§4 固体^{3}Heの核磁性

鹿児島大学集中講義（2007年6月20-22日）
東京大学大学院理学系研究科　福山 寛

C. Kittel, Introduction to Solid State Physics (4th ed, 1971)

520　15. 反磁性と常磁性

μである．単位体積当り，N個の原子の磁化は，したがって

$$M=(N_1-N_2)\mu=N\mu\cdot\frac{e^x-e^{-x}}{e^x+e^{-x}}=N\mu\tanh x \qquad (15\cdot 20)$$

であり，ここに $x\equiv\mu B/k_BT$ である．(15・11) で示した関数Lと (15・20) に示す tanh との違いは，連続的な配向をとるか，量子化された配向をとるかの違いによるものである．弱磁場展開も違った関数となる．

$x\ll 1$ に対しては $\tanh x\fallingdotseq x$ であるのでキュリーの式

$$M\fallingdotseq N\mu(\mu B/k_BT) \qquad (15\cdot 21)$$

となる．ガドリニウム塩中の常磁性イオンの結果は図 15・5 (a) に，また固体 He^3 中の He^3 原子核のを図 15・5 (b) に示す．

15. 反磁性と常磁性　521

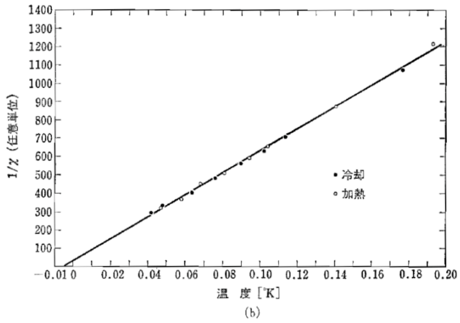


図 15・5 (b) 分子容積 23.6 cm^3/mol^{-1} の固体 He^3 の逆帯磁率．帯磁率は He^3 原子核による．〔P. B. Pipes と W.M. Fairbanks, Phys, Rev. Letters, 23, 520 (1969) による.〕

Until this age, at least in textbook, nuclear magnetism of solid ^{3}He had been known as a typical example of **paramagnetism**.

Pomeranchuk (1950)　prediction of $T_c \approx$ **0.1 μK**
dipole-dipole interactions ($E_{\text{d-d}}$)

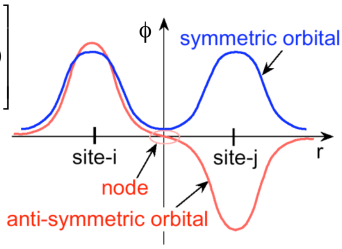
Bernades-Primakoff (1960)　prediction of $T_c \approx$ **100 mK**
direct **exchange** interactions (J)

Heisenbergモデル（有効スピンハミルトニアン）

フェルミオン2粒子系の波動関数 (Ψ) :

$$\Psi_{\text{singlet}}(r_1,r_2)=\frac{1}{\sqrt{2}}\{\phi_i(r_1)\phi_j(r_2)+\phi_i(r_2)\phi_j(r_1)\}\frac{1}{\sqrt{2}}\{\alpha(1)\beta(2)-\beta(1)\alpha(2)\}$$

$$\Psi_{\text{triplet}}(r_1,r_2)=\frac{1}{\sqrt{2}}\{\phi_i(r_1)\phi_j(r_2)-\phi_i(r_2)\phi_j(r_1)\}\begin{bmatrix}\alpha(1)\alpha(2)\\ \alpha(1)\beta(2)+\beta(1)\alpha(2)\\ \beta(1)\beta(2)\end{bmatrix}$$



これと同じ固有値を与える有効スピンハミルトニアン (H_{eff}):

$$H_{\text{eff}}=\frac{1}{4}(E_s+3E_t)-(E_s-E_t)\boldsymbol{S}_1\bullet\boldsymbol{S}_2$$

