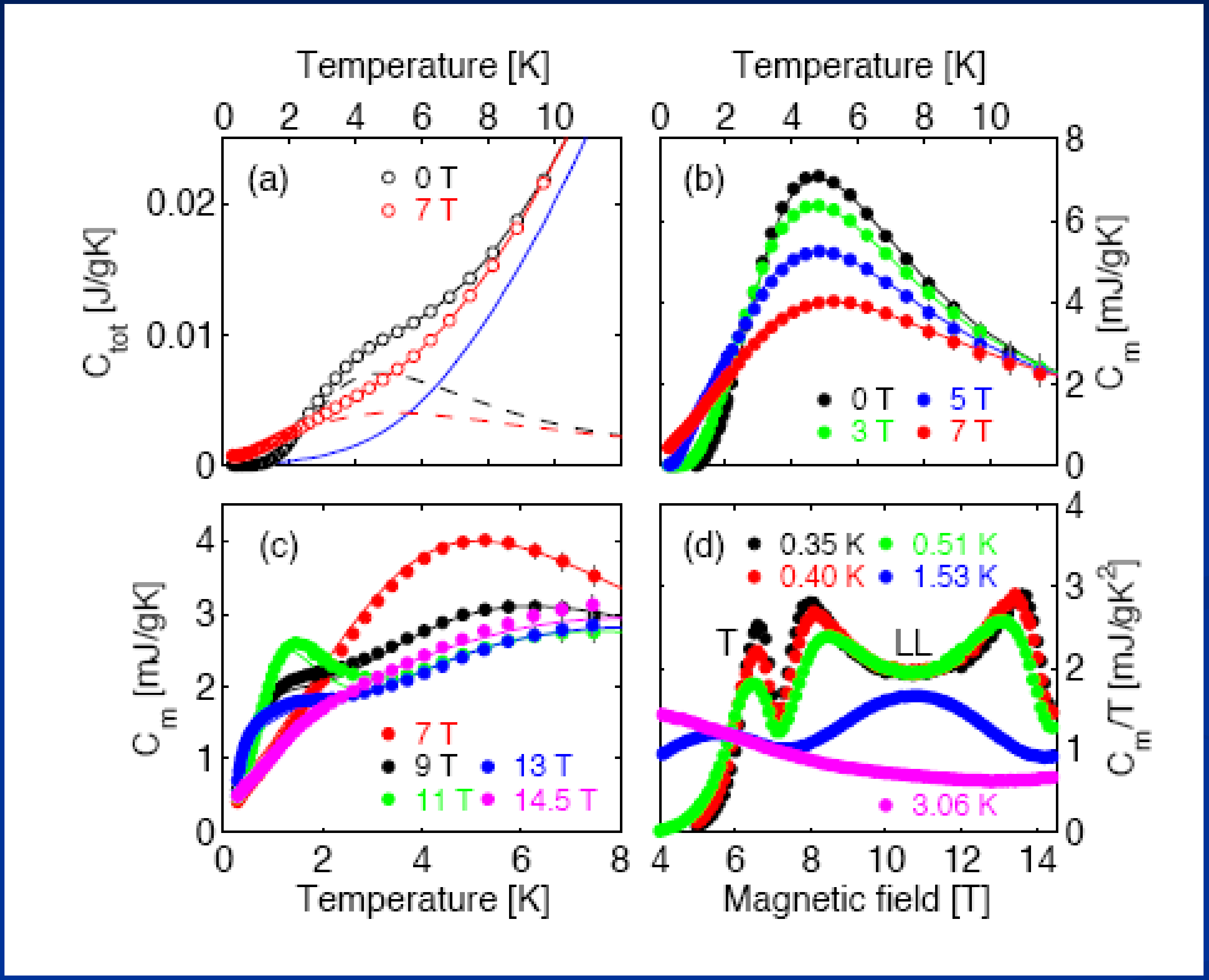

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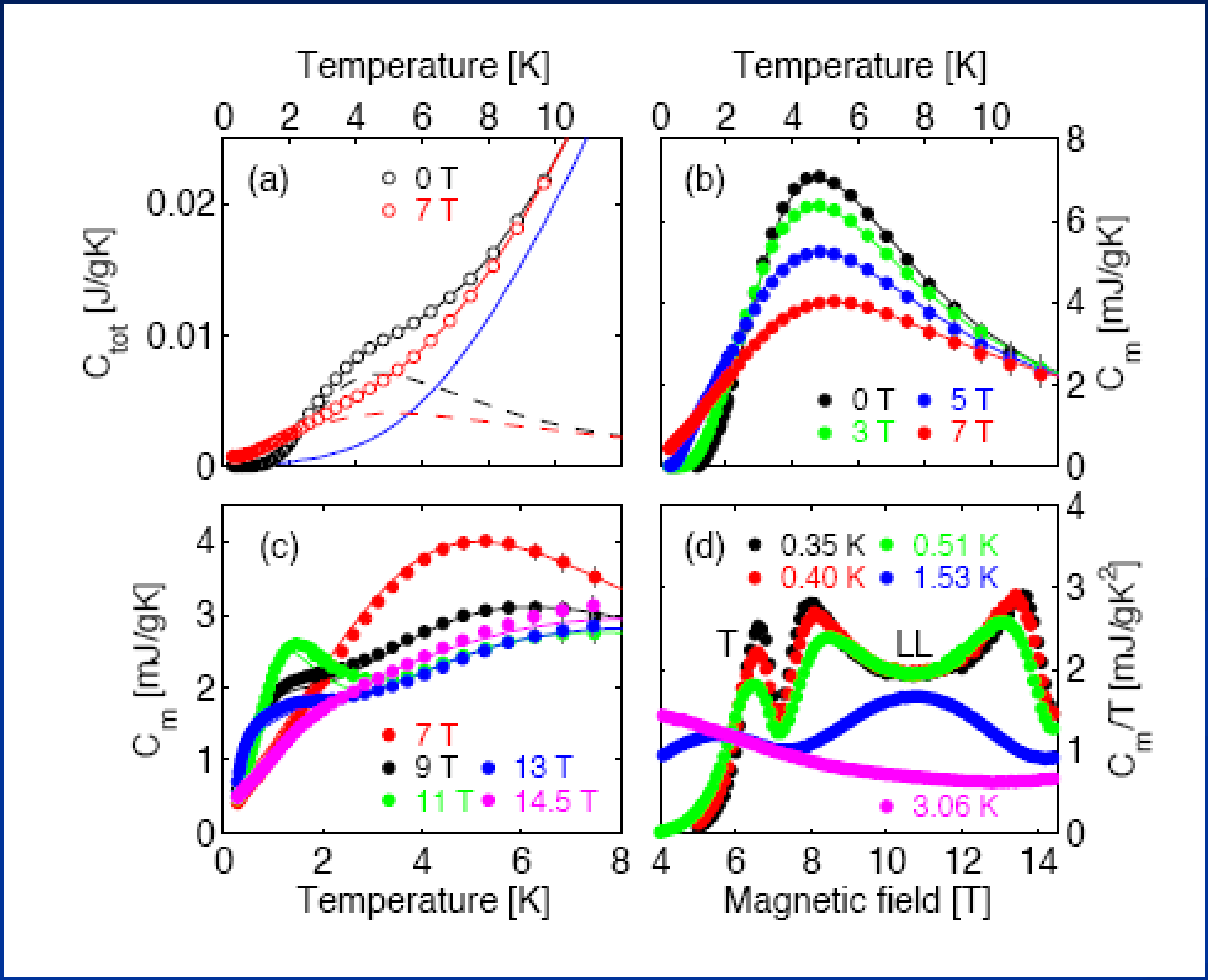

Ch. Rüegg et al.,

PRL 101, 247202 (2008)

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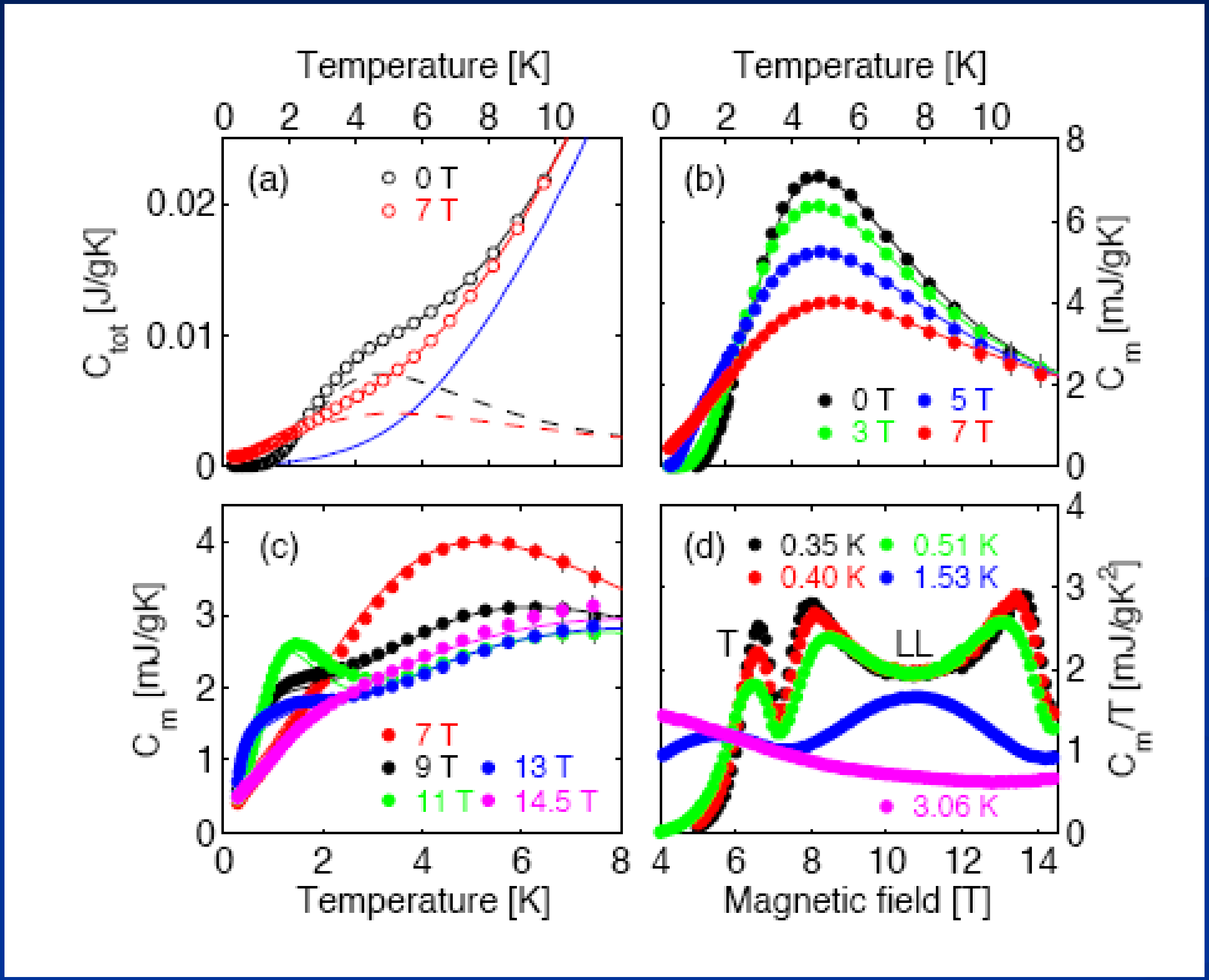

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Luttinger liquid: $C_v \propto \gamma T$

Peak : end of LL

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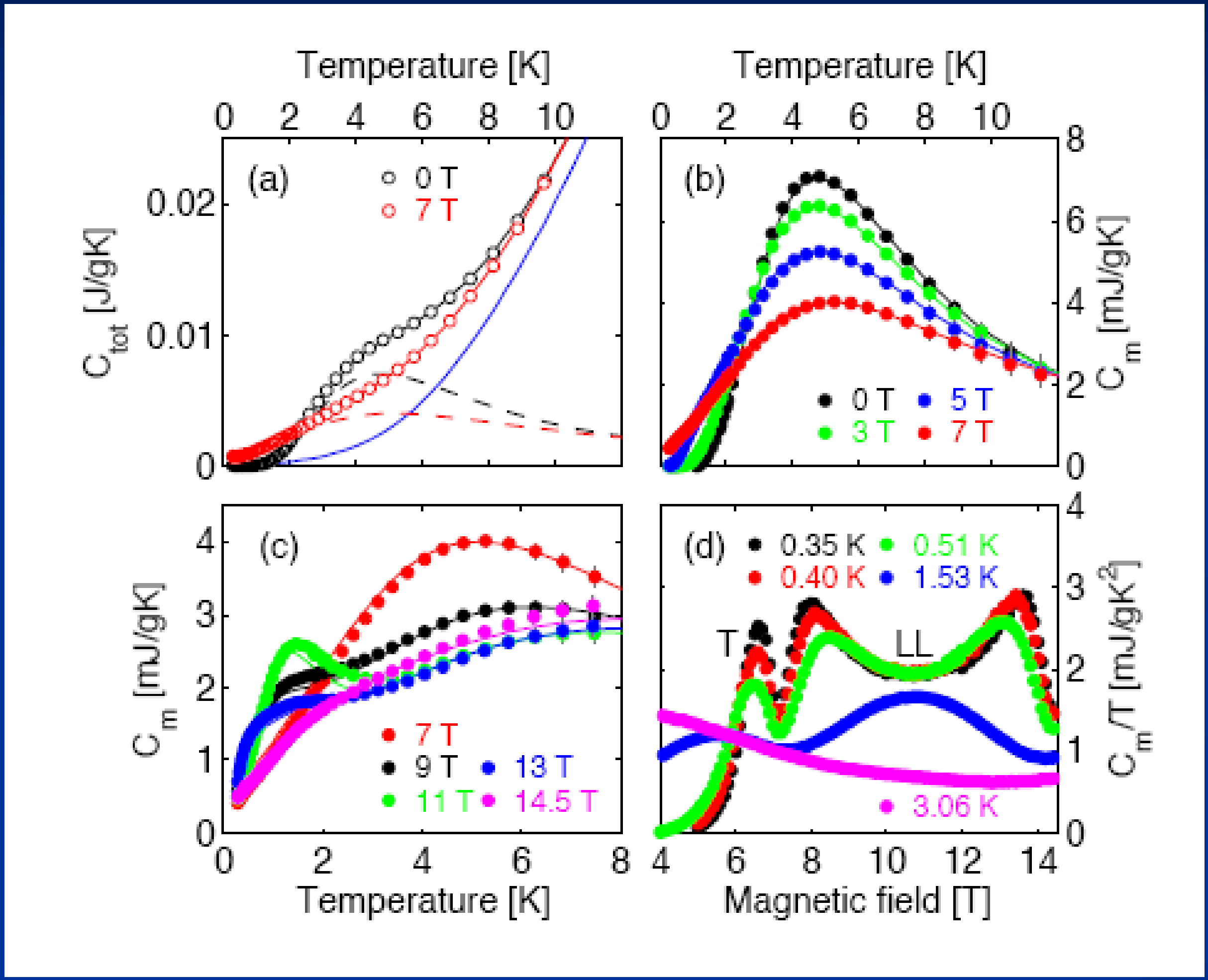

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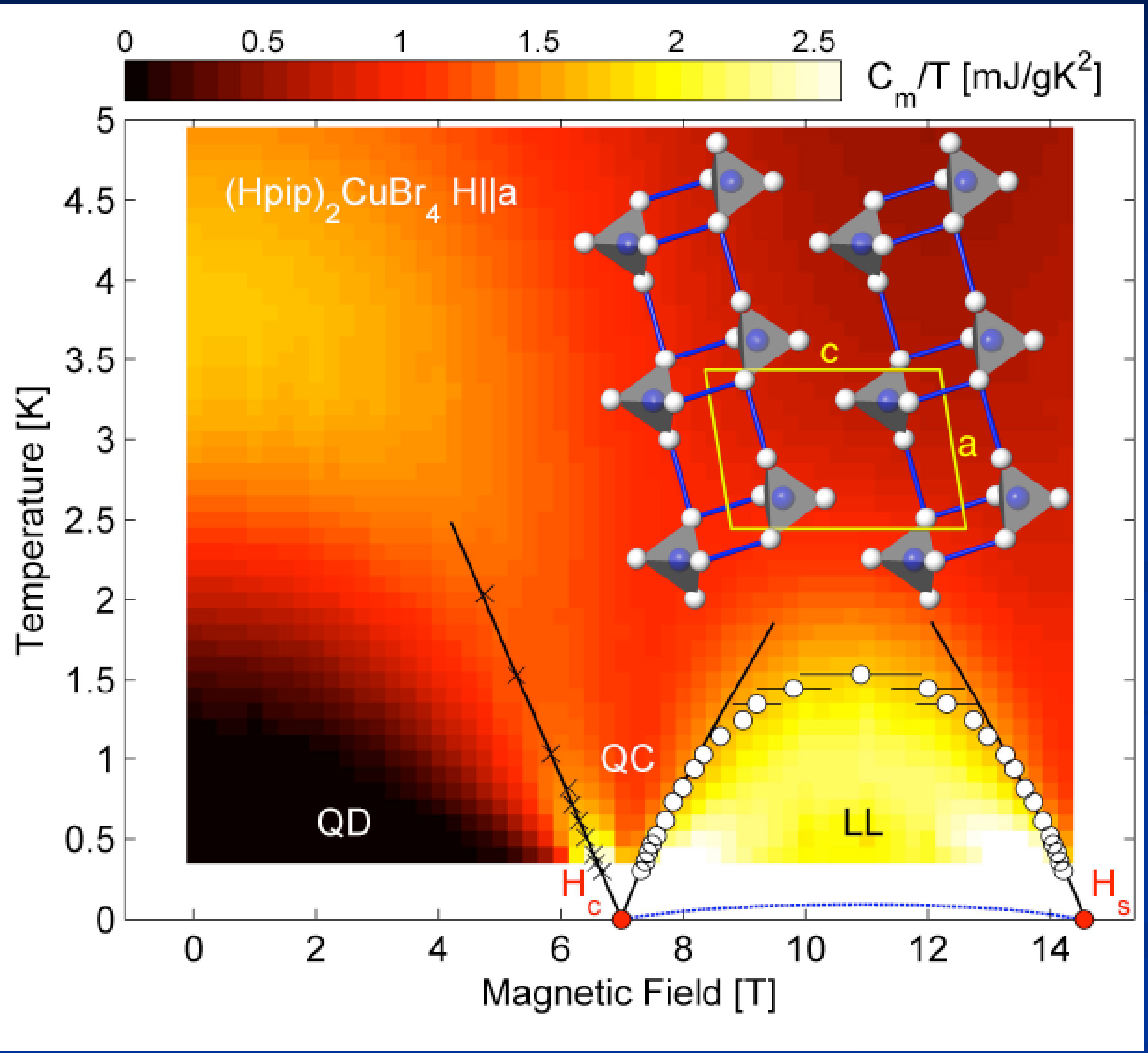

Ch. Rüegg et al., PRL 101, 247202 (2008)