$2J\equiv E_s - E_a$
: 交換相互作用

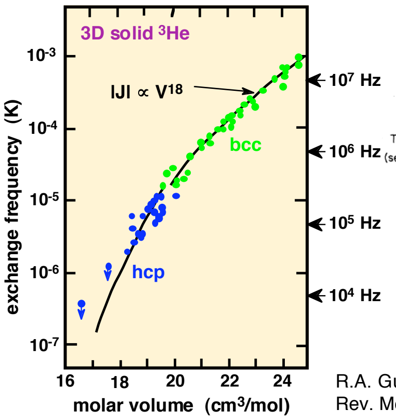
$J_{ij}>0$: FM (Coulomb interaction)
$J_{ij}<0$: AFM (solid ^{3}He)

Heisenbergスピンハミルトニアン:

$$H_{\text{Heisenberg}}=-2J_{ij}\sum_{i<j}\boldsymbol{S}_i\bullet\boldsymbol{S}_j$$

固体^{3}Heの交換相互作用に関する初期の研究

交換相互作用(J)の体積依存性

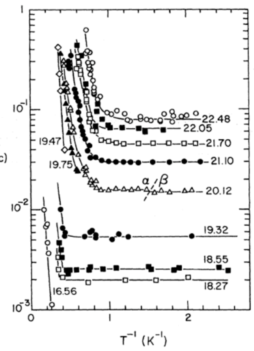


strong volume dependence

$|J| \propto V^{18}$

$J \approx$ -1 mK at melting pressure

スピン—スピン
緩和時間(T_2)

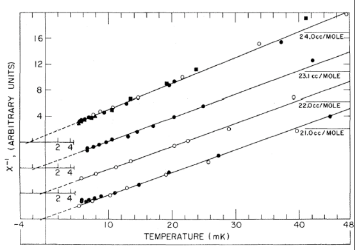


R.A. Guyer, R.C. Richardson and L.I. Zane, Rev. Mod. Phys. **43**, 532 (1971)

exchange narrowing

$T_2 \propto |J|$

帯磁率



W.P. Kirk et al., PRL **23**, 833 (1969)

antiferromagnetic

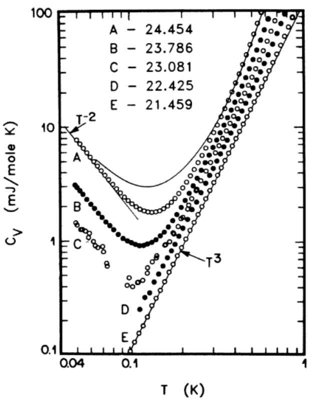
θ_W (= -4J) < 0

固体^{3}Heの交換相互作用に関する初期の研究 (II)

比熱

exchange term: $C_V \propto J^2/T^2$

phonon term: $C_V \propto T^3$



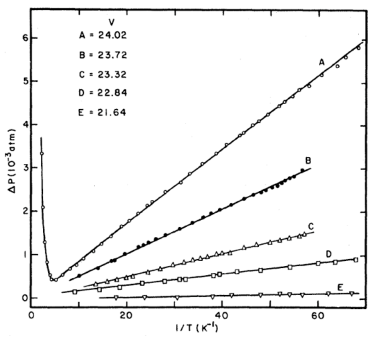
D.S. Greywall, PRB **15**, 2604 (1977)

Large energy separation
$h\omega_D/J \approx 10^4$

定積圧力

exchange term: $P_V \propto (\partial|J|/\partial V)/T$

phonon term: $P_V \propto T^4$

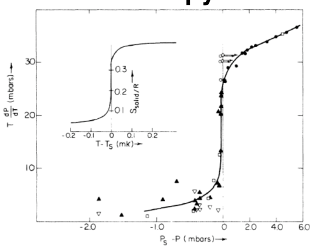


M.F. Panczyk and E.D. Adams, PR **187**, 321 (1969)