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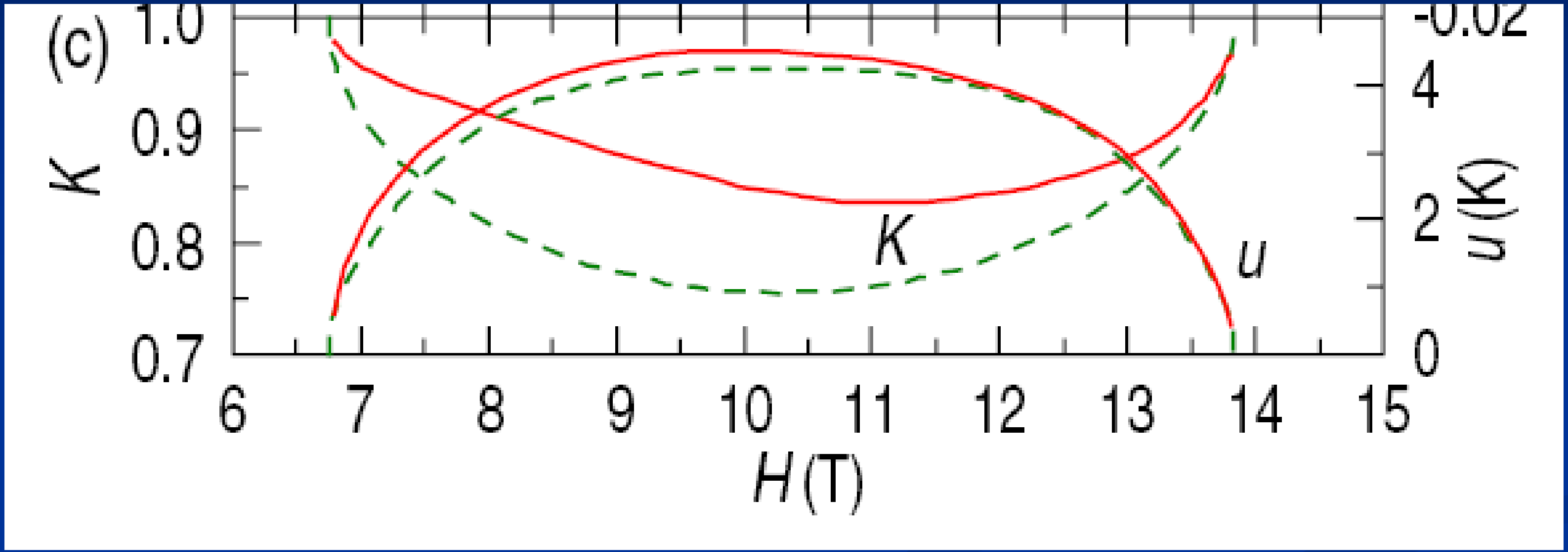

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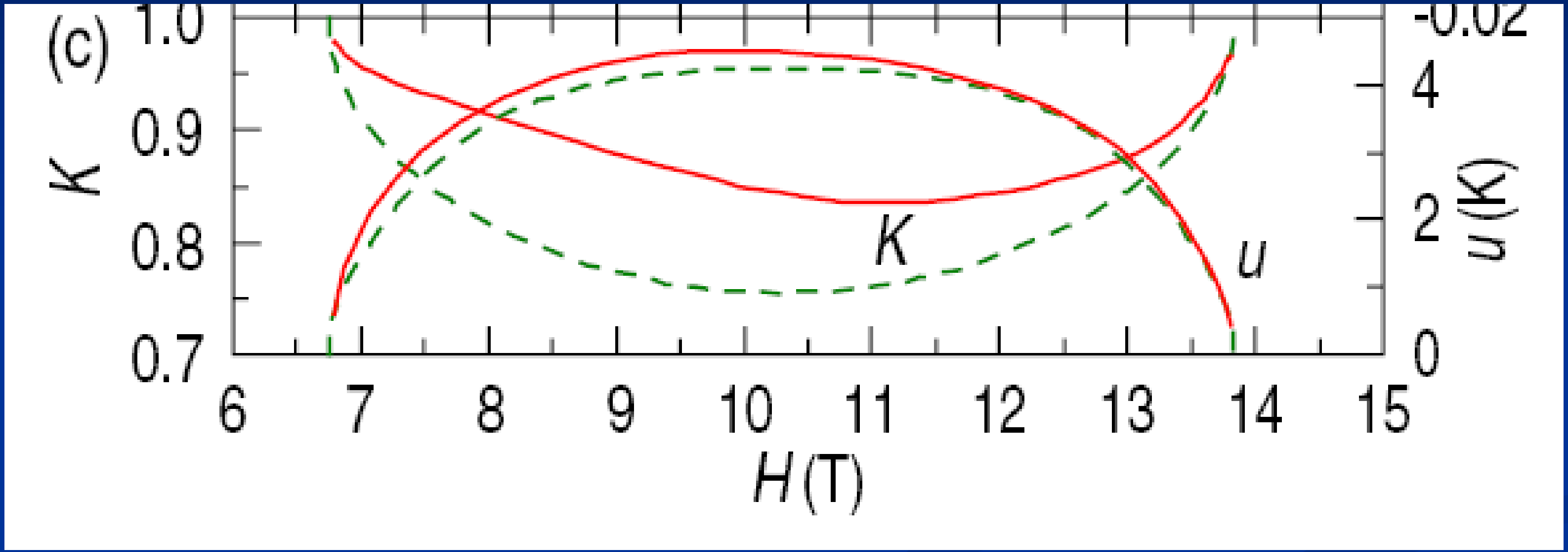

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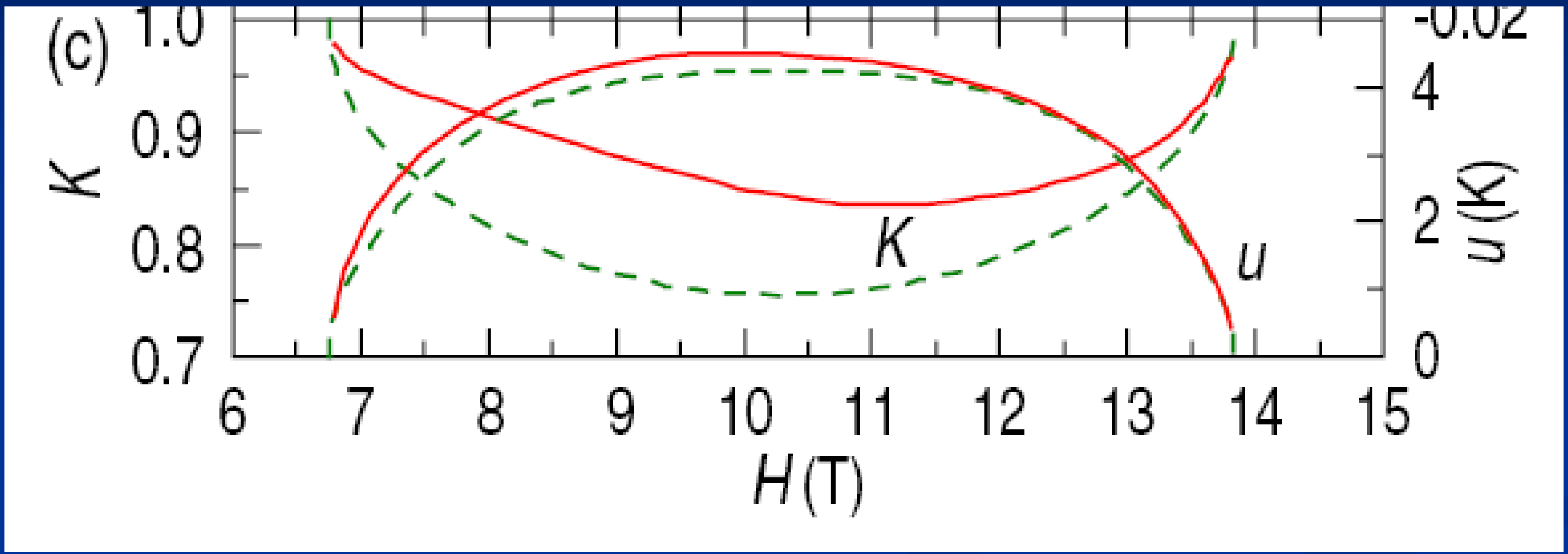

M. Klanjsek et al., PRL 101 137207 (2008)

Red : Ladder (DMRG)

Green: Strong coupling ($J_r \to \infty$) (BA)

Correlation functions

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M. Klanjsek et al., PRL 101 137207 (2008)