固体^{3}Heの核磁気秩序

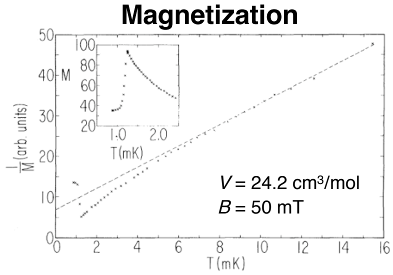
- Strong first-order AFM transition at T_N = 1 mK
  $\Delta S = 0.43 ln2$ (not spin-Peierls transition)
- AFM spin-waves in ordered phase
  meting pressure $\propto T^4$ $(C \propto T^3)$
- Anomalous behaviour in paramagnetic phase

Entropy



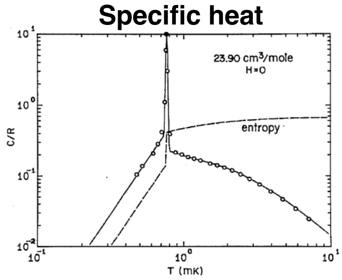
W.P. Halperin et al., PRL **32**, 927 (1974)

Magnetization



T.C. Prewitt and J.M. Goodkind, PRL **39**, 1283 (1977)

Specific heat



D.S. Greywall and P.A. Busch, PRB **36**, 6853 (1987)

磁気相図

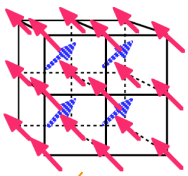
H. Fukuyama et al., cond-mat/0505177
H. Fukuyama et al., Physica B **169**, 197 (1991)

二つのAFM秩序相:

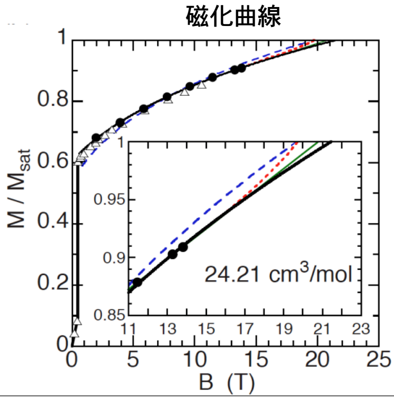
HFP ($B \geq$ 0.45 T) CNAF phase
- large magnetization ($M \geq 0.6\ M_{sat}$)
- positive (dT_c/dB)

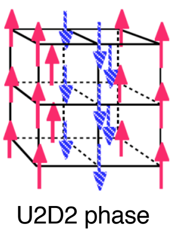
LFP ($B \leq$ 0.45 T) U2D2 phase

CNAF phase
(HFP)

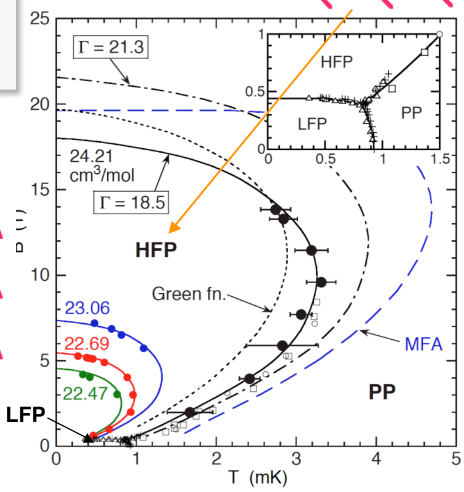


磁化曲線





U2D2 phase
(LFP)

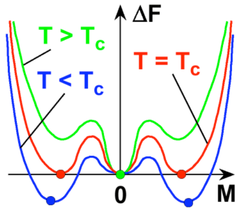


§4.1 リング交換モデル

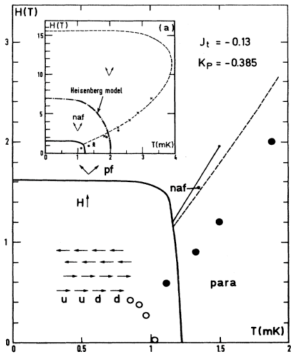
J.H. Hetherington and F.D.C. Willard, PRL **35**, 1442 (1975)
M. Roger,et. al., Rev. Mod. Phys. **55**, 1 (1983)