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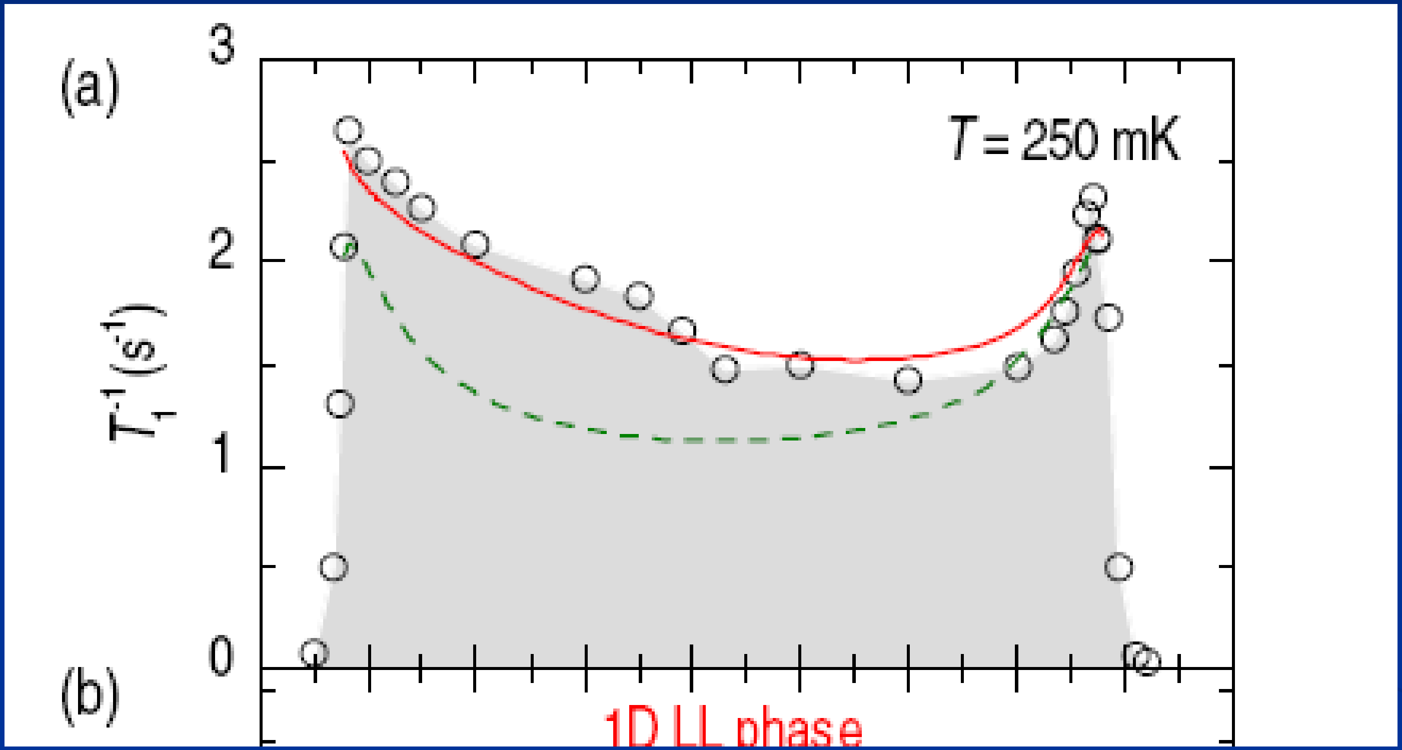

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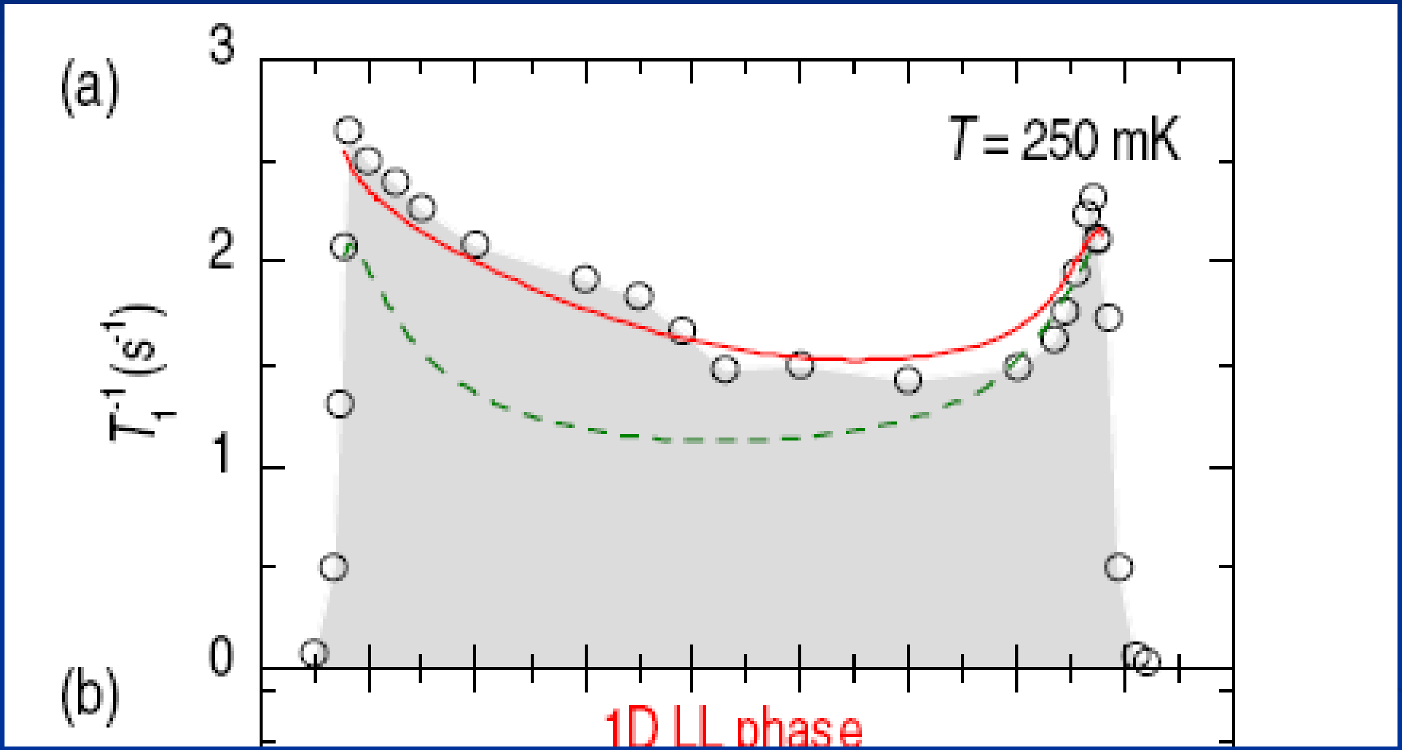

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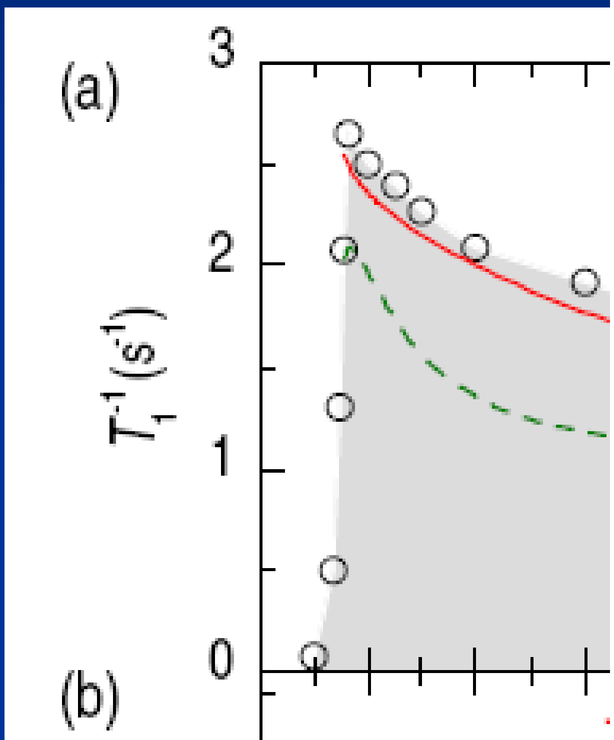