1. 大きな**4体交換相互作用**は1次相転移を説明できる

$$F = E - TS$$
$$= F_0 + \left(2k_BT - a\right)M^2 + \left(\frac{4}{3}k_BT - b\right)M^4 + \cdots$$
$$b \approx J_4\left(\sigma_i \bullet \sigma_j\right)\left(\sigma_k \bullet \sigma_l\right)$$



2. 固体^{3}Heの実験的な磁気相図は、3体交換(FM)と4体交換(AFM)の競合として理解できる。



Two parameter model:
- three-spin exchange (T_1) ≈ -0.13 mK
- planar four-spin exchange (K_P) ≈ -0.39 mK

リング交換の寄与は大きい!

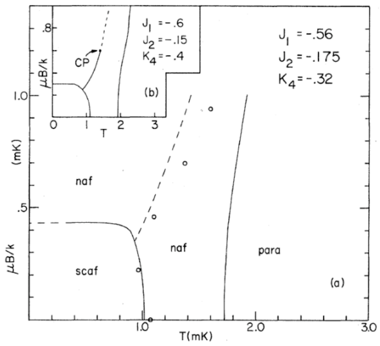
続き .... リング交換モデル（現象論）

VOLUME 35, NUMBER 21 PHYSICAL REVIEW LETTERS 24 NOVEMBER 1975

Two-, Three-, and Four-Atom Exchange Effects in bcc ^{3}He

J. H. Hetherington and F. D. C. Willard
Physics Department, Michigan State University, East Lansing, Michigan 48824
(Received 22 September 1975)

We have made mean-field calculations with a Hamiltonian obtained from two-, three-, and four-atom exchange in bcc solid ^{3}He. We are able to fit the high-temperature experiments as well as the phase diagram of Kummer *et al.* at low temperatures. We find two kinds of antiferromagnetic phases as suggested by Kummer's experiments.



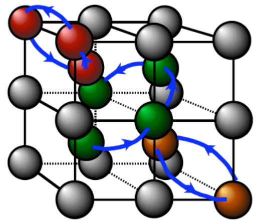
J.H. Hetherington and F.D.C. Willard, Phys. Rev. Lett. **35**, 1442 (1975)

リング交換 (RE) スピンハミルトニアン

D.J. Thouless, Proc. Phys. Soc. **86**, 893 (1965), *ibid.* **86**, 905 (1965)
M. Roger, J.H. Hetherington and J.M. Delrieu, Rev. Mod. Phys. **55**, 1 (1983)

$$H_{\mathrm{eff}} = \sum_{P} (-1)^{P} J_{P} P$$

P: permutation operator
***J**$_P$*: exchange energy



Two-spin exchange (AFM)

$$P_1 = P_{\mathrm{ij}} = \frac{1}{2}\left(1 + \sigma_{\mathrm{i}} \bullet \sigma_{\mathrm{j}}\right)$$ ← Heisenberg type

Three-spin exchange (FM)

$$P_{\mathrm{ijk}} = P_{\mathrm{jj}} P_{\mathrm{ij}} = \frac{1}{4}\left(1 + \sigma_{\mathrm{i}} \bullet \sigma_{\mathrm{j}}\right)\left(1 + \sigma_{\mathrm{i}} \bullet \sigma_{\mathrm{k}}\right)$$

$$P_2 = P_{\mathrm{ijk}} + P_{\mathrm{ijk}}^{-1} = \frac{1}{2}\left(1 + \sigma_{\mathrm{i}} \bullet \sigma_{\mathrm{j}} + \sigma_{\mathrm{j}} \bullet \sigma_{\mathrm{k}} + \sigma_{\mathrm{k}} \bullet \sigma_{\mathrm{i}}\right)$$