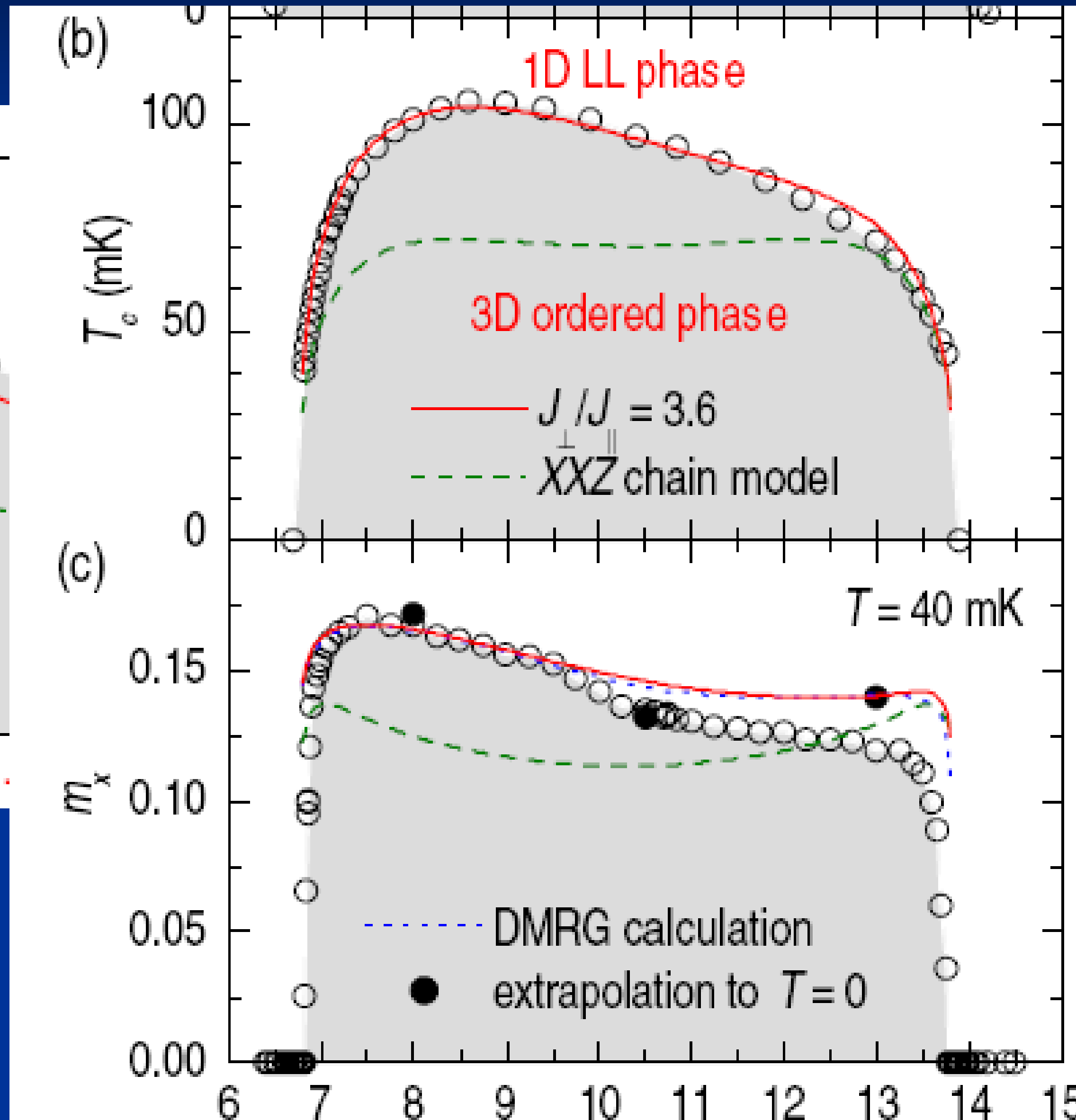

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Neutrons

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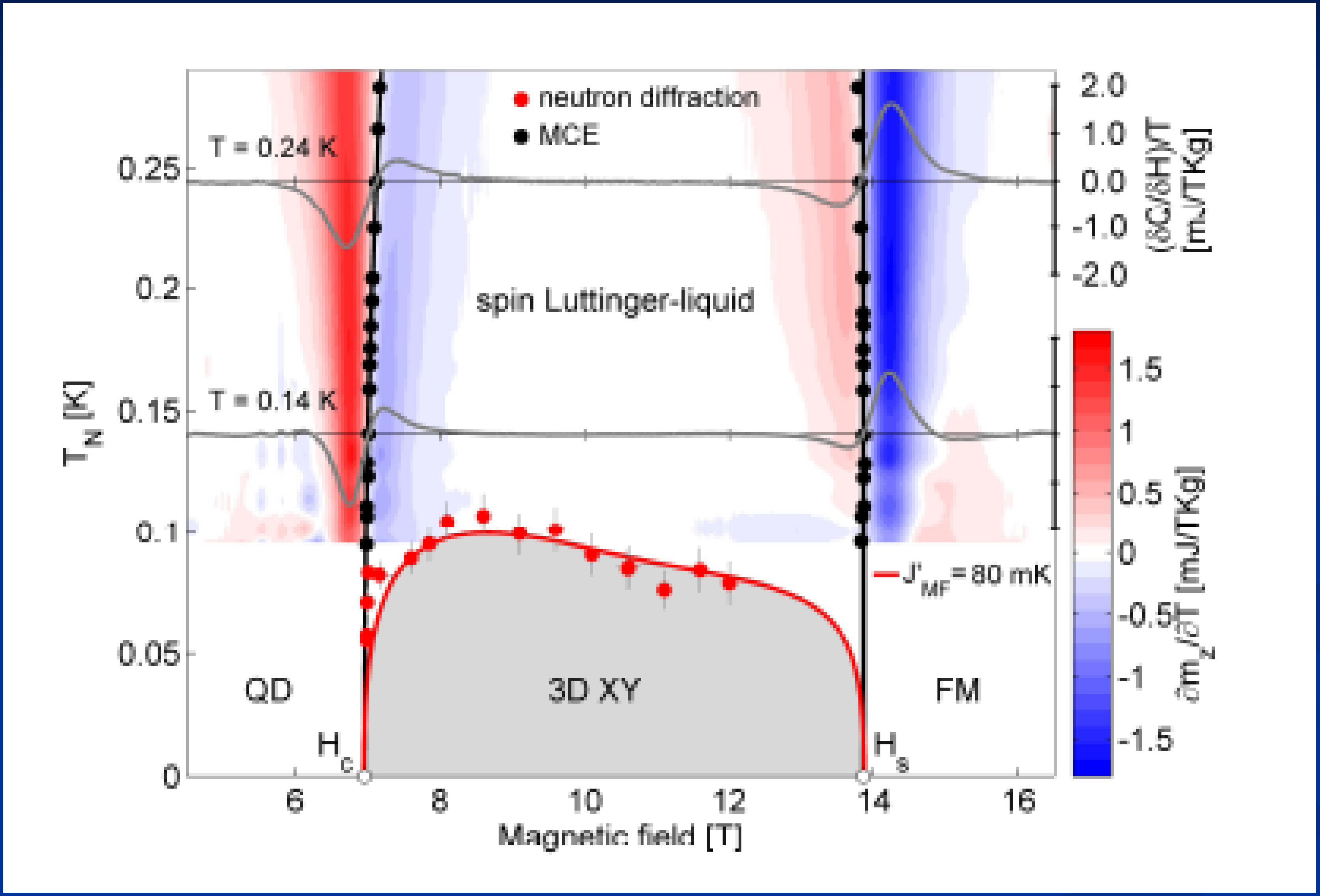
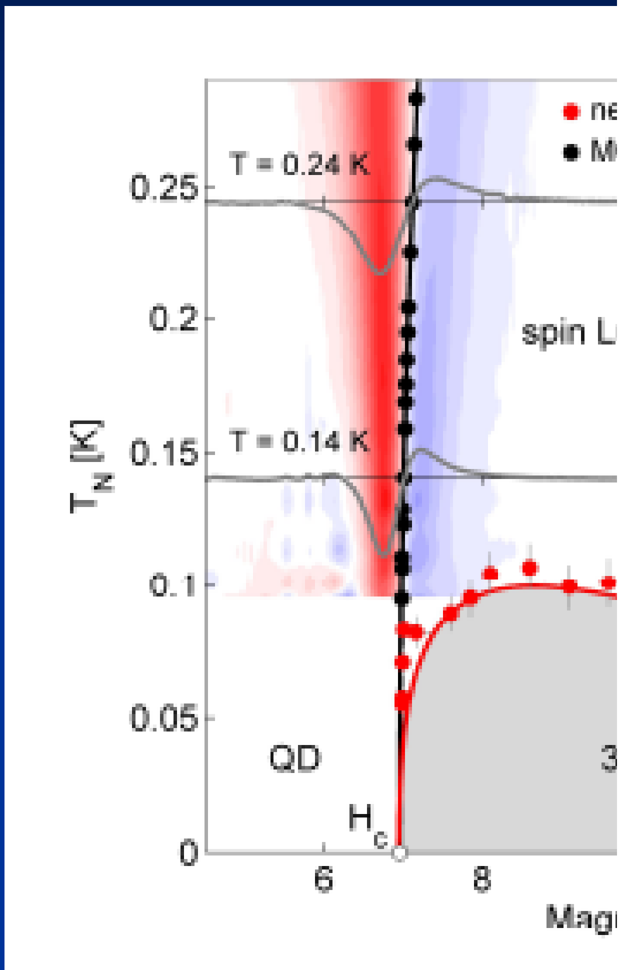
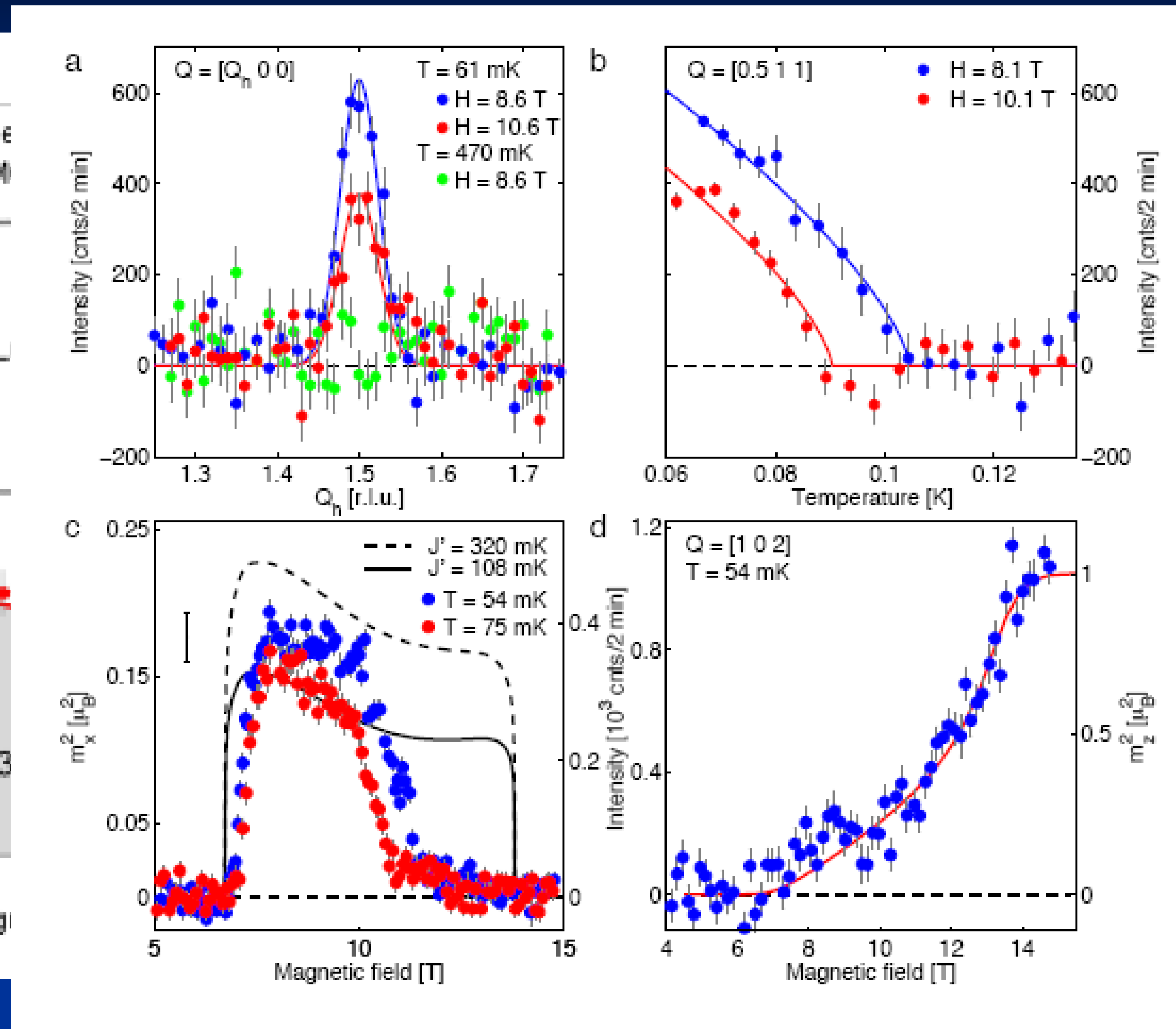


neutron diffraction
MCE
T = 0.24 K
T = 0.14 K
spin Luttinger-liquid
J'_{MF} = 80 mK
QD
3D XY
FM
H_c
H_s
T_N [K]
Magnetic field [T]
(δC/δH)/T [mJ/TKg]
$\partial m_z/\partial T$ [mJ/TKg]

Neutrons



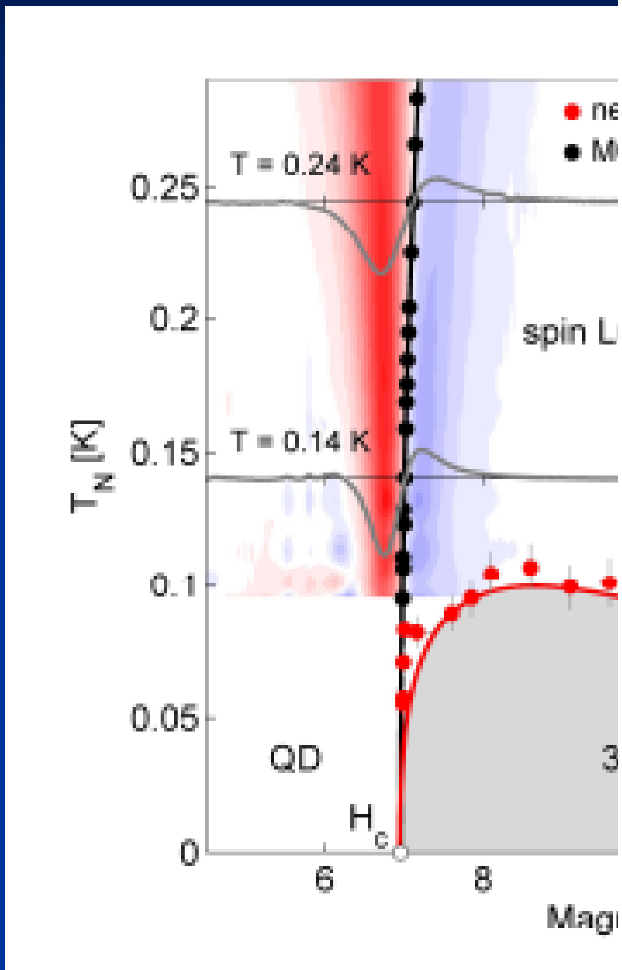


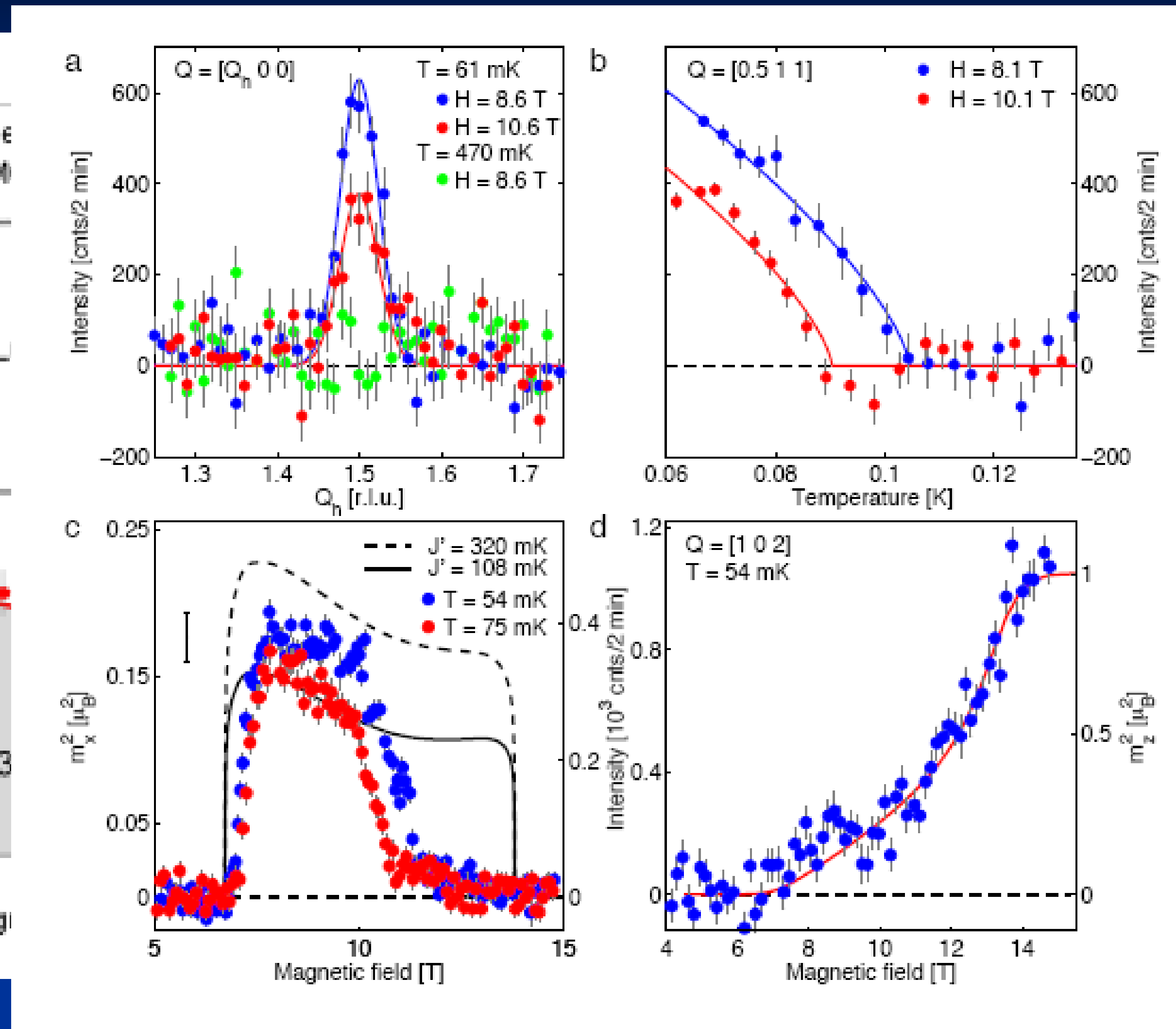
B. Thielemann et al.,

PRB 79, 020408(R) (2009)

J' = 27 mK

Neutrons





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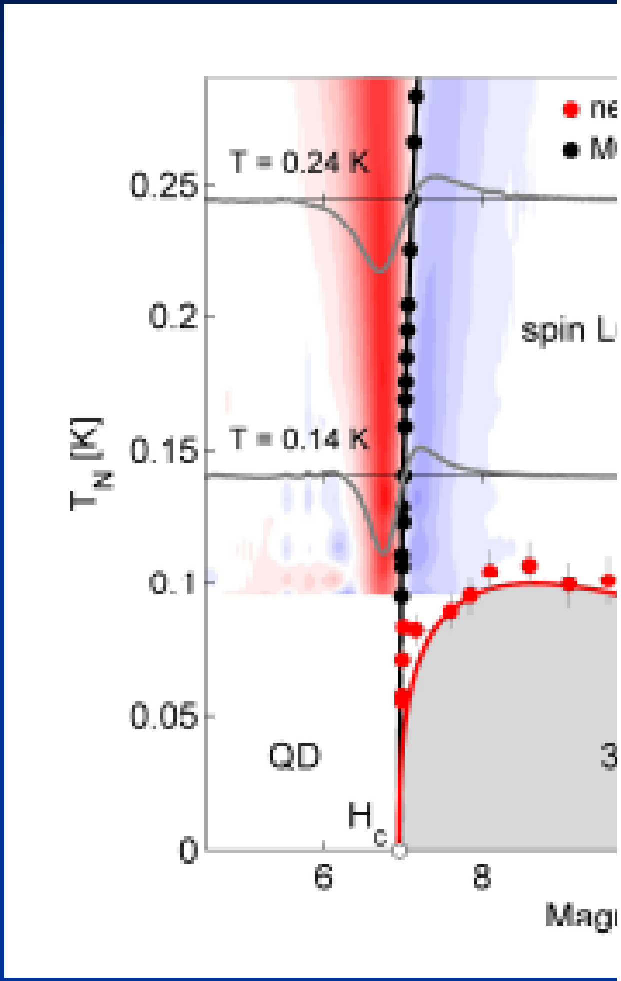