↑ Heisenberg type

Four-spin exchange (AFM)

$$P_{\mathrm{ijkl}} = P_{\mathrm{ijk}} P_{\mathrm{il}}$$

$$P_3 = P_{\mathrm{ijkl}} + P_{\mathrm{ijkl}}^{-1} = \frac{1}{4}\left(1 + \sum_{\mu<\nu} \sigma_{\mu} \bullet \sigma_{\nu} + G_{\mathrm{ijkl}}\right)$$

$$G_{\mathrm{ijkl}} \equiv \left(\sigma_{\mathrm{i}} \bullet \sigma_{\mathrm{j}}\right)\left(\sigma_{\mathrm{k}} \bullet \sigma_{\mathrm{l}}\right) + \left(\sigma_{\mathrm{i}} \bullet \sigma_{\mathrm{l}}\right)\left(\sigma_{\mathrm{j}} \bullet \sigma_{\mathrm{k}}\right) - \left(\sigma_{\mathrm{i}} \bullet \sigma_{\mathrm{k}}\right)\left(\sigma_{\mathrm{j}} \bullet \sigma_{\mathrm{l}}\right)$$ ← new terms

続き .... リング交換 (RE) スピンハミルトニアン

Five-spin exchange (FM)

$$P_{\mathrm{ijklm}} = P_{\mathrm{ijkl}} P_{\mathrm{im}}$$

$$P_5 = P_{\mathrm{ijklm}} + P_{\mathrm{ijklm}}^{-1} = \frac{1}{8}\left(1 + \sum_{\mu<\nu} \sigma_{\mu} \bullet \sigma_{\nu} + \sum_{\mu<\nu<\eta<\varepsilon} G_{\mu\nu\eta\varepsilon}\right)$$

↑ 4-spin terms

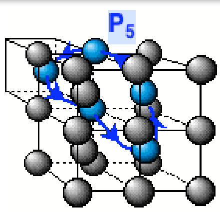
Six-spin exchange (AFM)

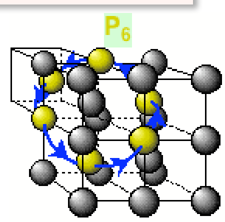
$$P_{\mathrm{ijklmn}} = P_{\mathrm{ijklm}} P_{\mathrm{in}}$$

$$P_6 = P_{\mathrm{ijklmn}} + P_{\mathrm{ijklmn}}^{-1} = \frac{1}{16}\left(1 + \sum_{\mu<\nu} \sigma_{\mu} \bullet \sigma_{\nu} + \sum_{\mu<\nu<\eta<\varepsilon} G_{\mu\nu\eta\varepsilon} + S_{\mathrm{ijklmn}}\right)$$

$$S_{\mathrm{ijklmn}} \equiv \left\{\left(\sigma_{\mathrm{i}} \bullet \sigma_{\mathrm{j}}\right)\left(\sigma_{\mathrm{k}} \bullet \sigma_{\mathrm{l}}\right)\left(\sigma_{\mathrm{m}} \bullet \sigma_{\mathrm{n}}\right) + \left(\sigma_{\mathrm{j}} \bullet \sigma_{\mathrm{k}}\right)\left(\sigma_{\mathrm{l}} \bullet \sigma_{\mathrm{ml}}\right)\left(\sigma_{\mathrm{n}} \bullet \sigma_{\mathrm{i}}\right) + \cdots\right\}$$

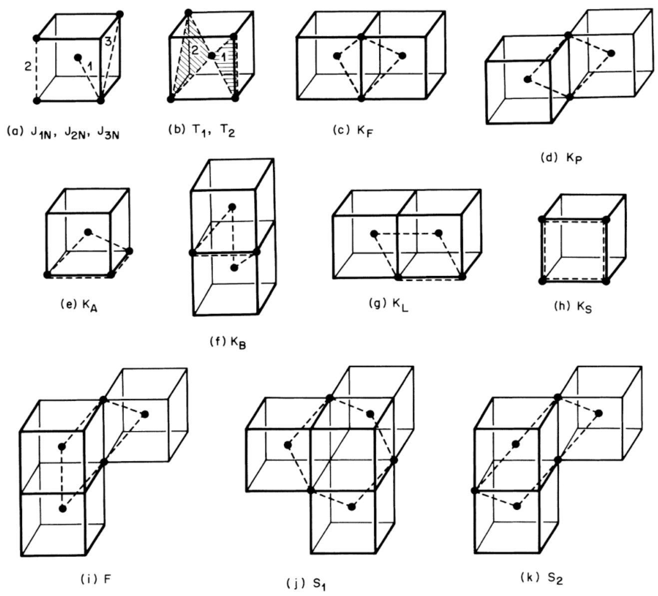
↑ new terms





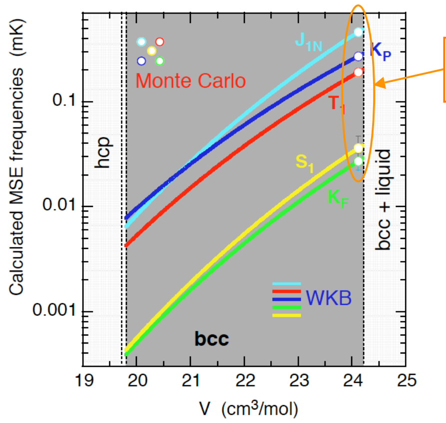
リング(原子)交換

H. Godfrin and D.D. Osheroff, PRB **38**, 4492 (1988)



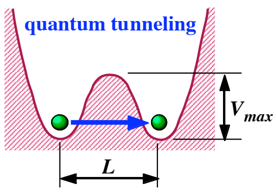
リング交換相互作用のWKB計算

M. Roger, PRB **30**, 6432 (1984)
M. Roger and J.H. Hetherington, PRB **41**, 200 (1990)



- Path-integral Monte Carlo (PIMC) calculations
  D.M. Ceperley and G. Jacucci, PRL **58**, 1648 (1987)

$$J_P = C_P s_P \exp\left(-A_P / g\right)$$



$A_P \approx L\sqrt{V_{\max}}$: action

$g = \hbar\left(8m\sigma^2\varepsilon\right)^{-1/2}\left(a/\sigma\right)^5$

C_P: prefactor

s_P: symmetry factor

ε, σ: parameters in L-J potential

a: lattice constant

$\Gamma(J_{1N}) \approx 18$: two-spin
$\Gamma(T_1) \approx 16$: three-spin
$\Gamma(K_P) \approx 15$: four-spin
$\Gamma(K_F) \approx 18$: four-spin
$\Gamma(S_1) \approx 19$: six-spin

Different MSEs should have different Grüneisen constants.

$$\Gamma\left(J_P\right) \equiv \frac{\partial \ln J_P}{\partial \ln V} \approx \frac{5A_P}{3g}$$

続き .... リング交換相互作用のWKB計算

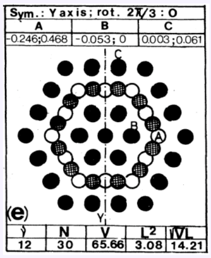
2D systems with r^{-12} potential

M. Roger, PRB **30**, 6432 (1984)

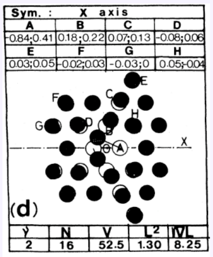
$$J_P = C_P s_P \exp(-A_P/g), \quad A_P \approx L\sqrt{V_{\max}}$$

$$\Gamma(J_P) \equiv \frac{\partial \ln J_P}{\partial \ln V} \approx \frac{5A_P}{3g}$$

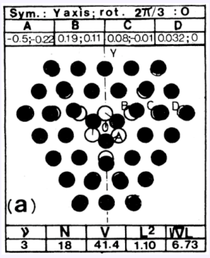
Twelve-spin
$V_{\max}$ = 65.7
L^2 = 3.08
$\boldsymbol{A_P}$ **= 14.21**
N = 30