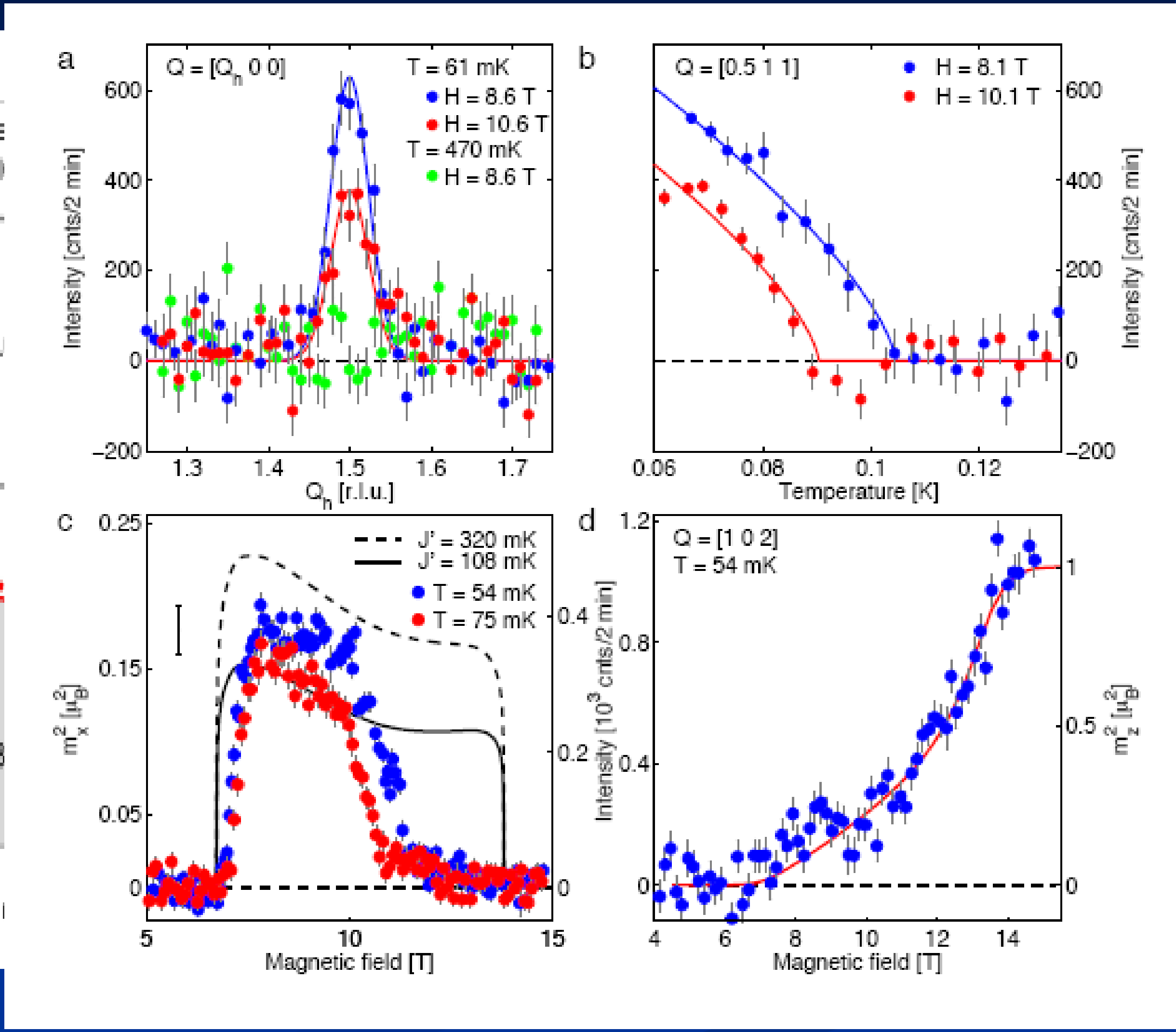

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Beyond Luttinger liquid

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- High energy correlations (LL valid for ω, $T \ll J$)

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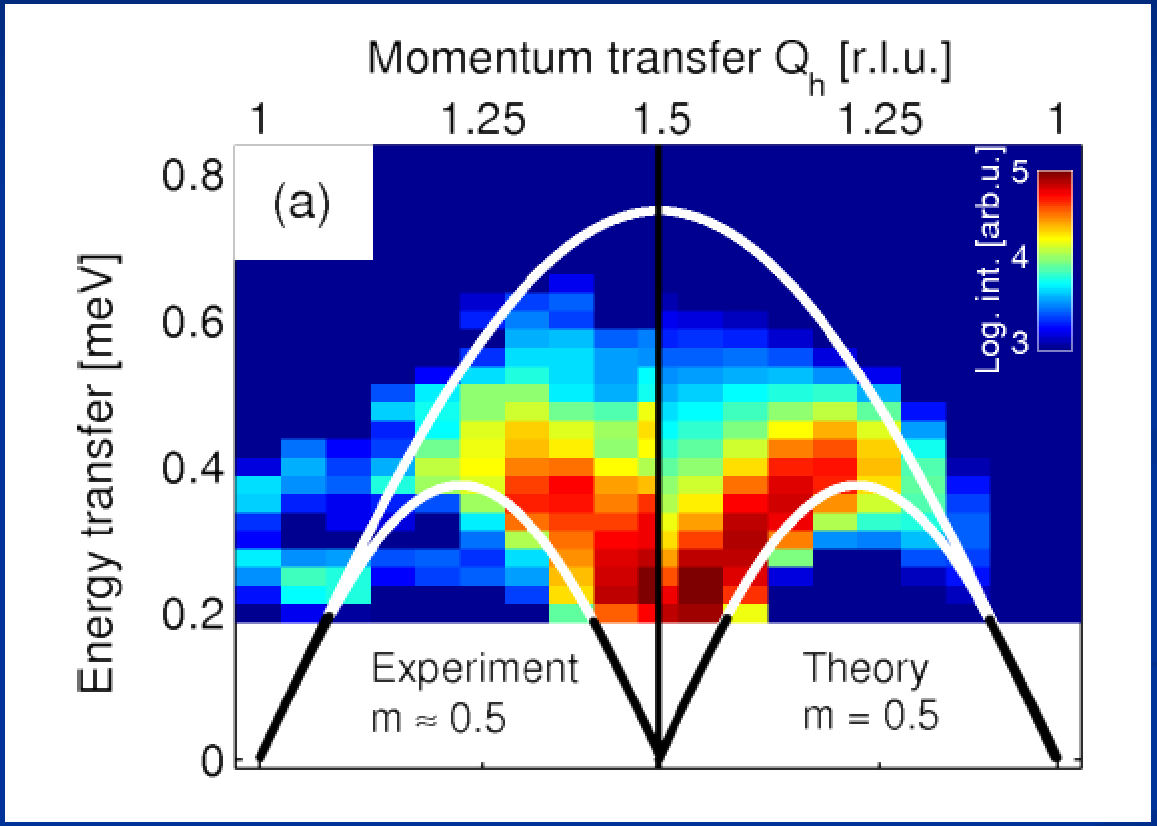

B. Thielemann et al. PRL 102 107204 (2009)

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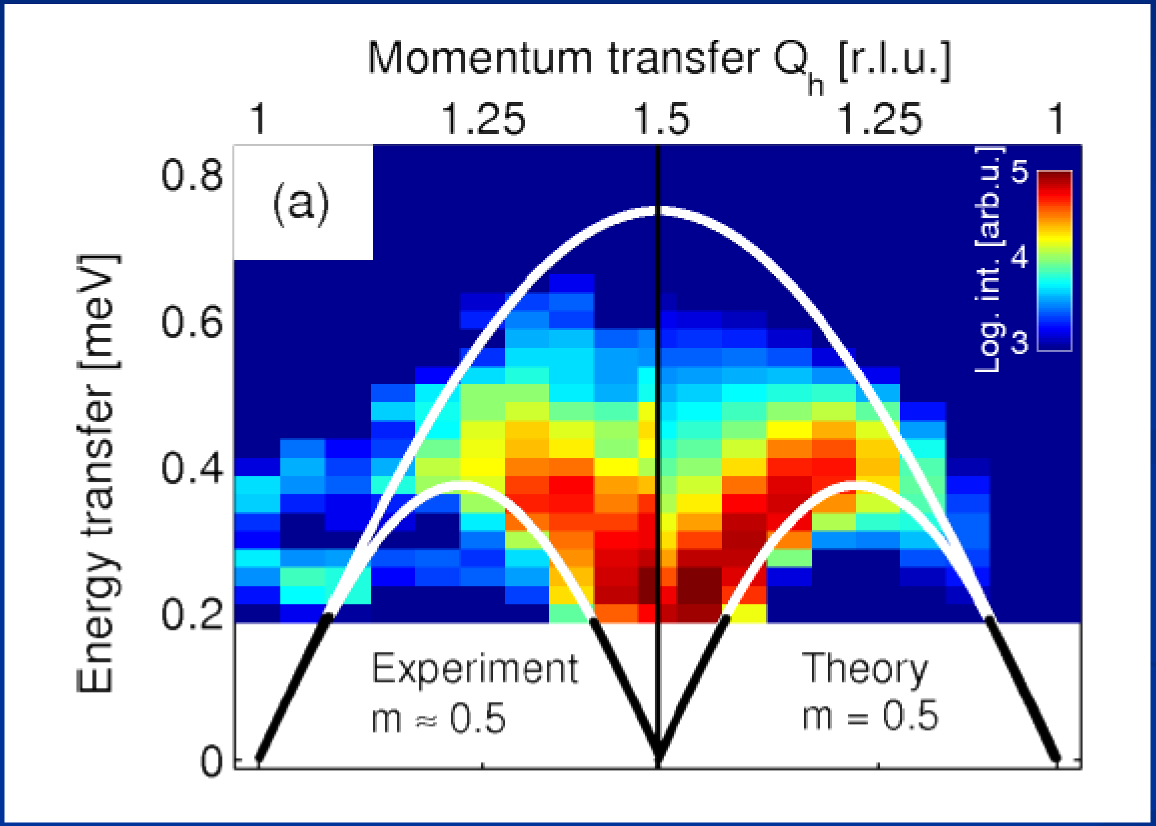