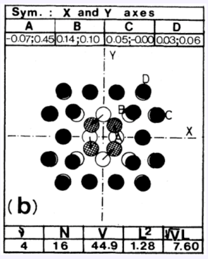
Two-spin
$V_{\max}$ = 52.5
L^2 = 1.30
$\boldsymbol{A_P}$ **= 8.25**
N = 16



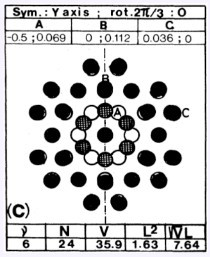
Three-spin
$V_{\max}$ = 41.4
L^2 = 1.10
$\boldsymbol{A_P}$ **= 6.73**
N = 18



Four-spin
$V_{\max}$ = 44.9
L^2 = 1.28
$\boldsymbol{A_P}$ **= 7.60**
N = 16



Six-spin
$V_{\max}$ = 35.9
L^2 = 1.63
$\boldsymbol{A_P}$ **= 7.64**
N = 24



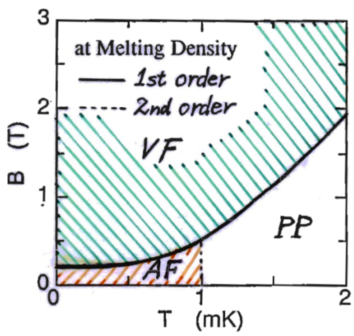
リング交換以外のモデル

空格子点モデル

A.F. Andreev et al., JETP Lett. **26**, 36 (1978)

FM polaron due to zero-point vacancy (ZPV)

- Nagaoka theorem for bipartite lattice (bcc)
- absence of upper critical field (B_{c2}) for HFP

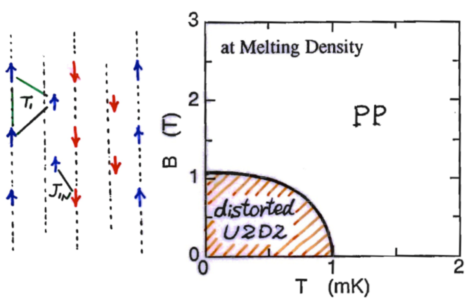


スピンPeierlsモデル

R.A. Guyer and P. kumar, JLTP **47**, 321 (1982)

Anisotropic lattice distortion $\approx 10^{-2}$

- no explanation for HFP
- $|J|$ ($\approx$ 1 mK) is too small compared to θ_D ($\approx$ 10 K).

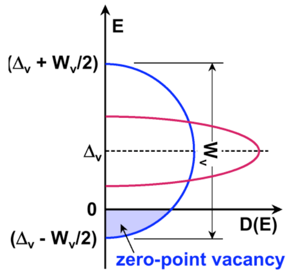


強磁性的スピンポーラロン

First of all, ZPV (or hole) should exist in solid ^{3}He.

A.F. Andreev and I.M. Lifshitz, Sov. Phys. JETP **29**, 1107 (1969)
H. Matsuda and T. Tsuneto, Suppl. Prog. Theor. Phys. **46**, 411 (1970)

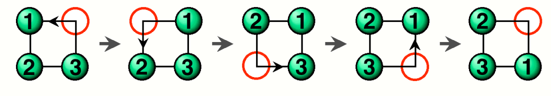
⇒ S. Miyashita (O-03)

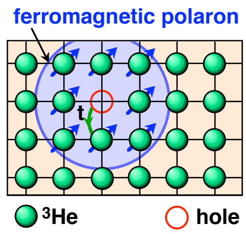


Nagaoka theorem requires FM ground state for bipartite lattices with single hole,

Y. Nagaoka, PR **147**, 392 (1966)

because hopping back of a hole to original position always results in odd particle permutation.





FM plolaron is created in AFM background at T = 0.