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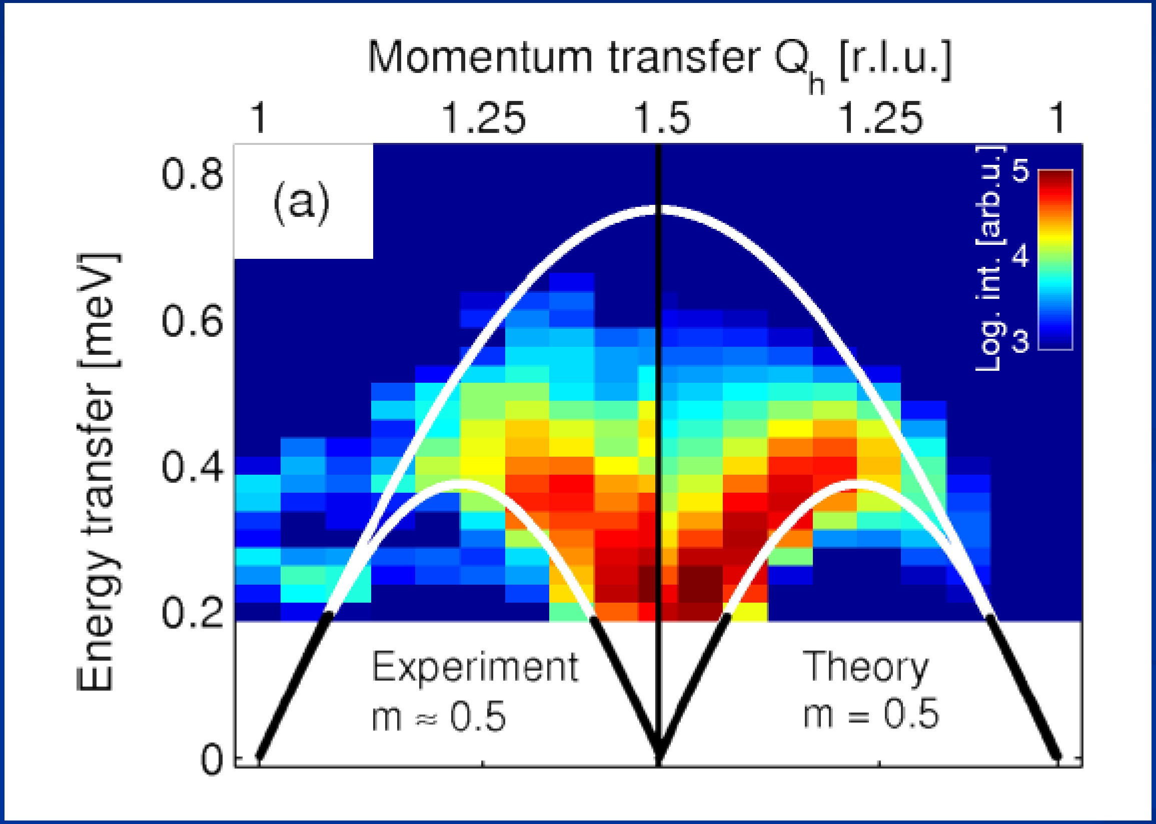

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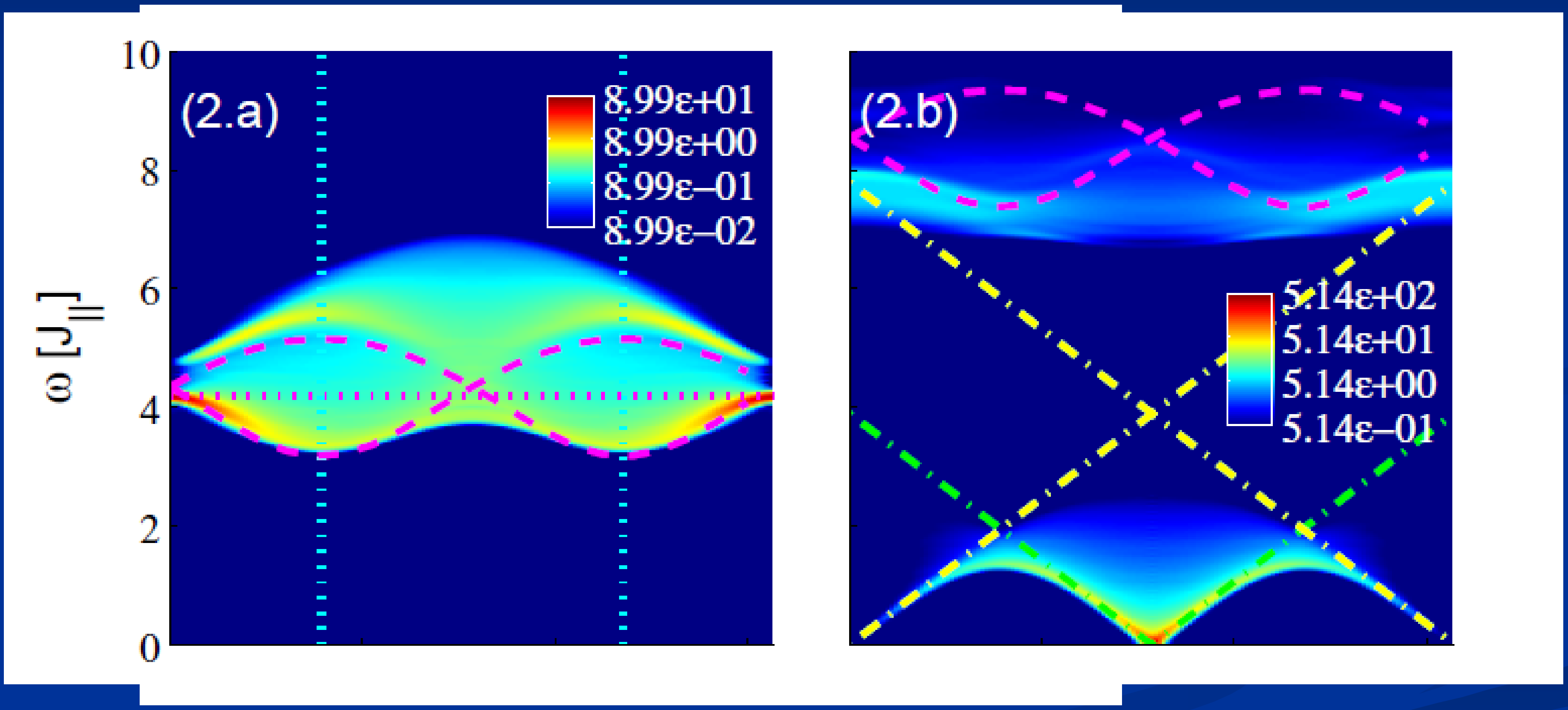

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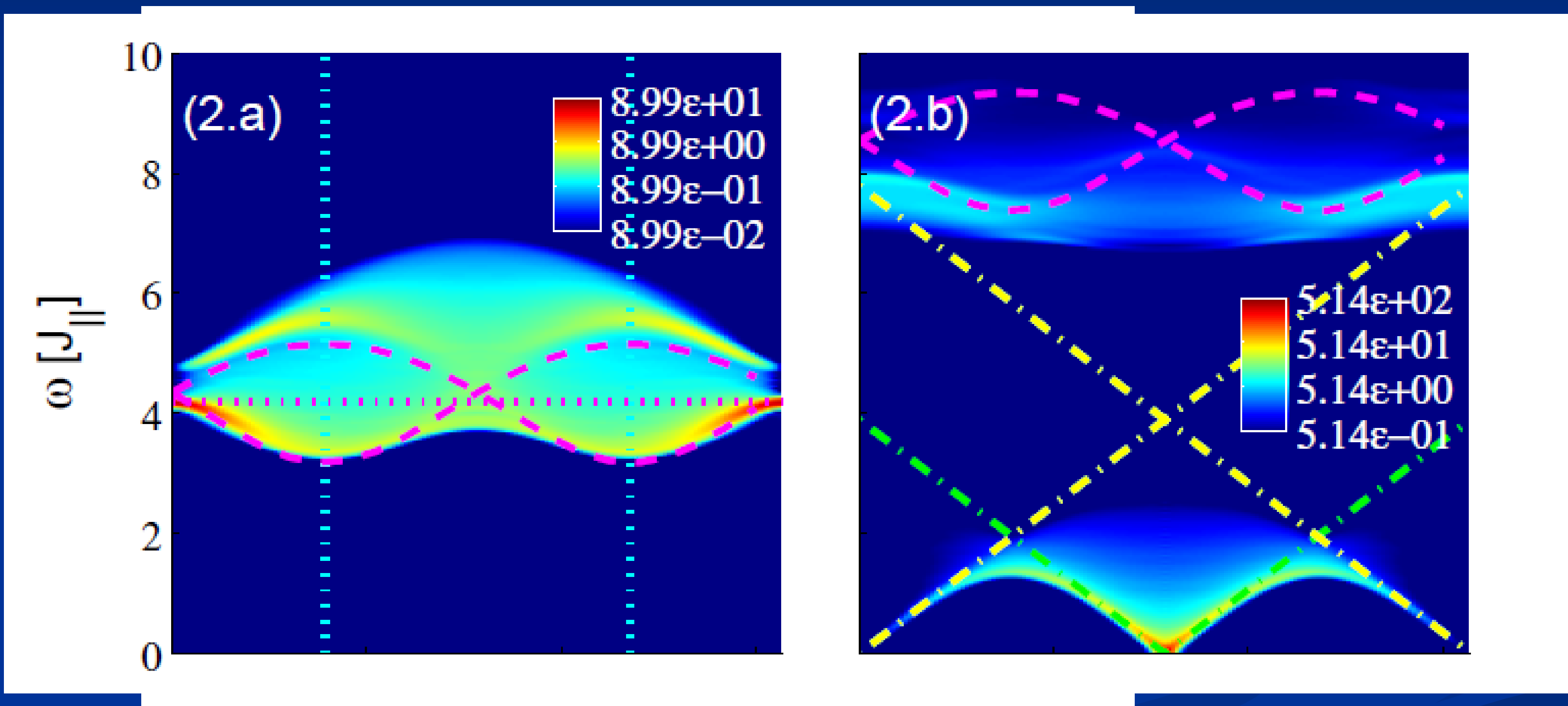

Time dependent DMRG

B. Thielemann et al. PRL 102 107204 (2009)

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- Other materials, impurities and doping